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Deformationally strong singularities in modified gravity

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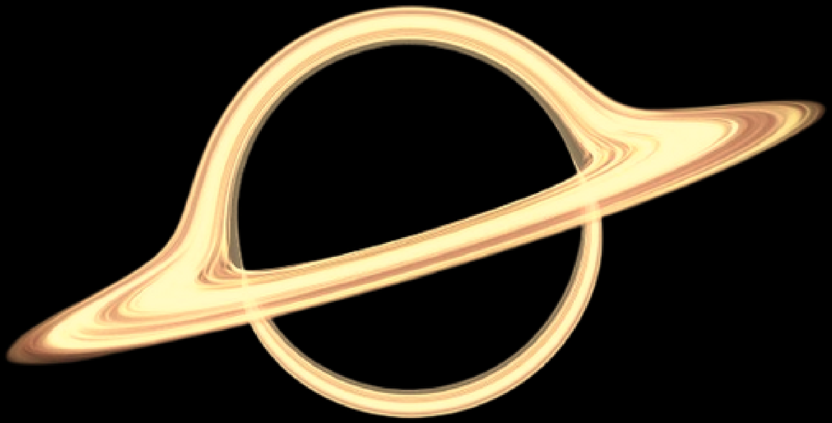
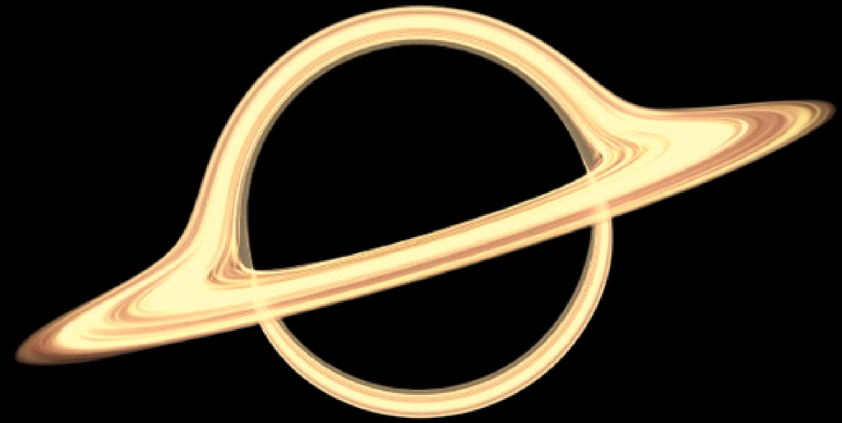
Summary

- A primer on singularities
- Modified gravity and Wormhole solution
- Singularity Strength
- Conclusions



$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$



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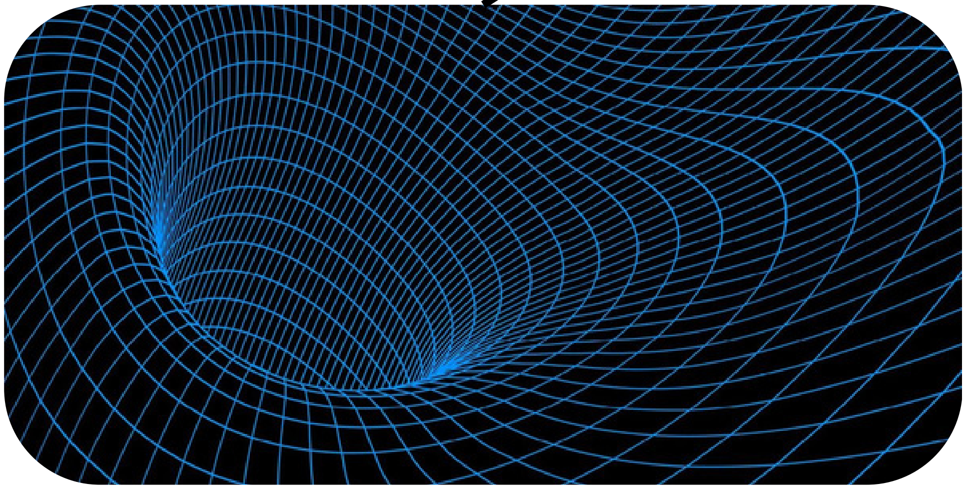
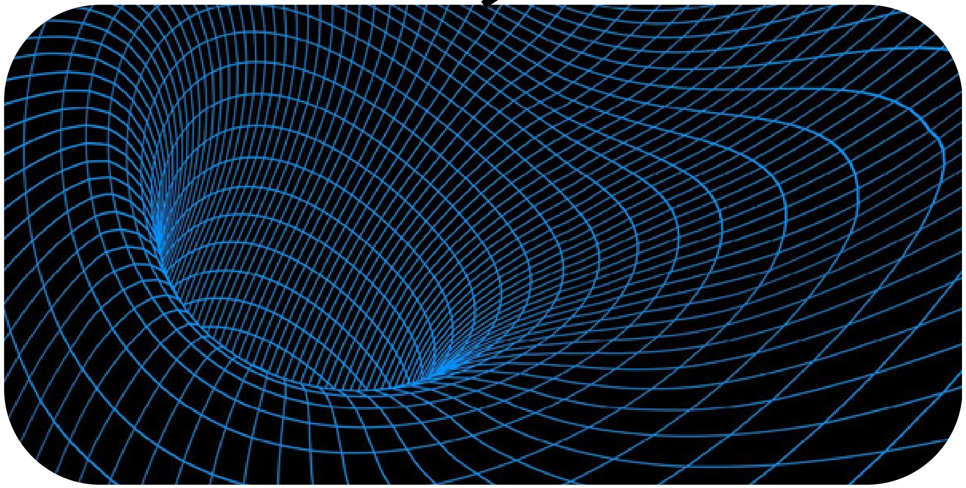
$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48M^2}{r^6}$$

It becomes unbounded as one approached the origin: Curvature singularity



Unbounded curvature scalars

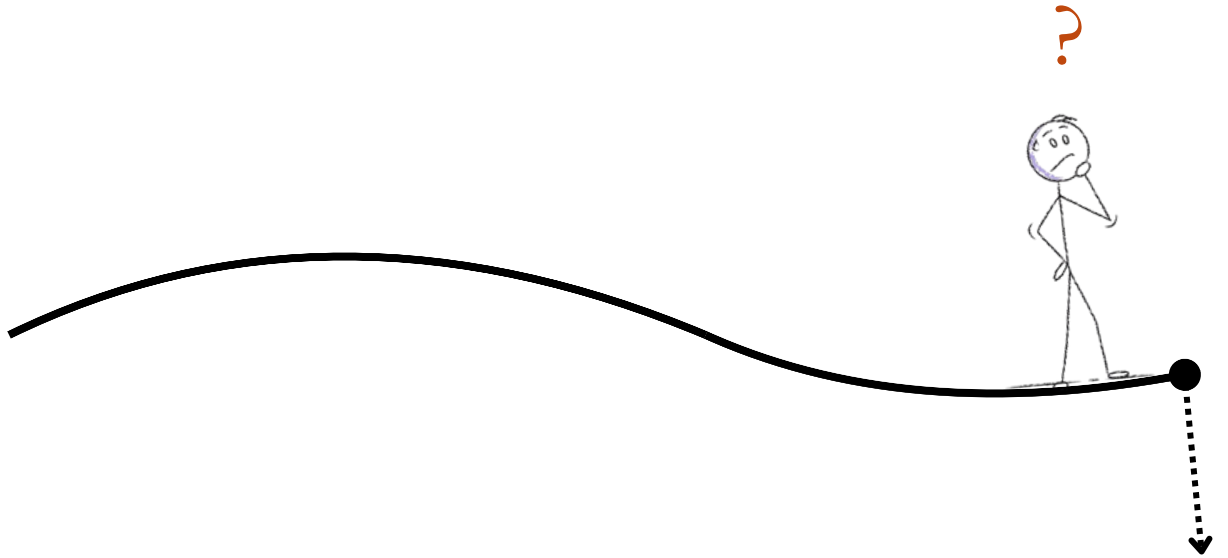
Singularity



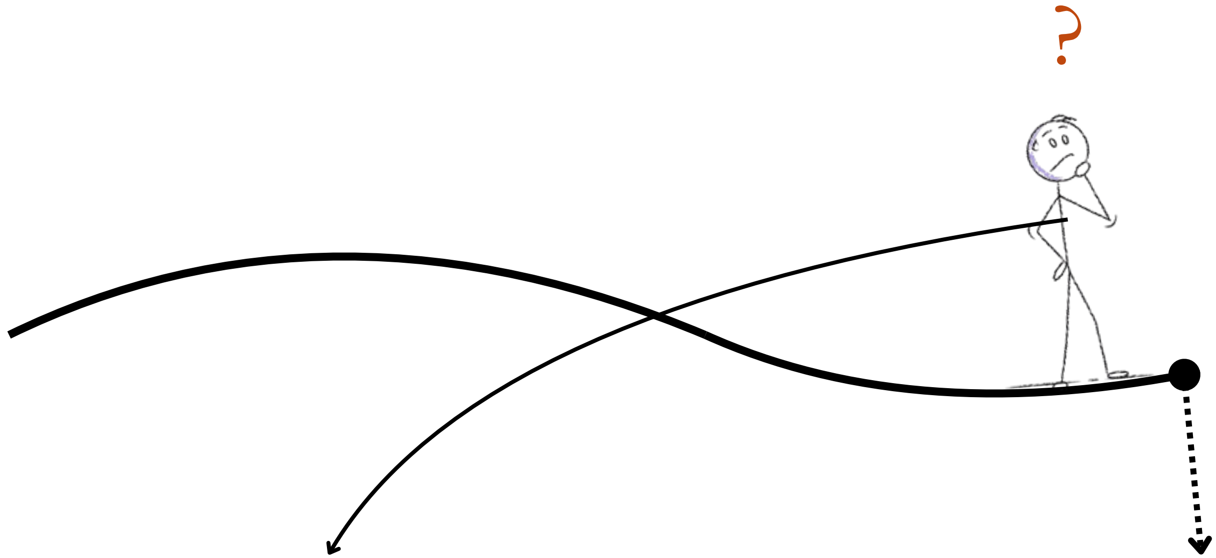
Unbounded
curvature scalars

Singularity

A spacetime is singular only if it has unbounded scalar curvature?

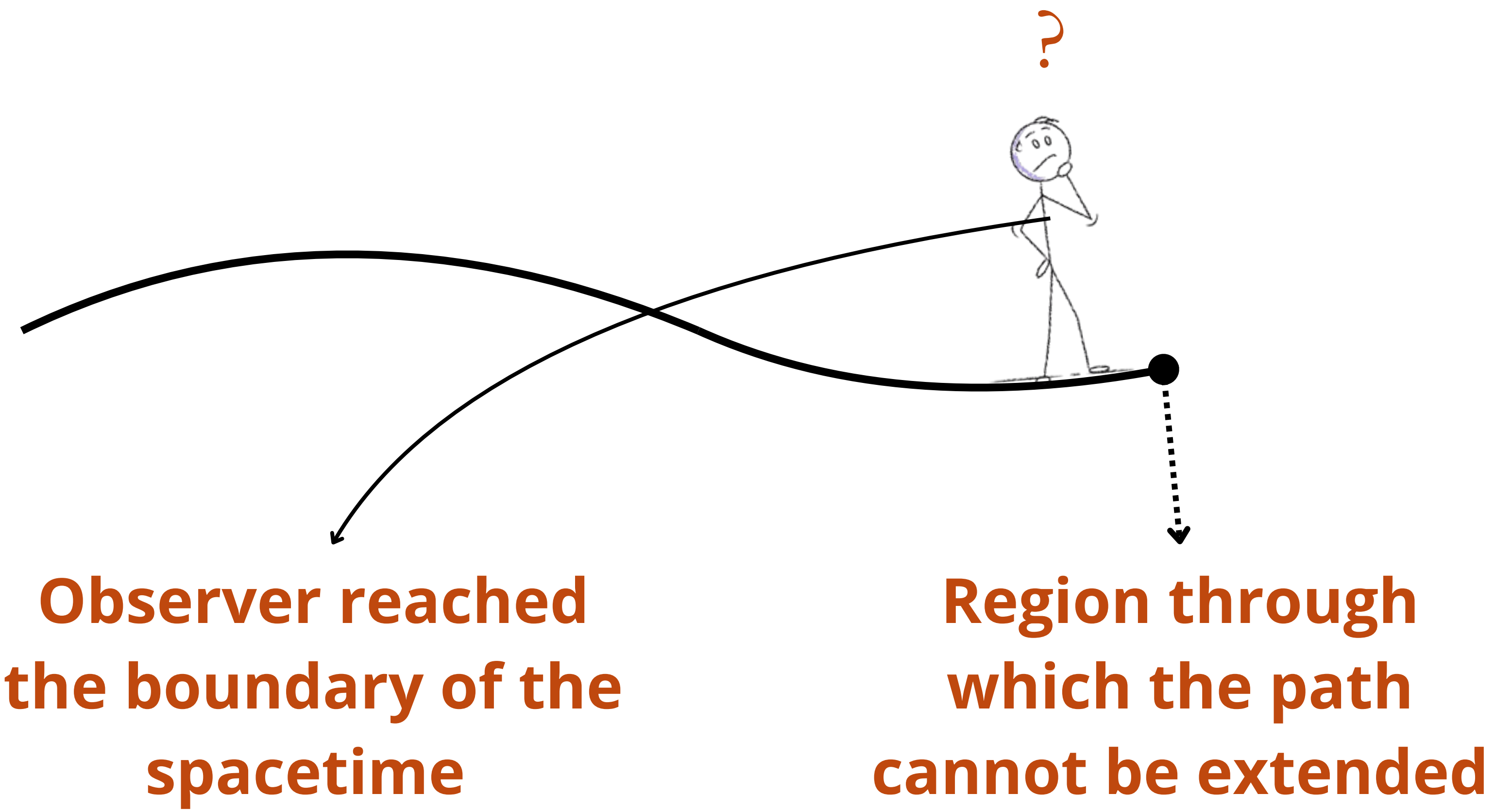


Region through which the path cannot be extended



Observer reached the boundary of the spacetime

Region through which the path cannot be extended



The presence of incomplete paths is a better definition of spacetime singularity

Let us consider

$$S_{\mathrm{EiBI}} = \frac{1}{\epsilon\kappa^2}\int d^4x \left(\sqrt{-|g_{\mu\nu} + \epsilon R_{(\mu\nu)}|} - \lambda\sqrt{-g}\right)$$

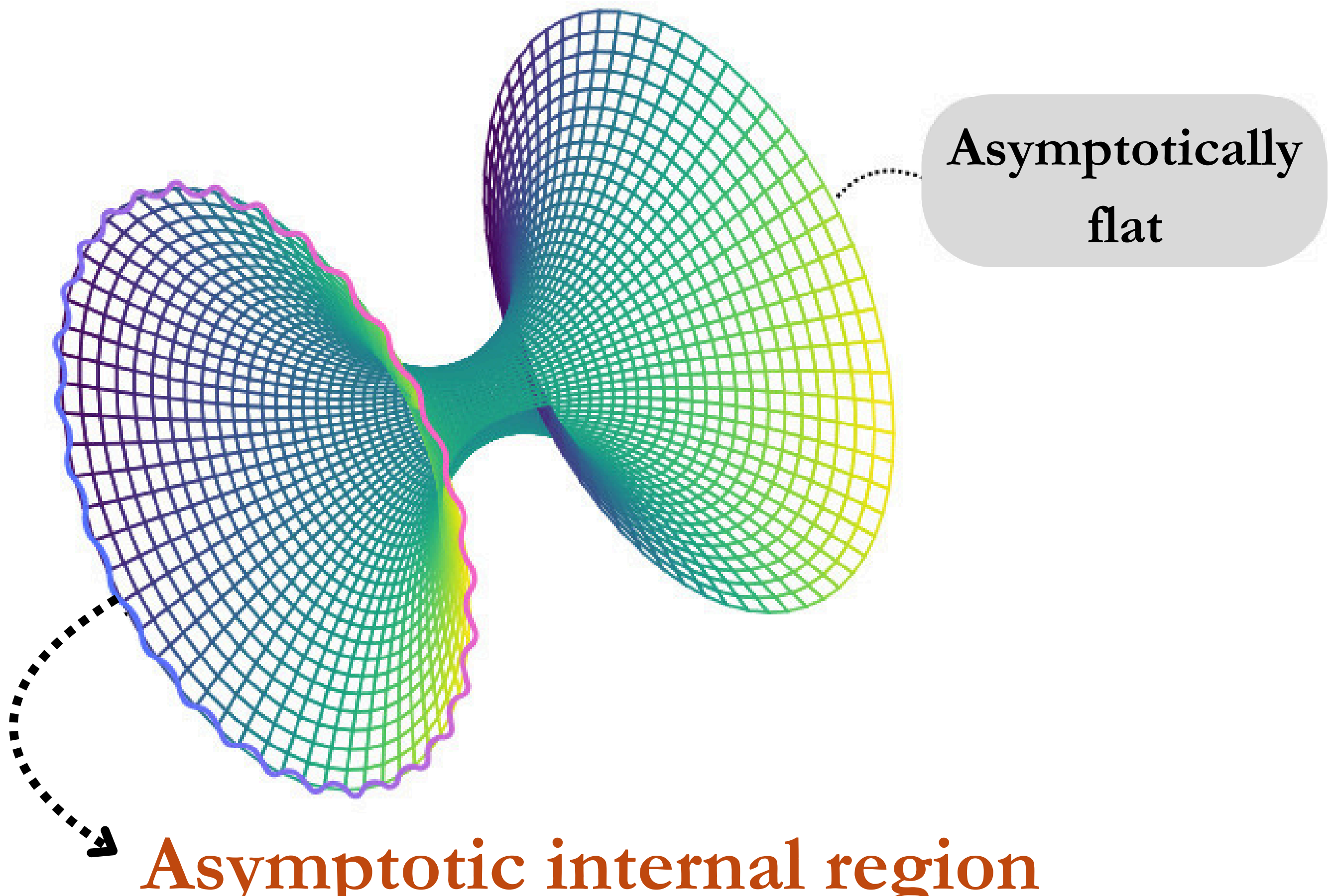
EiBI gravity

$$S_{\mathrm{QP}} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}(R + aR^2 + bQ)$$

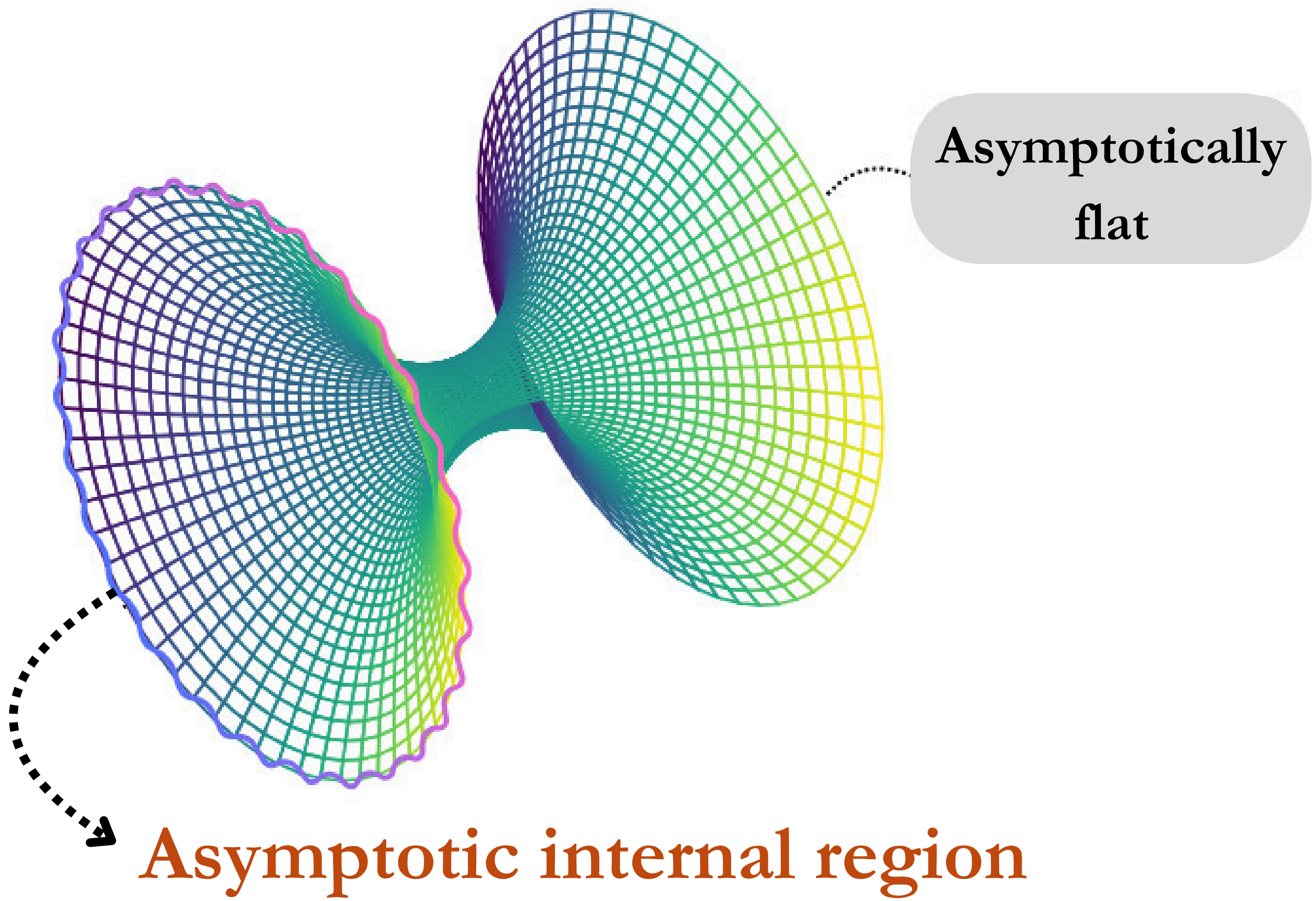
Quadratic Palatini gravity

+

Scalar fields



Asymptotically flat
Asymptotic internal region



$$ds^2 = -\frac{\alpha}{r^4}dt^2 + \frac{\beta}{r^2}dr^2 + r^2 d\Omega^2$$

Is it a regular spacetime?

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$$\lim_{r\to\infty} g^{\mu\nu}R_{\mu\nu} = -6/\beta$$
$$\lim_{r\to\infty} R_{\mu\nu}R_{\mu\nu} = 36/\beta^2$$
$$\lim_{r\to\infty} R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} = 108/\beta^2$$

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When to stop calculating?

ZM invariant	Degree	Asymptotic limit ($r \to \infty$)
I_1	2	$48/\beta^2$
I_2	2	0
I_3	3	$-96/\beta^3$
I_4	3	0
I_5	1	$-6/\beta$
I_6	2	$36/\beta^2$
I_7	3	$-216/\beta^3$
I_8	4	$1296/\beta^4$
I_9	3	0
I_{10}	3	0
I_{11}	4	$216/\beta^4$
I_{12}	4	0
I_{13}	5	0
I_{14}	5	0
I_{15}	4	$27/2\beta^4$
I_{16}	5	$-27/2\beta^5$
I_{17}	5	0

So... is it regular? We have to look for incomplete paths!

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For radial motion

$$\frac{\alpha\beta}{4}\dot{u}^2 = E^2$$ ➜ **Massless particles**

$$\frac{\alpha\beta}{4}\dot{u}^2 = E^2 - \alpha u^2$$ ➜ **Massive particles**

$$u(\lambda) \equiv \frac{1}{r(\lambda)^2}$$

For the massless case

$$u(\lambda) = \pm \frac{2E\lambda}{\sqrt{\alpha\beta}} + u_0$$

$$\lambda \to \frac{\sqrt{\alpha\beta}}{2E} u_0, \;\; u(\lambda) \to 0$$

The particle reaches the asymptotic region in finite affine time

For the massive case

$$u(\tau) = \frac{E}{\sqrt{\alpha}} \cos\left(\frac{2(\tau - \tau_0)}{\sqrt{\beta}}\right)$$

$$\tau \to \tau_0 + (\pi\sqrt{\beta}/4), \;\; u(\tau) \to 0$$

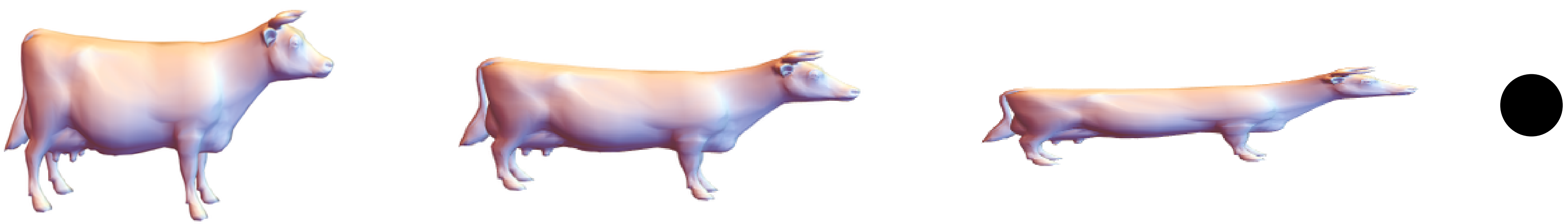
The particle reaches the asymptotic region in finite affine time

The spacetime has incomplete paths

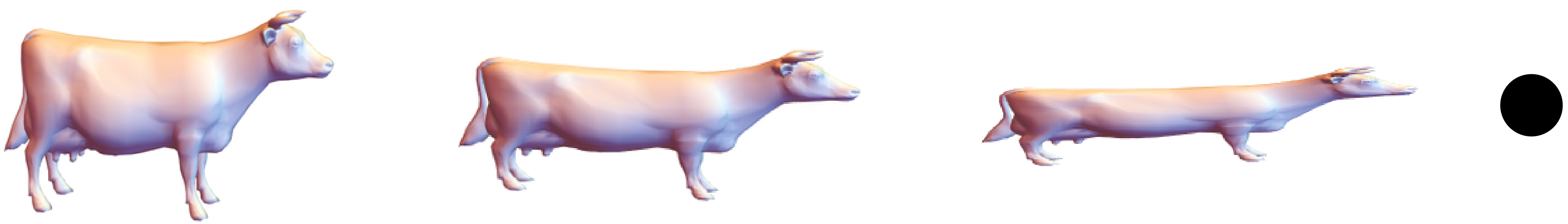
The spacetime has incomplete paths

Path incompleteness says nothing about the nature of the singularity

What happens when an object approaches the singularity?

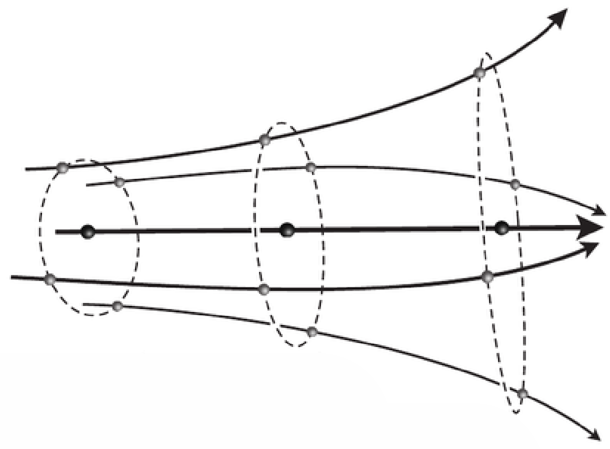


The singularity is strong if



$$V \to 0$$

The singularity is strong if



$$V \to 0$$

This is a statement about Jacobi fields!

Is this the case for our spacetime?

$$\int_{\tau_0}^{\tau} R_{\mu\nu} v^\mu v^\nu d\tau' \rightarrow \textbf{Converges}$$

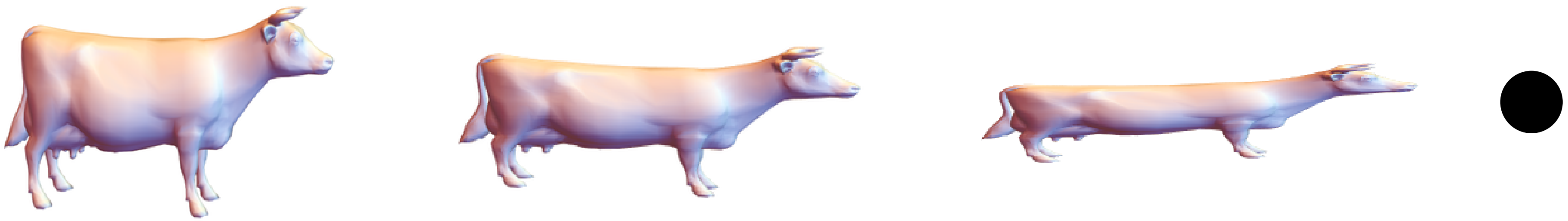
as one approaches the singularity

+

$$R_{\mu\nu} v^\mu v^\nu \geq 0$$

Not our case!

What if



$$V \to \text{not zero}$$

The singularity is not strong, even though objects may still be destroyed at the singularity

Deformationally strong singularity

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It concerns the individual Jacobi fields

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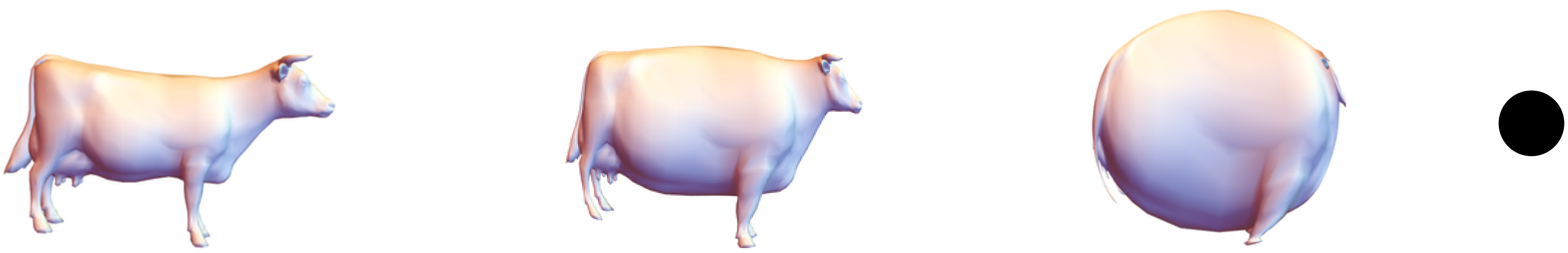
It concerns the individual Jacobi fields

Which is our case!

$$\vec{J}_{\rm ang} \sim r$$



$$V \to \infty$$

Scalar curvature invariants are insufficient to point out the pathological nature of a spacetime

Path completeness is better suited

Not being a strong singularity does not imply in the non-destruction of observers

The study of individual Jacobi fields is better suited

Thank you!