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Rotating black holes in Einstein-Maxwell-dilaton theory

Electric Charged

Etevaldo Costa, C. Herdeiro, E. Radu

The action of the EMd model is given by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left\{ R - 2\partial_\mu \Phi \partial^\mu \Phi - e^{2\gamma\Phi} F_{\mu\nu} F^{\mu\nu} \right\} ,$$

with γ an arbitrary parameter. The field equations are obtained by varying the action

$$\begin{aligned} R_{\mu\nu} - \frac{g_{\mu\nu}}{2} R &= 2T_{\mu\nu} , \\ \nabla_\nu \left(e^{2\gamma\Phi} F^{\mu\nu} \right) &= 0 , \\ \nabla_\nu \nabla^\nu \Phi &= \frac{\gamma e^{2\gamma\Phi}}{2} F_{\mu\nu} F^{\mu\nu} . \end{aligned}$$

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1. $\gamma = 0$: Einstein–Maxwell system with an uncoupled scalar.
2. $\gamma = 1$: emerges in a low energy limit of string theory (not a right truncation).
3. $\gamma = \sqrt{3}$: Kaluza-Klein compactification of 5-d theory to 4-d.

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- Solutions are invariant under the simultaneous sign change $(\gamma, \Phi) \to -(\gamma, \Phi)$.

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The system possesses several symmetries, for instance.

- Solutions are invariant under the simultaneous sign change $(\gamma, \Phi) \to -(\gamma, \Phi)$.
- Discrete duality “rotation” $(\mathcal{F}, \Phi) \to (e^{2\gamma\Phi} \star \mathcal{F}, -\Phi)$.

What already exists in the literature?

- **Analytical Solutions**:
  - Spherically symmetric electric for any γ.

    D. Garfinkle, G.T. Horowitz, A. Strominger (1990)
  - Rotating electric solutions for $\gamma = 0, \sqrt{3}$.

    J. H. Horne, G. T. Horowitz (1992)

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  - Weakly charged.

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- **Approximated Solutions**:
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    R. Casadio et.al (1997)
- **Numerical Solutions**:
  - Dyonic solutions, but not the full parameter space.

    B. Kleihaus, J. Kunz, F. Navarro-Lérida (2003)

Motivation

- Overview of the parameter space of the solutions for different values of γ.
- Devote some attention to extremal rotating black holes.

Preliminaries

We are interested in asymptotically flat solutions which are axisymmetric and stationary in four dimensions.

- Circularity is an imposition of the equations of motion.

$$\eta \wedge \xi \wedge R(\xi) = \xi \wedge \eta \wedge R(\eta) = 0\,, \qquad (R(\kappa)_\mu = R_{\mu\nu}\kappa^\nu)\,.$$

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{-2F_0} r^2 \sin^2\theta \left(d\varphi - \frac{W}{r^2}dt\right)^2\,, \quad N \equiv 1 - \frac{r_H}{r}\,.$$

$$\mathcal{A}_\mu dx^\mu = \left(\mathcal{A}_t - \mathcal{A}_\varphi \sin\theta \frac{W}{r^2}\right) dt + \mathcal{A}_\varphi \sin\theta d\varphi\,, \qquad \Phi = \Phi(r,\theta)\,.$$

Preliminaries

We are interested in asymptotically flat solutions which are axisymmetric and stationary in four dimensions.

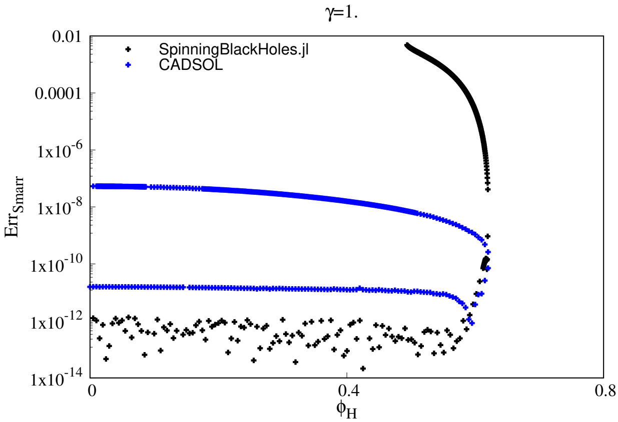
- Circularity is an imposition of the equations of motion.

- Solutions satisfy the mass formula.

$$M = 2\Omega_H J + \frac{\kappa}{4\pi G} A + \phi_{\mathcal{H}} Q_e \, .$$

Numerical Results

Solutions were constructed with the professional package CADSOL. We also performed calculations using the recent package SpinningBlackHoles.jl P. G. S. Fernandes, D. J. Mulryne (2023).



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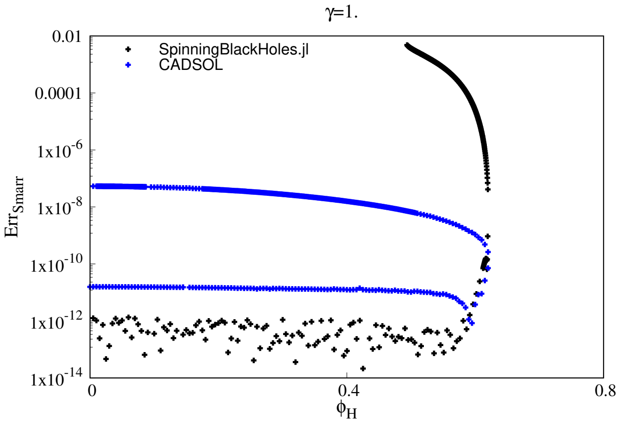
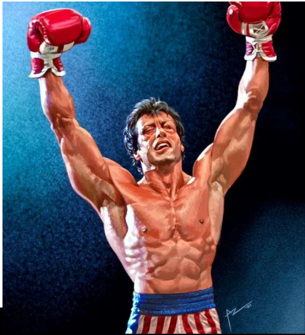
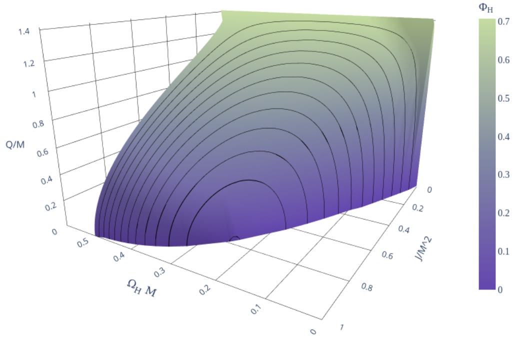


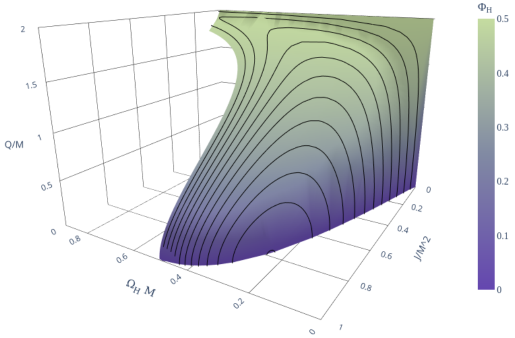


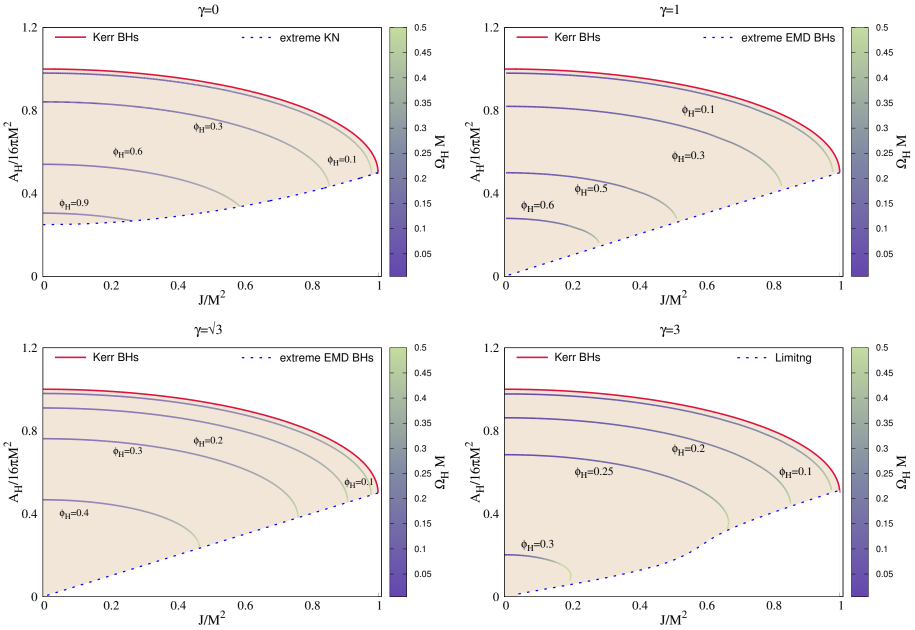
Numerical Results

$\gamma = 1$



$\gamma = \sqrt{3}$




γ=0
Kerr BHs
extreme KN
ϕ_H=0.1
ϕ_H=0.3
ϕ_H=0.6
ϕ_H=0.9
γ=1
Kerr BHs
extreme EMD BHs
ϕ_H=0.1
ϕ_H=0.3
ϕ_H=0.5
ϕ_H=0.6
γ=√3
Kerr BHs
extreme EMD BHs
ϕ_H=0.1
ϕ_H=0.2
ϕ_H=0.3
ϕ_H=0.4
γ=3
Kerr BHs
Limitng
ϕ_H=0.1
ϕ_H=0.2
ϕ_H=0.25
ϕ_H=0.3
$A_H/16\pi M^2$
J/M^2
$\Omega_H M$

Issue with extremality

- Some recent literature shows that extremal BHs with regular horizons are rare.

  G. T. Horowitz, M. Kolanowski, J. E. Santos (2023), G. T. Horowitz, J. E. Santos (2024)

- Pathologies in this model are not quite apparent from the scalars.

- We failed to construct the near-horizon geometry.

Conclusion

1. Not a mere copy of Einstein-Maxwell.

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1. Not a mere copy of Einstein-Maxwell.
2. We did not find any indication so far for the violation of the Kerr bound.
3. Our study of extremal solutions corroborates with recent literature.

Scenes from the next episodes

Dyonic Solutions.

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