Quasibound states of dirty wormholes

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Outline

Wormholes

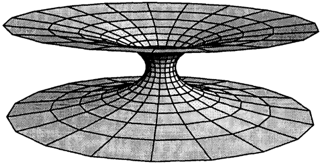


Figure 1: Wormhole connecting two different universes (M. Visser, 1995).

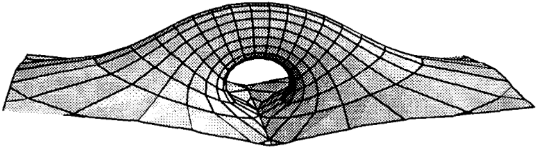


Figure 2: Wormhole connecting the same universe (M. Visser, 1995).

Astrophysical environments

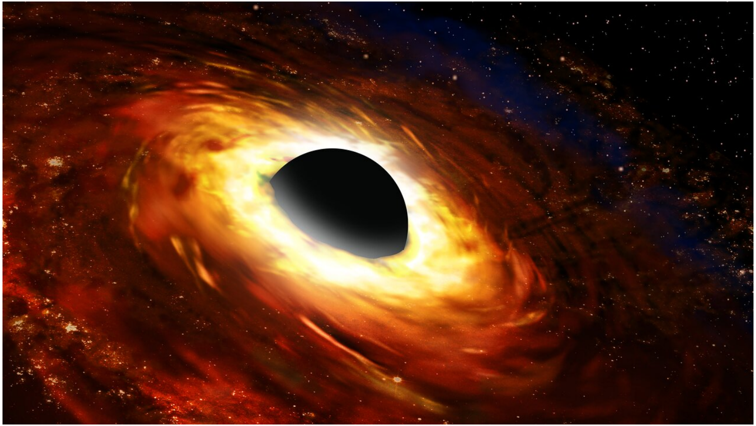


Figure 3: Black hole with accretion disk (NOIRLab).

Dirty wormhole description

To model a dirty wormhole, we use a thin-shell Schwarzschild wormhole

$$ds^2 = -f_\pm(r_\pm)dt^2 + f_\pm(r_\pm)^{-1}dr_\pm^2 + r_\pm^2 d\Omega^2, \tag{1}$$

where $\pm$ represents the side of the wormhole. We introduce a mass function $m(r_+)$, such that

$$f_+(r_+) \to 1 - \frac{2m(r_+)}{r_+}. \tag{2}$$

The form of the mass function $m(r_+)$ depends on the model of thick shell of matter we are considering.

We chose the mass function as (R. Konoplya, 2019)

$$m(r_+) = \begin{cases} M, & r_+ < r_s, \\ M + R(r_+, r_s, \Delta r_s)\Delta M, & r_s \leq r_+ \leq r_s + \Delta r_s, \\ M + \Delta M, & r_+ > r_s + \Delta r_s, \end{cases} \tag{3}$$

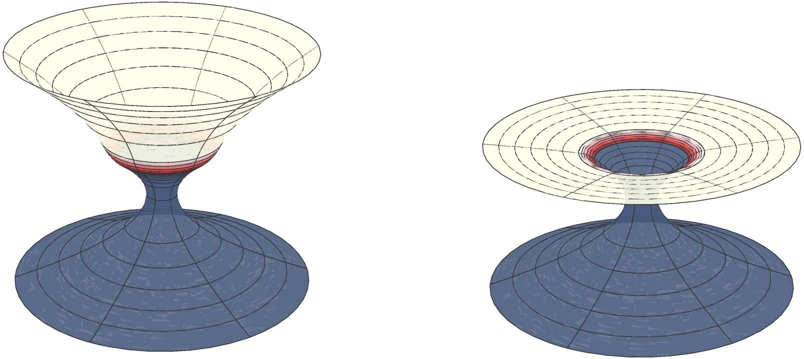
with

$$R(r_+, r_s, \Delta r_s) = \left(3 - 2\frac{r_+ - r_s}{\Delta r_s}\right)\left(\frac{r_+ - r_s}{\Delta r_s}\right)^2, \tag{4}$$

where r_s, Δr_s and ΔM are the radius, thickness and mass of the shell, respectively. It is useful to define a global coordinate $r_\star \in (-\infty, \infty)$, implicitly defined through

$$dr_\star = \pm\frac{dr_\pm}{f_\pm(r_\pm)}, \tag{5}$$

where the positive (negative) sign describes the upper (lower) side of the wormhole throat.



$r_0/M = 2.002, r_s/M = 6, \Delta r_s/M = 2, \Delta M/M = 2.2$

$r_0/M = 2.002, r_s/M = 6, \Delta r_s/M = 2, \Delta M/M = -0.9$

Figure 4: Embedding diagrams of dirty wormholes.

Quasibound states

To calculate the quasibound states of the dirty wormhole, we need to solve the Schrödinger-like equation

$$\frac{d^2\psi}{dr_\star^2} + (\omega^2 - V)\psi = 0, \tag{6}$$

where V is the effective potential of the scalar field. The quasibound modes are characterized by complex frequencies with small imaginary part, which are solutions of the eigenvalue problem (6) when the boundary conditions

$$\psi(r_\star) \approx \begin{cases} e^{i\omega r_\star}, & r_\star \to +\infty, \\ e^{-i\omega r_\star}, & r_\star \to -\infty, \end{cases} \tag{7}$$

are considered. Eq. (7) represents purely outgoing scalar waves at both sides of the dirty wormhole spacetime. In order to determine those complex frequencies, we employ the direct integration method (P. Pani, 2013).

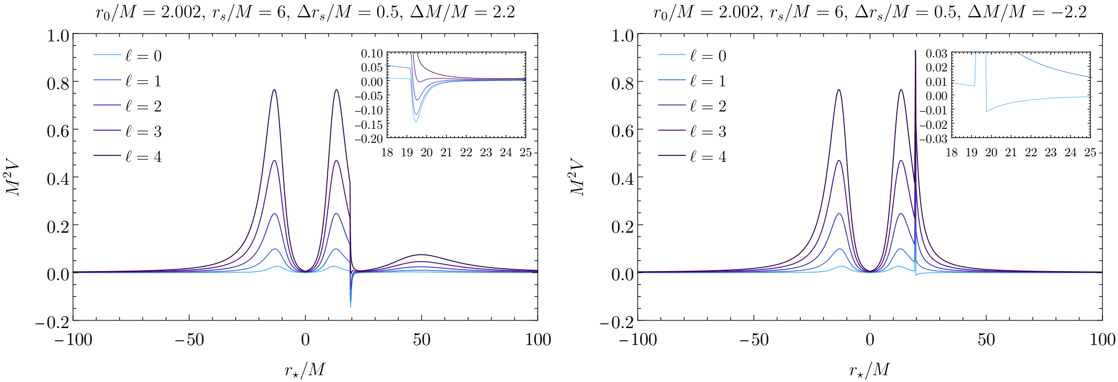


Figure 5: Effective potential for the scalar field of some dirty wormhole configurations.

Table 1: Quasibound frequencies of a thin-shell Schwarzschild wormhole with throat located at $r_0 = 2.002M$.

$r_0 = 2.002M$		
ℓ	$M\omega_r$	$-M\omega_i$
0	0.1055	7.25×10^{-3}
1	0.1481	9.71×10^{-5}
	0.2608	4.04×10^{-3}
2	0.1807	2.34×10^{-7}
	0.3205	6.28×10^{-5}
	0.4284	2.08×10^{-3}

Table 2: Quasibound frequencies of dirty wormholes with throat located at $r_0 = 2.002M$ and thick shells of matter located between $6M$ and $6.5M$.

$\Delta M/M = -3$			$\Delta M/M = 2.2$		
ℓ	$M\omega_r$	$-M\omega_i$	ℓ	$M\omega_r$	$-M\omega_i$
0	0.1071	5.21×10^{-3}	0	0.0407	4.52×10^{-3}
				0.1099	9.45×10^{-3}
1	0.1481	7.38×10^{-5}	1	0.0901	2.92×10^{-3}
	0.2609	2.81×10^{-3}		0.1480	3.27×10^{-4}
				0.2621	3.99×10^{-3}
2	0.1807	2.01×10^{-7}	2	0.1253	3.51×10^{-4}
	0.3205	4.12×10^{-5}		0.1807	3.76×10^{-6}
	0.4281	1.79×10^{-3}		0.3205	9.29×10^{-5}
				0.4288	1.80×10^{-3}

Conclusions

- We studied a thin-shell wormhole surrounded by a spherical thick shell of matter.
- For large values of the thick shell mass, new quasibound states arise due to the potential well of the thick shell.
- The others parameters of the thick shell do not change significantly the quasibound frequencies.

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