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OC12:

Gravitational bremsstrahlung and the Fulling-Davies-Unruh thermal bath

Belém -Pará - Brazil

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Belém-Pará
Thursday the 21st of November, 2024

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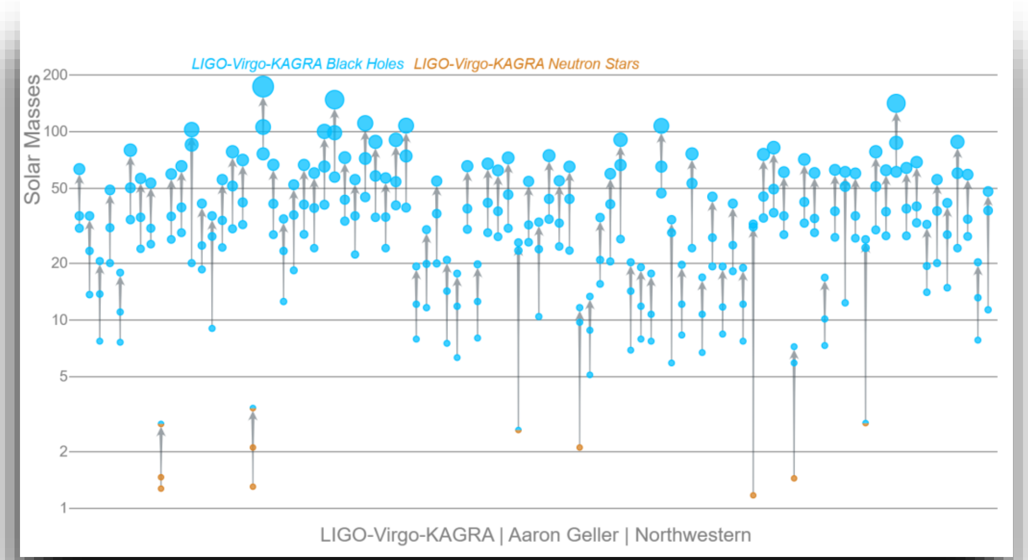
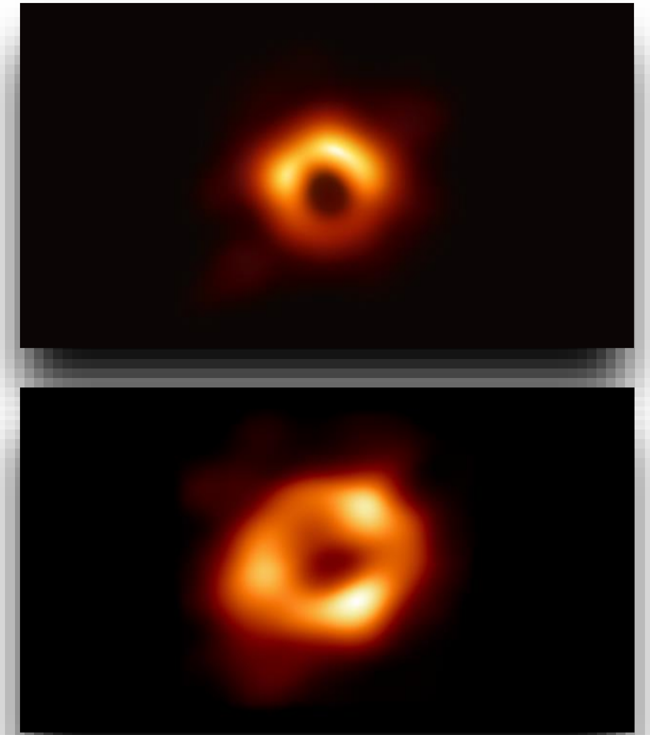
- LIGO-Virgo-KAGRA



- EHT



GWs catalogue and the first shadows of BHs. Top: M87* BH. Bottom: Sgr A* BH.

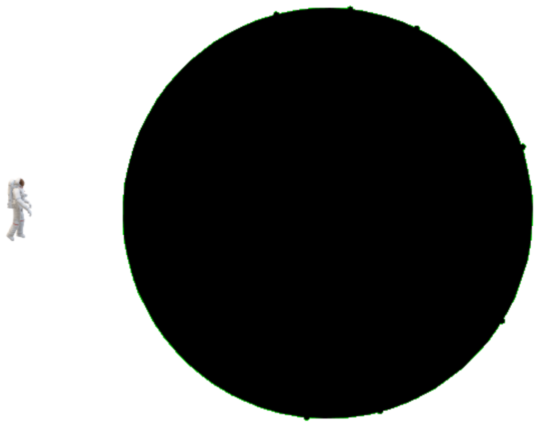
Strong evidence for the existence of BHs: LIGO-Virgo-KAGRA + EHT

Graphic representation of the masses of announced GW detections in a Stellar Graveyard

- Quantum gravity?

- Prime results: GR 🤝 QFT
  - Hawking effect

$$T_H = \frac{\hbar c^3}{8\pi k_B G M}$$



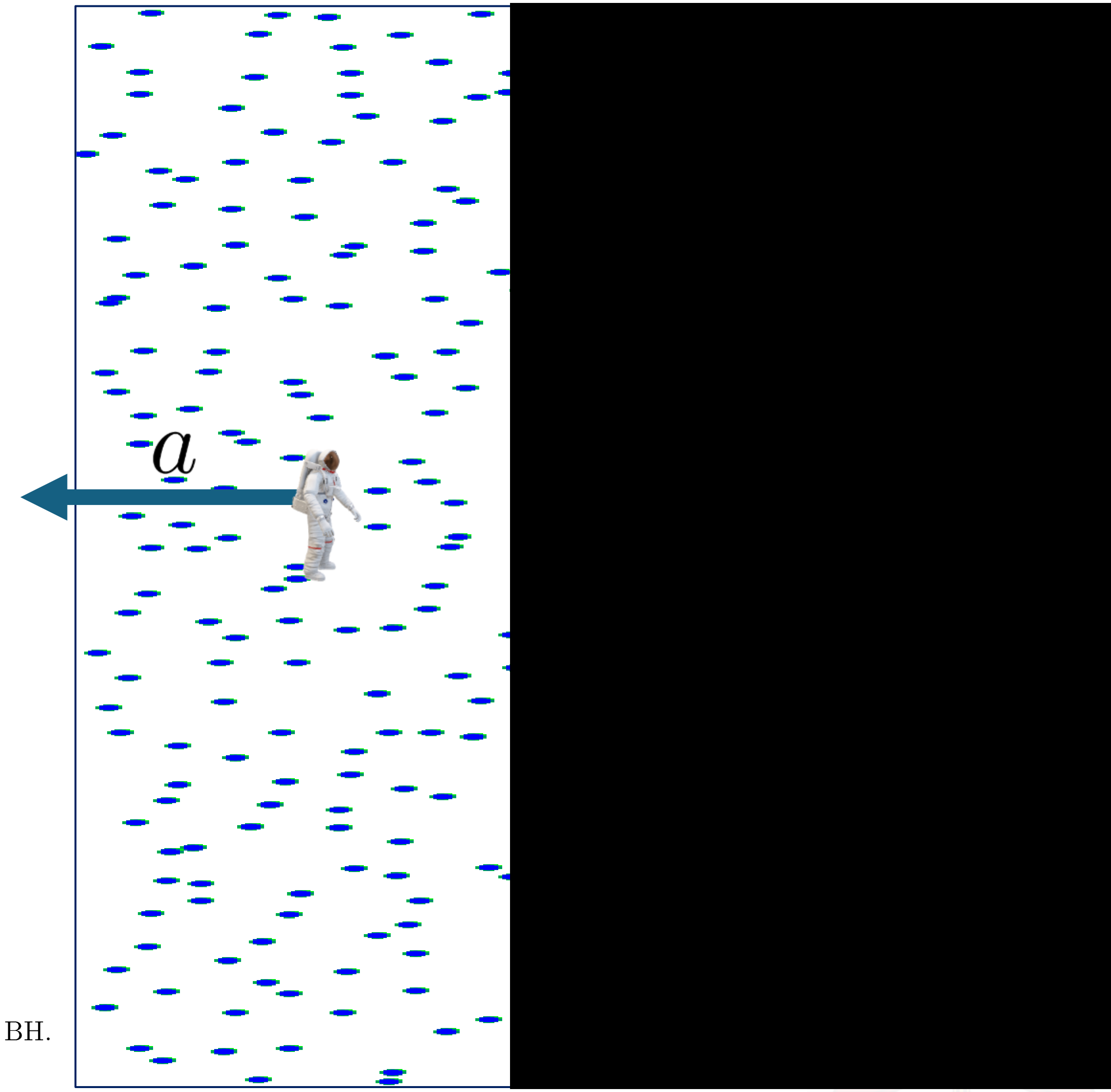
Pictorial representation of Hawking radiation.

- Prime results: GR 🤝 QFT
  - Hawking effect:

$$T_H = \frac{\hbar \kappa}{2\pi c k_B}$$

  - Unruh effect:

$$T_U = \frac{\hbar a}{2\pi c k_B}$$



Representation of Hawking radiation in the vicinity of the BH.

REVIEWS OF MODERN PHYSICS, VOLUME 80, JULY–SEPTEMBER 2008

The Unruh effect and its applications

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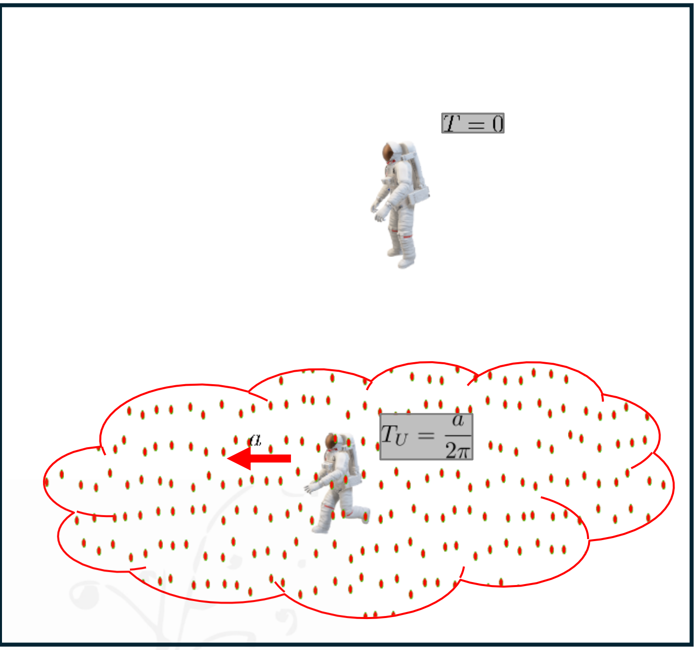
$$\langle 0_M | \hat{N}_M | 0_M \rangle = 0$$



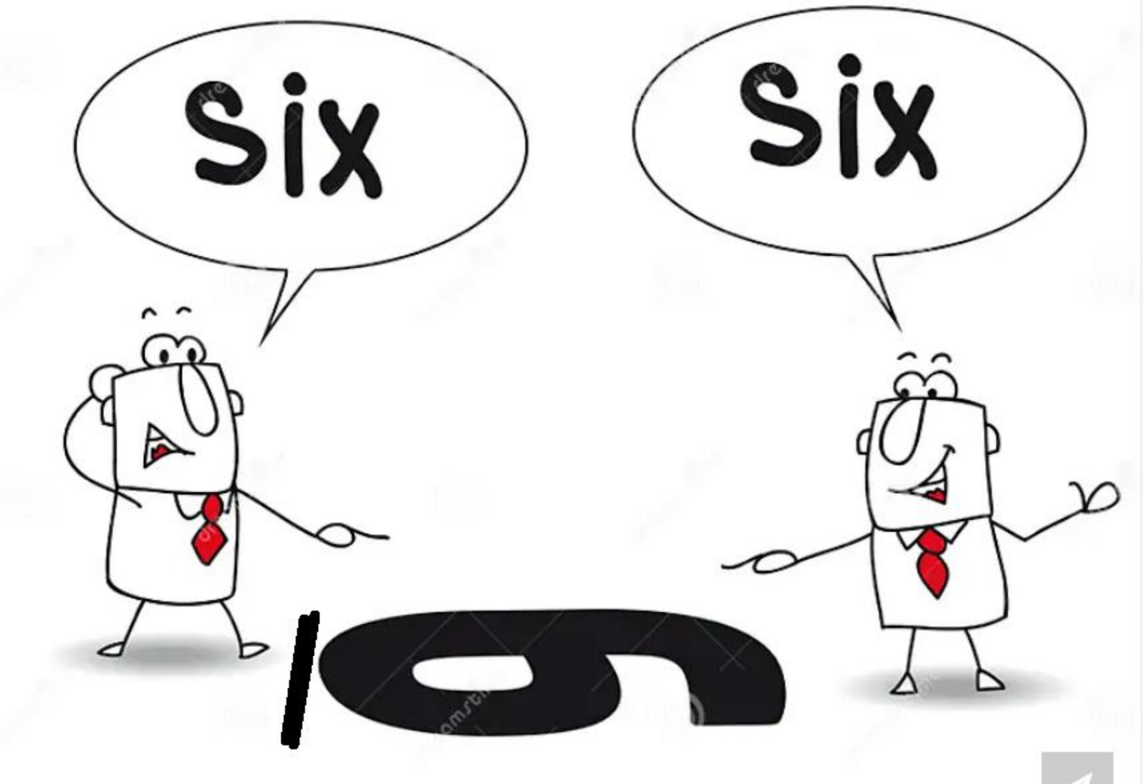
A pictorial representation of the Unruh effect.

$$\langle 0_M | \hat{N}_R | 0_M \rangle = n(\omega)$$

Nº of Rindler particles in $|0_M\rangle$:

$$n(\omega) = \frac{1}{e^{\frac{2\pi\omega}{a}} - 1}$$

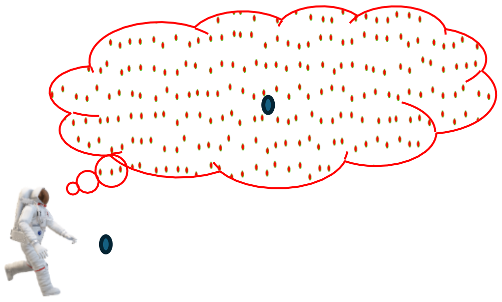
This term first appeared in Bose, S. N. Z. Physik 26, 178–181 (1924);



Adapted from web

❑ Radiation in flat spacetime

Accelerated observer:

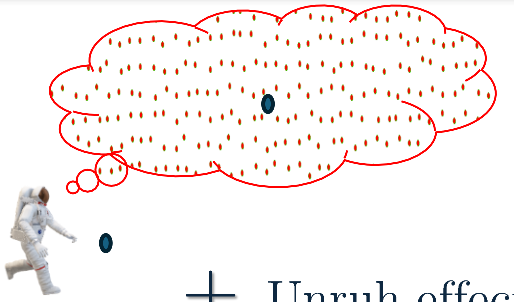


Inertial observer:

The response to the quantum field must be equal!

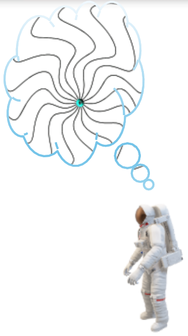
\+ Unruh effect:

$$T_U = \frac{\hbar a}{2\pi c k_B}$$



+ Unruh effect:

$$T_U = \frac{\hbar a}{2\pi c k_B}$$



- All observers must agree with the response rate:

$$R_{\text{em}}^{\lambda \mathbf{k}_\perp} = \int_0^\infty d\omega \frac{|A_{\text{em}}^{\lambda \mathbf{k}_\perp}|^2}{T} \qquad \mathcal{A}_{\text{em}}^{(\omega,\mathbf{k}_\perp)} = \langle 0_{\text{F}} | \, \hat{a}_{(\omega,\mathbf{k}_\perp)} \widehat{S}_{\text{int}} \, | 0_{\text{F}} \rangle$$

- Interaction term describing the one-graviton processes:

$$\widehat{S}_{\text{int}} = \sqrt{8\pi G} \int T^{\mu\nu} \widehat{h}_{\mu\nu} \sqrt{-g} \, d^4x$$

We are analyzing gravitational radiation OR how the source interacts with gravitons from the thermal bath.

Theoretical description of bremsstrahlung:

Spontaneous emission

$$R^{\lambda \mathbf{k}_\perp}_{em} = \int_0^\infty d\omega \frac{|A^{\lambda \mathbf{k}_\perp}_{em}|^2}{T} = 0$$

Absorption/induced emission

$$R^{\lambda \mathbf{k}} = \int_0^\infty d\omega \frac{|A^{\lambda \mathbf{k}_\perp}|^2}{T} n(\omega) \neq 0$$

Nº of Rindler photons with zero energy

$$n(\omega) = \frac{1}{e^{\frac{2\pi\omega}{a}} - 1}$$

RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 45, NUMBER 10 15 MAY 1992

Bremsstrahlung and zero-energy Rindler photons

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(Received 29 January 1992)

The bremsstrahlung effect is analyzed in the frame coaccelerating with the charge. In particular, it

PHYSICAL REVIEW D VOLUME 46, NUMBER 8 15 OCTOBER 1992

Bremsstrahlung and Fulling-Davies-Unruh thermal bath

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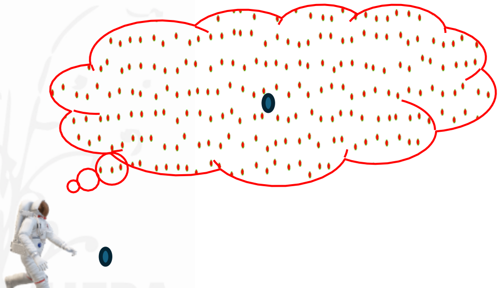
The Enrico Fermi Institute, The University of Chicago, 5640 Ellis Avenue, Chicago, Illinois 60637-1433

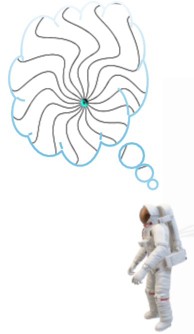
(Received 26 May 1992)

Polarization — Transverse momentum

$$R^{\lambda \mathbf{k}_\perp}_{\text{em}} = \int_0^\infty d\omega \frac{|A^{\lambda \mathbf{k}_\perp}_{\text{em}}|^2}{T}$$

$$R^{\text{tot}}_{k_\perp} = \frac{q^2}{4\pi^3 a} \left| K_1\left(\frac{k_\perp}{a}\right)\right|^2$$





A pictorial representation of the bremsstrahlung.

Gravitational bremsstrahlung in flat spacetime

Rindler spacetime

- Minkowski coordinates:

$$(t, z, x, y)$$

- *Right* Rindler wedge:

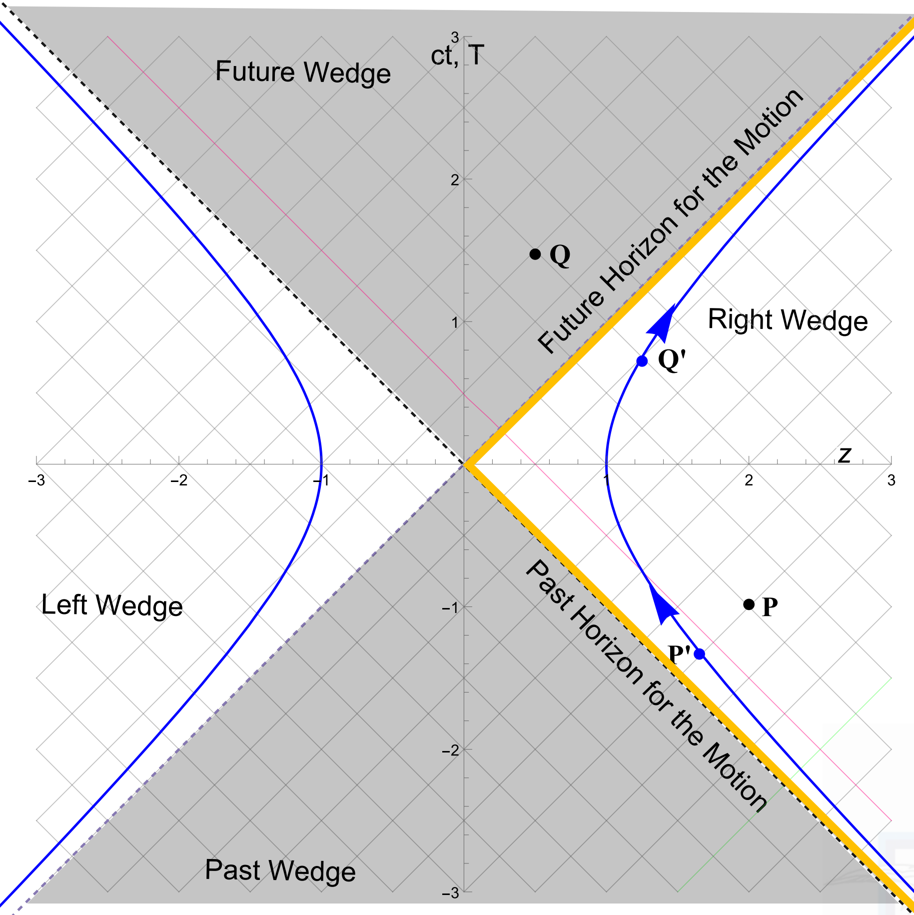
$$\begin{aligned} t &= a^{-1}e^{a\xi}\sinh a\eta, \\ z &= a^{-1}e^{a\xi}\cosh a\eta, \end{aligned}$$

where $z > |t|$ and $-\infty < \eta, \xi < \infty$.

- Accelerated-observer spacetime:

$$d\tau^2 = e^{2a\xi}\left(d\eta^2 - d\xi^2\right) - dx^2 - dy^2$$

Proper acceleration $\alpha := ae^{-a\xi}$.
Proper time $\tau = e^{a\xi}\eta$.



Minkowski spacetime diagram depicting hyperbolic trajectories and the four Rindler wedges.

- Inertial observer (Minkowski coordinates):

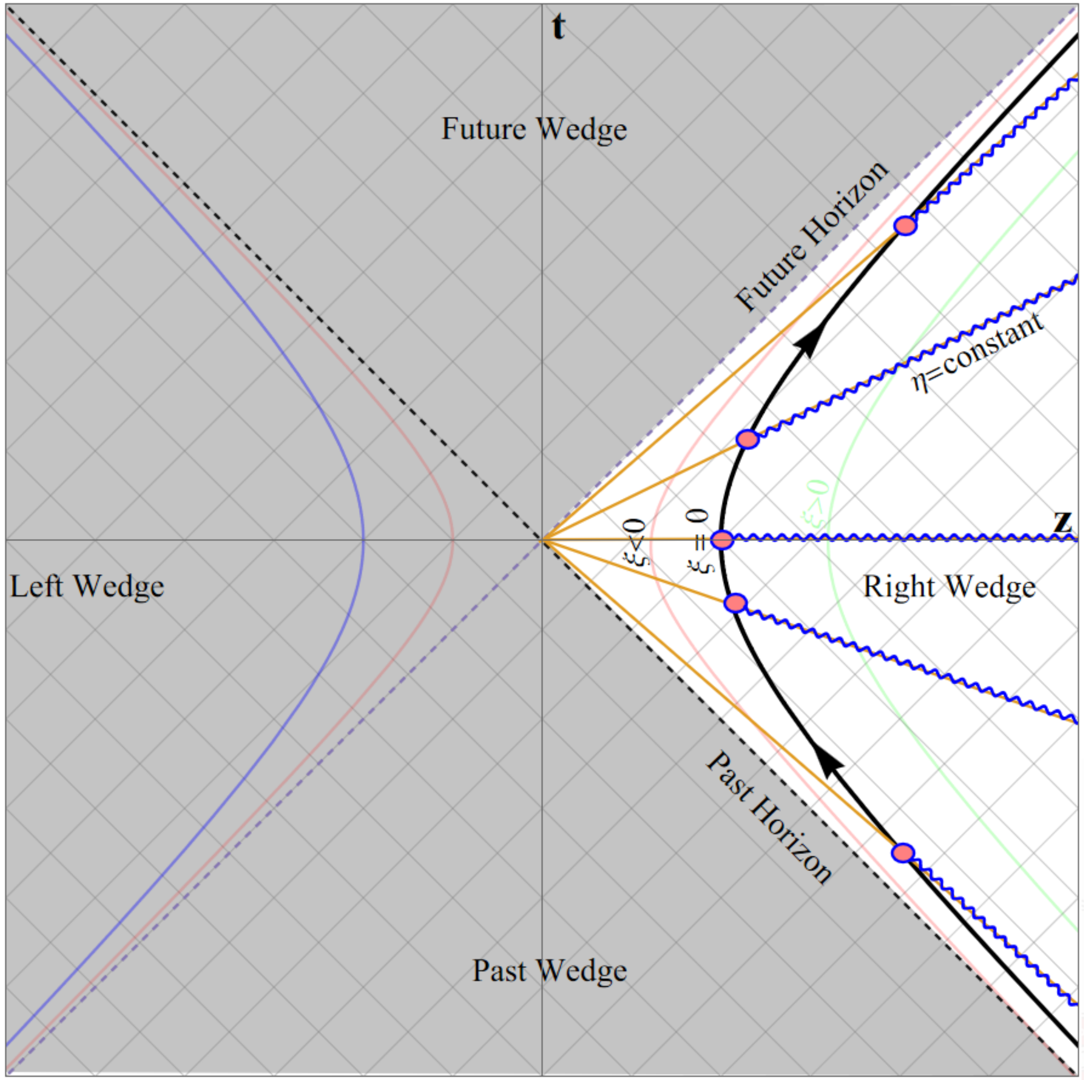
$$T^{tt} = \mu\left[\cosh^2 a\eta\,\delta(\xi) + a\theta(\xi)\right]\delta^{(2)}(\mathbf{x}_\perp),$$
$$T^{tz} = \mu\sinh a\eta\cosh a\eta\,\delta(\xi)\delta^{(2)}(\mathbf{x}_\perp),$$
$$T^{zz} = \mu\left[\sinh^2 a\eta\,\delta(\xi) - a\theta(\xi)\right]\delta^{(2)}(\mathbf{x}_\perp).$$

- Conservation equation:

$$\nabla_\mu T^{\mu\nu} = 0$$

- Accelerated observer (Rindler coordinates):

$$T^{\eta\eta} = \mu\left[\delta(\xi) + (1-\beta)ae^{-(2+\beta)a\xi}\theta(\xi)\right]\delta^{(2)}(\mathbf{x}_\perp),$$
$$T^{\xi\xi} = -\mu a e^{-(2+\beta)a\xi}\theta(\xi)\delta^{(2)}(\mathbf{x}_\perp).$$



Minkowski spacetime with the stress-energy depicted in the right Rindler wedge.

- The total emission probability:

$$\mathcal{P}^M = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k} \overline{\mathcal{T}^{\mu\nu}(k)} \sum_\lambda \epsilon^\lambda_{\mu\nu} \epsilon^\lambda_{\alpha\beta} \mathcal{T}^{\alpha\beta}(k)$$

Where $\epsilon^\lambda_{\mu\nu}$ is the Minkowski polarization matrix:

Considering $k^\mu = (k, 0, 0, k)$

$$\epsilon^{\lambda_1}_{11} = -\epsilon^{\lambda_1}_{22} = \epsilon^{\lambda_2}_{12} = \epsilon^{\lambda_2}_{21} = \frac{1}{\sqrt{2}}$$

- With fixed transverse momentum:

$$\begin{aligned} \mathcal{P}^{\mathrm{M}}_{\mathbf{k}_\perp} &= \int \frac{dk_z}{(2\pi)^3 2k} \left(\overline{\mathcal{T}^{\mu\nu}} \mathcal{T}_{\mu\nu} - \frac{1}{2} \overline{\mathcal{T}} \mathcal{T} \right) \\ &= \int \frac{dk_z}{(2\pi)^3 2k} \left(\frac{1}{2} \left| \mathcal{T}^{tt} + \mathcal{T}^{zz} \right|^2 - 2 \left| \mathcal{T}^{tz} \right|^2 \right), \end{aligned}$$

$$\mathcal{T}^{\mu\nu}(\mathbf{k}) = \int e^{i(kt - \mathbf{k}\cdot\mathbf{x})} T^{\mu\nu}(x)\, d^4x$$

- The differential response rate:

$$\mathcal{R}^{\mathrm{M}}_{\mathbf{k}_\perp} = \frac{\mathcal{P}^{M}_{\mathbf{k}_\perp}}{T_0} = \mu^2 \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \cosh 2a\sigma \exp\left\{ \frac{2i}{a} \left[k' \sinh\left(\frac{a}{2}\sigma \right) \right] \right\} d\sigma \right] \frac{dk'_z}{4k'(2\pi)^3}$$

Convergence factor by letting $\sigma \mapsto \sigma + 2i\epsilon$

$$\mathcal{R}^{\mathrm{M}}_{\mathbf{k}_\perp} = \mu^2 \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} \cosh 2a\sigma \exp\left[\frac{ik'}{a} \left(e^{ia\epsilon} e^{a\sigma/2} - e^{-ia\epsilon} e^{-a\sigma/2} \right) \right] d\sigma \right\} \frac{dk'_z}{4k'(2\pi)^3}$$

$$s_\pm = \frac{k' + k'_z}{k_\perp} e^{\pm a\sigma/2}$$

$$\int_0^\infty ds s^{\nu-1} \exp\left[\frac{i\mu}{2} \left(s - \frac{\beta^2}{s} \right) \right] = 2\beta^\nu e^{i\nu\pi/2} K_\nu(\beta\mu)$$

- The differential response rate:

$$\mathcal{R}^{\mathrm{M}}_{\mathbf{k}_\perp} = \frac{\mu^2}{8\pi^3 a} \left| K_2\left(\frac{k_\perp}{a} \right) \right|^2$$

- GWs in Rindler spacetime:

PHYSICAL REVIEW D **103,** 083503 (2021)

Gravitational waves in Kasner spacetimes and Rindler wedges in Regge-Wheeler gauge: Formulation of Unruh effect

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$$h_{\mu\nu}^{(\omega,\mathbf{k}_\perp)}(\eta,\xi,\mathbf{x}_\perp) = H_{\mu\nu}^{(\omega,k_\perp)}(\xi)e^{-i\omega\eta+i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}$$

Positive frequency (∂_η)

Parity: *Even* | *Odd.*

Even-parity modes:

$$\tilde{h}_{\mu\nu}^{R,\mathrm{e}} = \begin{pmatrix} -\frac{\mathcal{K}^2(\xi)}{\kappa^2}\varphi^{R,\mathrm{e}} - \frac{1}{\kappa^2}(\omega^2\varphi^{R,\mathrm{e}} + 2a\partial_\xi\varphi^{R,\mathrm{e}}) & -\frac{2i\omega}{\kappa^2}(\partial_\xi\varphi^{R,\mathrm{e}} - a\varphi^{R,\mathrm{e}}) & 0 & 0 \\ * & -e^{2a\xi}\varphi^{R,\mathrm{e}} - \frac{2}{\kappa^2}(a\partial_\xi\varphi^{R,\mathrm{e}} - \partial_\xi^2\varphi^{R,\mathrm{e}}) & 0 & 0 \\ * & * & \varphi^{R,\mathrm{e}} & 0 \\ * & * & * & \varphi^{R,\mathrm{e}} \end{pmatrix}$$

$$\tilde{h}_{\mu\nu}^{R,\mathrm{o}} = \begin{pmatrix} 0 & 0 & -\frac{ik_z}{\kappa^2}\partial_\xi\varphi^{R,\mathrm{o}} & \frac{ik_y}{\kappa^2}\partial_\xi\varphi^{R,\mathrm{o}} \\ * & 0 & -\frac{\omega k_z}{\kappa^2}\varphi^{R,\mathrm{o}} & \frac{\omega k_y}{\kappa^2}\varphi^{R,\mathrm{o}} \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix}$$

$$\nabla_\alpha \nabla^\alpha \varphi^{(\omega,\mathbf{k}_\perp)} = 0$$

$$\varphi^{(\omega,\mathbf{k}_\perp)}(\eta,\xi,\mathbf{x}_\perp) = \phi^{(\omega,k_\perp)}(\xi) e^{-i\omega\eta + i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}$$

Normalized scalar modes:

$$\phi^{(\omega,k_\perp)}(\xi) = \sqrt{\frac{\sinh(\pi\omega/a)}{4\pi^4 a}} K_{i\omega/a}\left(\frac{k_\perp e^{a\xi}}{a}\right)$$

- Explicitly, we have:

$$\begin{aligned} H^{(\omega,k_\perp)}_{\eta\eta}(\xi) &= H^{(\omega,k_\perp)}_{\xi\xi}(\xi) \\ &= \frac{1}{\sqrt{2}}\left[-e^{2a\xi} + \frac{2}{k_\perp^2}\left(\omega^2 + a\partial_\xi\right)\right]\phi^{(\omega,k_\perp)}(\xi), \end{aligned}$$

$$H^{(\omega,k_\perp)}_{\eta\xi}(\xi) = \frac{\sqrt{2}i\omega}{k_\perp^2}\left(\partial_\xi - a\right)\phi^{(\omega,k_\perp)}(\xi),$$

$$H^{(\omega,k_\perp)}_{ij}(\xi) = -\frac{\delta_{ij}}{\sqrt{2}}\phi^{(\omega,k_\perp)}(\xi),$$

- The differential response rate:

$$\mathcal{R}^{\mathrm{R}}_{\mathbf{k}_\perp} = \frac{\mu^2}{8\pi^3 a}\left|K_2\left(\frac{k_\perp}{a}\right)\right|^2$$

- The consistent description of physiscs (in $|0_M\rangle$) in the accelerated frame requires the Unruh effect.

- In particular, we have fundamentally described the bremsstrahlung in relation to the Unruh effect.

❑ Acknowledgments

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CAPES

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Quantum Fields in Curved Spacetimes

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❑ Bibliography:

- L. Parker, Particle Creation in Expanding Universes, Phys. Rev. Lett. **21**, 562 (1968).
- L. Parker, Quantized fields and particle creation in expanding universes. I, Phys. Rev. **183**, 1057 (1969).
- L. Parker, Quantized fields and particle creation in expanding universes. II, Phys. Rev. D **3**, 346 (1971); **3**, E2546 (1971).
- S.W. Hawking, Particle creation by black holes, Commun. Math. Phys. 43, 199 (1975).
- W. Unruh, Notes on black-hole evaporation, Phys. Rev. D **14**, 870 (1976).
- J. Weber, Evidence for Discovery of Gravitational Radiation, Phys. Rev. Lett. **22**, 1320 (1969).
- C.W. Misner, Interpretation of Gravitational-Wave Observations, Phys. Rev. Lett. **28**, 994 (1972).
- C.W. Misner, R. A. Breuer, D. R. Brill, P. L. Chrzanowski, H. G. Hughes, and C. M. Pereira, Gravitational Synchrotron Radiation in the Schwarzschild Geometry, Phys. Rev. Lett. **28**, 998 (1972).
- M. Davis, R. Ruffini, J. Tiomno, and F. Zerilli, Can Synchrotron Gravitational Radiation Exist?, Phys. Rev. Lett. **28**, 1352 (1972).
- L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, Scalar radiation emitted from a source rotating around a black hole, Classical Quantum Gravity 17, 19 (2000); Classical Quantum Gravity **33**, 209502(C) (2016).
- V. Cardoso and J. P. Lemos, Scalar synchrotron radiation in the Schwarzschild-anti-de Sitter geometry, Phys. Rev. D **65**, 104033 (2002).
- J. P. B. Brito, R. P. Bernar, and L. C. B. Crispino, Synchrotron geodesic radiation in Schwarzschild-de Sitter spacetime, Phys. Rev. D **101**, 124019 (2020).
- J. P. B. Brito, R. P. Bernar, and L. C. B. Crispino, Radiation emitted by a source orbiting a Schwarzschild–anti–de Sitter black hole, Phys. Rev. D **104**, 124085 (2021).
- A. G. Riess *et al.*, Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. **116**, 1009 (1998).
- S. Perlmutter et al., Measurements of Ω and Λ from 42 high redshift supernovae, Astrophys. J. 517, 565 (1999).
- F. Kottler, Über die physikalischen grundlagen der Einsteinschen gravitationstheorie, Ann. Phys. (N.Y.) **361**, 401 (1918).
- P. J. E. Peebles, and B. Ratra, The cosmological constant and dark energy, Rev. Mod. Phys. **75**, 559 (2003).
- L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, Interaction of Hawking radiation and a static electric charge, Phys. Rev. D **58**, 084027 (1998).
- J. Castiñeiras, L. C. B. Crispino, R. Murta, and G. E. A. Matsas, Semiclassical approach to black hole absorption of electromagnetic radiation emitted by a rotating charge, Phys. Rev. D **71**, 104013 (2005).
- *XGAL: Time travelling through the Universe*, https://xgalweb.wordpress.com/leaps/. Accessed on July 24, 2024.
- *Masses in the Stellar Graveyard*, https://media.ligo.northwestern.edu/gallery/mass-plot. Accessed on July 24, 2024.
- Y. Sugiyama, K. Yamamoto, and T. Kobayashi, Gravitational waves in Kasner spacetimes and Rindler wedges in Regge-Wheeler gauge: formulation of Unruh effect, Phys. Rev. D **103**, 083503 (2021).
- L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, The Unruh effect and its applications, Rev. Mod. Phys. **80**, 787 (2008).
- A. Higuchi, G. E. A. Matsas e D. Sudarsky, Bremssstrahlung and zero-energy Rindler photons, Phys. Rev. D 45, R3308(R) (1992).
- A. Higuchi, G. E. A. Matsas e D. Sudarsky, Bremsstrahlung and Fulling-Davies-Unruh thermal bath, Phys. Rev. D 46, 3450 (1992).