Scattering of S^2 kinks

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Summary

- Introduction
- Nonlinear O(3) σ model in (1, 1) dimensions
- Geometric and direct coupling
- An extended sine-Gordon model with internal structure
- Scattering results
- Conclusion

Introduction

- Topological defects are field theory solutions with localized density energy that propagate freely without losing form.
- Topological map between the physical coordinate space and the internal field space.
- In (1,1) dimensions, we have the kink and antikink as the simplest topological defects.
- In high dimensionality, one can consider a multiplet multiple of fields with a geometric constraint, as done, for instance, in the nonlinear $O(3)$ sigma model.
- When coupled to gravity, the nonlinear $O(3)$ sigma model gives hairy black holes (C. Herdeiro, I. Perapechka, E. Radu, Ya. Shnir, Gravitating solitons and black holes with synchronised hair in the four dimensional O(3) sigma-model, J. High Energ. Phys. 2019, 111(2019)).

- In (2,1) dimensions, after spontaneous breaking symmetry, every finite energy field configuration corresponds to a mapping $S^2 \to S^2$. Unstable lump solutions, but their interactions were investigated (W. J. Zakrzewski, Nonlinearity 1991)
- In (1,1) dimensions, after explicit breaking symmetry, every finite energy field configuration corresponds to a mapping $S^1 \to S^2$ (Yu. Loginov, Phys. Atom. Nuclei 2011). Non-contractible loops in internal space.
- A massive nonlinear $O(3)$ sigma model in $(1,1)$-dimensions with quadratic potential (A. Alonso-Izquierdo, M. A. Gonzalez Leon, J. Mateos Guilarte, Phys. Rev. Lett. 2008, Phys. Rev. D 2009)
- An interesting aspect of these works is the use of spherical coordinates for the fields, leading to a curved metric for the configuration space. Here the metric components depend on the fields, meaning that there is a geometric constraint.

- Bubble universe collisions (Pontus Ahlqvist, Kate Eckerle and Brian Greene, Kink Collisions in Curved Field Space, J. High Energ. Phys. 2015, 59 (2015))
- Bubbles constraining eternal inflation (Matthew C. Johnson, Carroll L. Wainwright, Anthony Aguirre, Hiranya V. Peiris, Simulating the universe(s) III: observables for the full bubble collision spacetime, JCAP 07, 020 (2016))
- Deformations of sigma models in the plane R2 and in the sphere S2, as well as the transference of solutions between these two manifolds (A. Alonso-Izquierdo, A. J. Balseyro Sebastian, M. A. Gonzalez Leon, Transference of kinks between S2 and R2 Sigma models [hep-th/2410.01344]).

Nonlinear O(3) σ model in (1, 1) dimensions

$$S = \int dtdx[\frac{1}{2}\partial_\mu\vec{\varphi}\cdot\partial^\mu\vec{\varphi} - \nu(1-\vec{\varphi}\cdot\vec{\varphi})]$$

$$\vec{\varphi} = (\varphi^1, \varphi^2, \varphi^3)$$

$$\begin{aligned} \varphi^1 &= \sin\chi\cos\phi \\ \varphi^2 &= \sin\chi\sin\phi \\ \varphi^3 &= \cos\chi \end{aligned}$$

$$S = \int dtdx\left(\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}\sin^2\chi\partial_\mu\phi\partial^\mu\phi\right)$$

Two coupled real scalar fields (φ, χ) in an S^2 internal space

$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\chi. \end{pmatrix}$$

Geometric and direct coupling

$$S = \int dt dx \left(\frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \frac{1}{2} \sin^2 \chi \partial_\mu \phi \partial^\mu \phi - V(\phi, \chi) \right).$$

$$\begin{aligned} \partial_\mu (\sin^2 \chi \partial^\mu \phi) + V_\phi(\phi, \chi) &= 0, \\ \partial_\mu \partial^\mu \chi - \frac{1}{2} \sin(2\chi) \partial_\mu \phi \partial^\mu \phi + V_\chi(\phi, \chi) &= 0 \end{aligned}$$

Static solutions are solutions of the equations

$$\begin{aligned} \frac{d^2\chi}{dx^2} - \frac{1}{2} \sin(2\chi) \left(\frac{d\phi}{dx} \right)^2 &= V_\chi(\phi, \chi), \\ \frac{d}{dx} \left(\sin^2 \chi \frac{d\phi}{dx} \right) &= V_\phi(\phi, \chi) \end{aligned}$$

BPS method

$$\rho = \frac{1}{2}\sin^2\chi\left(\frac{d\phi}{dx}\right)^2 + \frac{1}{2}\left(\frac{d\chi}{dx}\right)^2 + V(\phi,\chi).$$

$$V(\phi,\chi) = \frac{1}{2}\frac{W_\phi^2}{\sin^2\chi} + \frac{1}{2}W_\chi^2.$$

$$\rho = \frac{1}{2}\sin^2\chi\left(\frac{d\phi}{dx} \mp \frac{W_\phi}{\sin^2\chi}\right)^2 + \frac{1}{2}\left(\frac{d\chi}{dx} \mp W_\chi\right)^2 + W_\phi\frac{d\phi}{dx} + W_\chi\frac{d\chi}{dx}.$$

$$\begin{aligned} \frac{d\phi}{dx} &= \pm\frac{W_\phi}{\sin^2\chi} \\ \frac{d\chi}{dx} &= \pm W_\chi \end{aligned}$$

BPS equations

Stability analysis

$$\begin{pmatrix} \tilde{\phi}(x,t) \\ \tilde{\chi}(x,t) \end{pmatrix} = \begin{pmatrix} \phi(x) \\ \chi(x) \end{pmatrix} + acos(\omega t) \begin{pmatrix} \eta_1(x) \\ \eta_2(x) \end{pmatrix}$$

$$\widehat{\mathcal{H}} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \sin(\chi)^2 \omega^2 \eta_1 \\ \omega^2 \eta_2 \end{pmatrix}$$

$$\widehat{\mathcal{H}} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

$$\begin{aligned} H_{11} &= -\sin(2\chi)\frac{d\chi}{dx}\frac{d}{dx} - \sin^2(\chi)\frac{d^2}{dx^2} + V_{\phi\phi} \\ H_{12} &= -\sin(2\chi)\frac{d\phi}{dx}\frac{d}{dx} - 2\cos(2\chi)\frac{d\phi}{dx}\frac{d\chi}{dx} + V_{\phi\chi} \\ H_{21} &= \sin(2\chi)\frac{d\phi}{dx}\frac{d}{dx} + V_{\chi\phi} \\ H_{22} &= -\frac{d^2}{dx^2} + \cos(2\chi)\left(\frac{d\phi}{dx}\right)^2 + V_{\chi\chi}. \end{aligned}$$

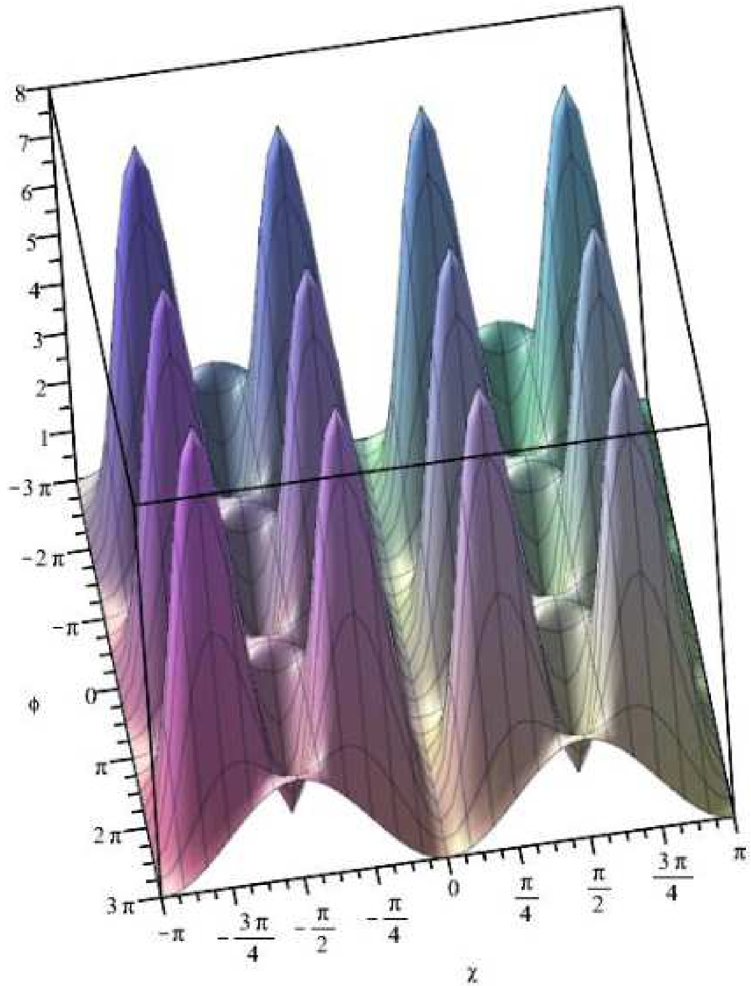
An extended sine-Gordon

$$W_\phi = 2\sin\left(\frac{\phi}{2}\right)\sin^2\chi.$$

$$V(\phi,\chi) = 2\sin^2\chi\left(\sin^2\left(\frac{\phi}{2}\right) + 16\cos^2\chi\cos^2\left(\frac{\phi}{2}\right)\right)$$

$$\phi = \pm 2n\pi,\ \chi = \pm(2n+1)\pi/2$$

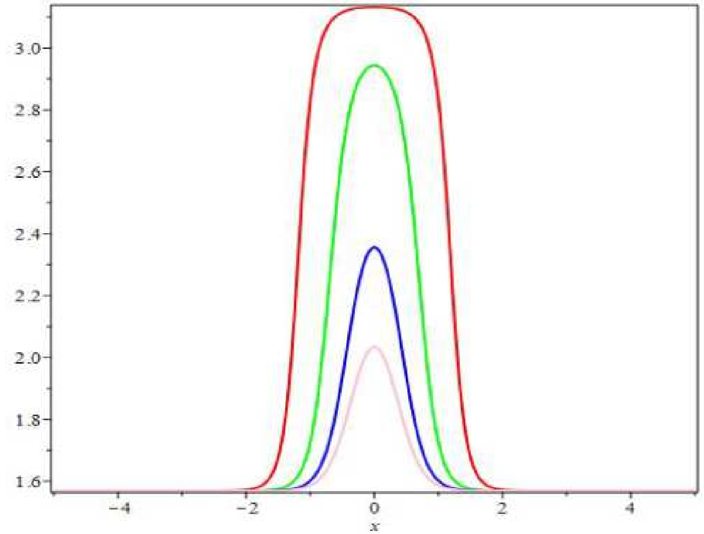
For S^2: $0 \leq \chi \leq \pi$

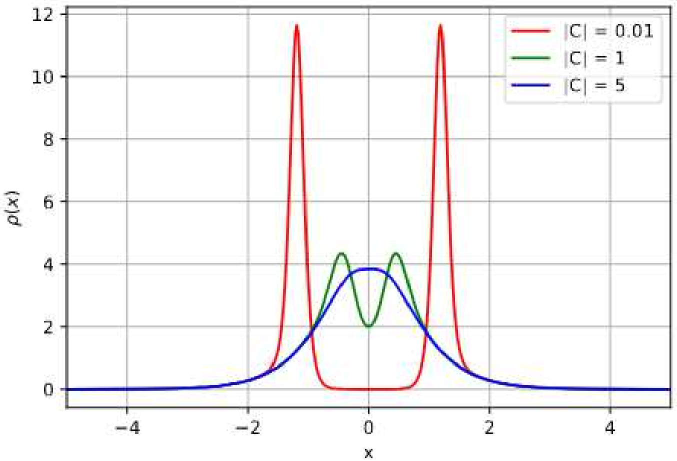


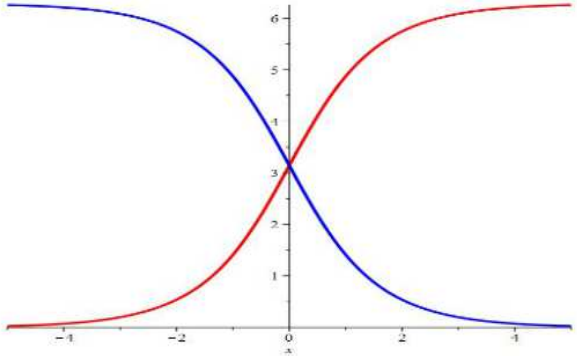
BPS equations and solutions

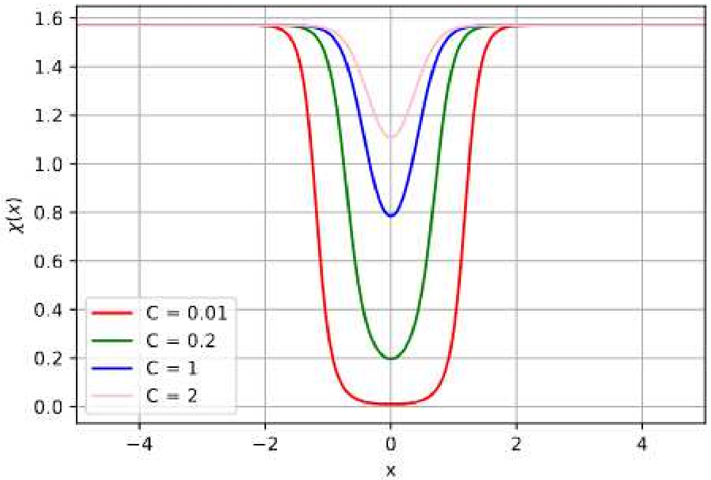
$$\frac{d\phi}{dx} = \pm 2\sin\left(\frac{\phi}{2}\right), \qquad \phi(x) = 4\arctan(e^{x}).$$

$$\frac{d\chi}{dx} = \mp 4\sin(2\chi)\cos\left(\frac{\phi}{2}\right). \qquad \chi(C,x) = \arctan(C\cosh^{8}(x)),$$









Soliton solutions in internal space

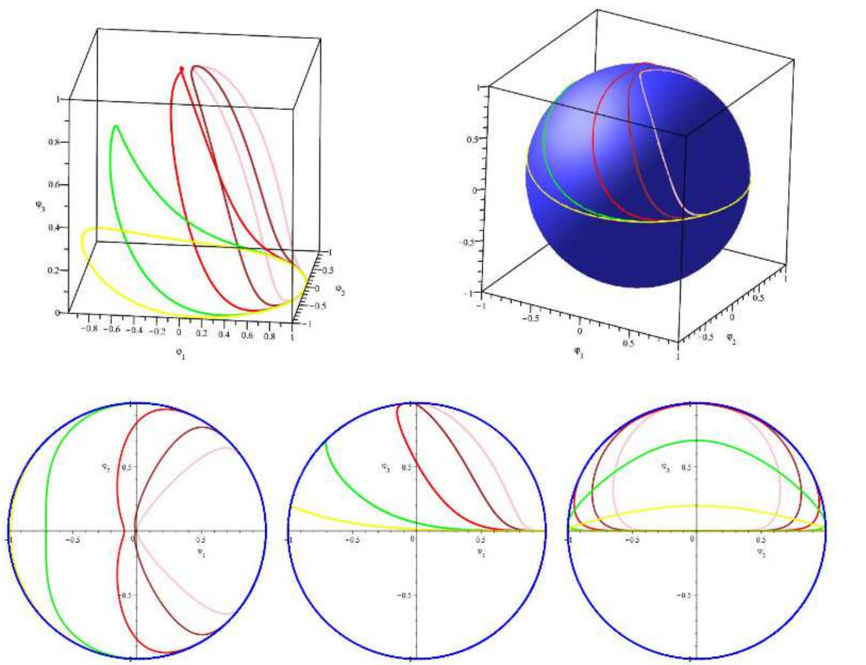


FIG. 4: Soliton solutions in internal space for $C = 10^{-3}$ (pink), $C = 10^{-2}$ (brown), $C = 0.1$ (red), $C = 1$ (green) and $C = 5$ (yellow) a) the soliton solutions are loops in internal space. b) The soliton loops viewed in the S^2_{int} internal space. c) solitons in phase space $\varphi_1 \times \varphi_2$. d) c) solitons in phase space $\varphi_1 \times \varphi_3$. e) solitons in phase space $\varphi_2 \times \varphi_3$.

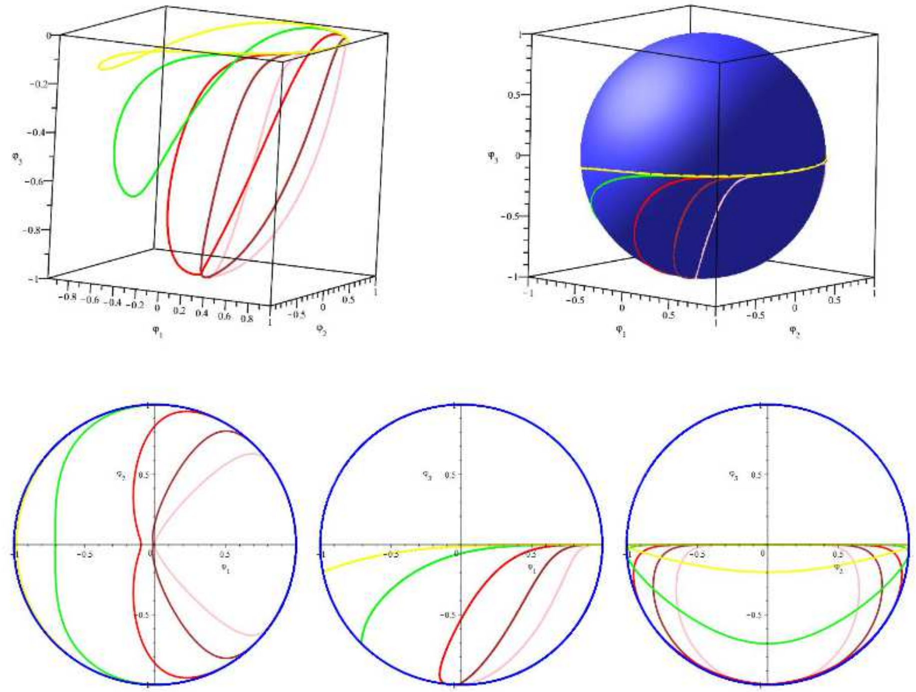


FIG. 5: Soliton solutions in internal space for $C = -10^{-3}$ (pink), $C = -10^{-2}$ (brown), $C = -0.1$ (red), $C = -1$ (green) and $C = -5$ (yellow) a) the soliton solutions are loops in internal space. b) The soliton loops viewed in the S^2_{int} internal space. c) solitons in phase space $\varphi_1 \times \varphi_2$. d) c) solitons in phase space $\varphi_1 \times \varphi_3$. e) solitons in phase space $\varphi_2 \times \varphi_3$.

Kink-antikink and antilump-antilump scattering

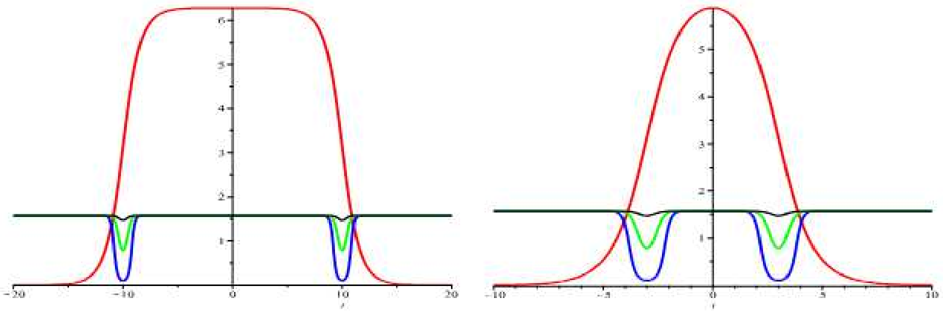


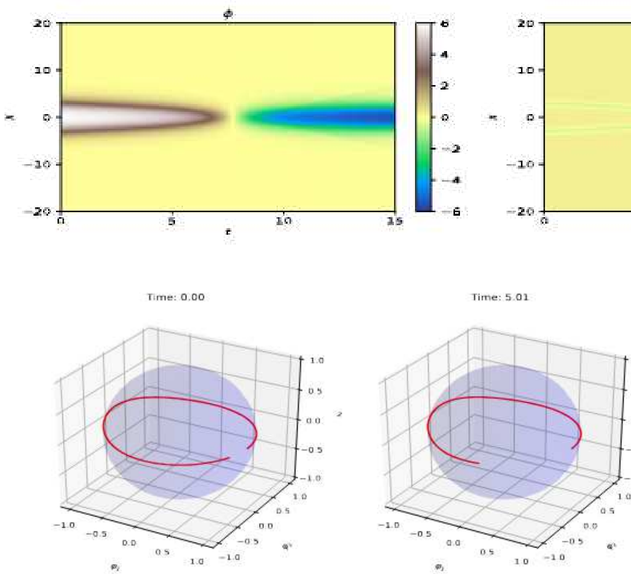
FIG. 7: Initial field configuration for a pair of defects for a) $x_0 = 10$ and b) $x_0 = 3$. For all figures, $\phi(x,t)$ (red) and $\chi(x)$ for $C = 0.1$ (blue), $C = 1$ (green) and $C = 10$ (black).

$$\chi(x,0,x_0,v) = \chi_K(\gamma(x+x_0-vt)) + \chi_{\bar{K}}(\gamma(x-x_0+vt)) - 1,$$
$$\dot{\chi}(x,0,x_0,v) = \dot{\chi}_K(\gamma(x+x_0-vt)) + \dot{\chi}_{\bar{K}}(\gamma(x-x_0+vt)),$$

$$\phi(x,0,x_0,v) = \phi_K(\gamma(x+x_0-vt)) + \phi_{\bar{K}}(\gamma(x-x_0+vt)) - \pi,$$
$$\dot{\phi}(x,0,x_0,v) = \dot{\phi}_K(\gamma(x+x_0-vt)) + \dot{\phi}_{\bar{K}}(\gamma(x-x_0+vt)),$$

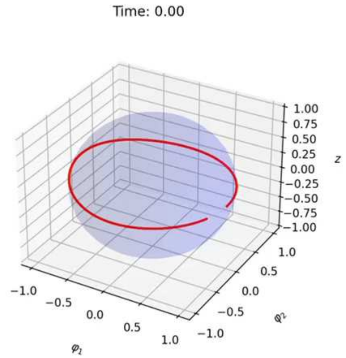
Kink-antikink and antilump-antilump scattering

$v = 0.2, C = 5$

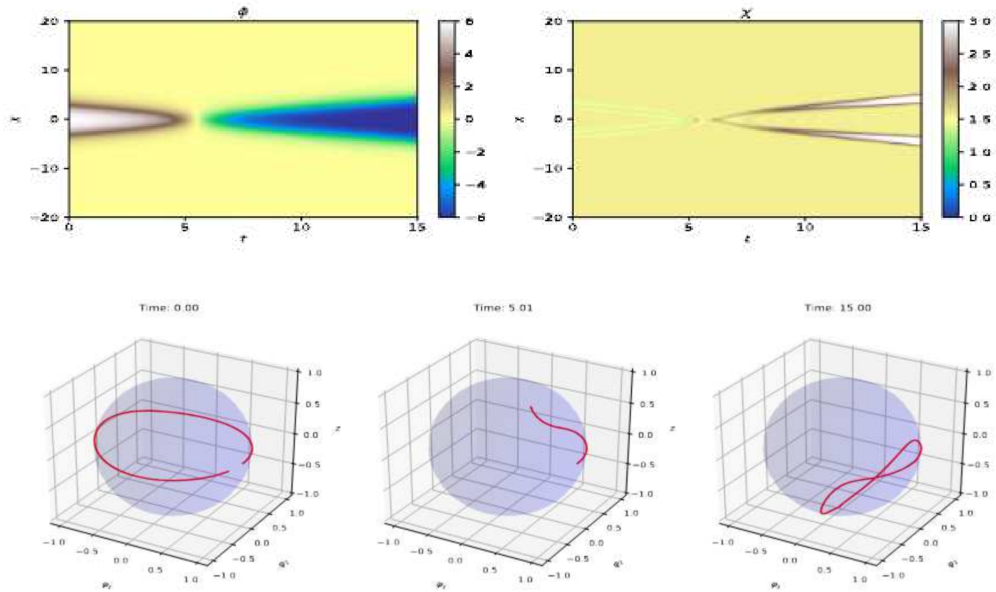


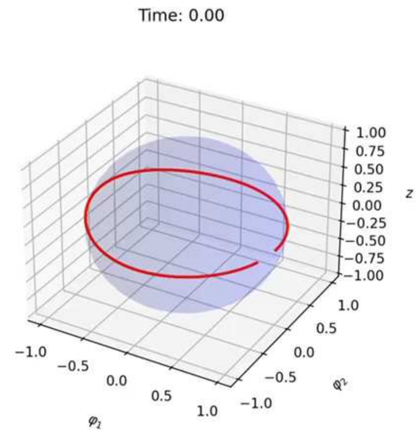
(a)

Time: 0.00 Time: 5.01 Time: 15.00



$v = 0.4, C = 5$





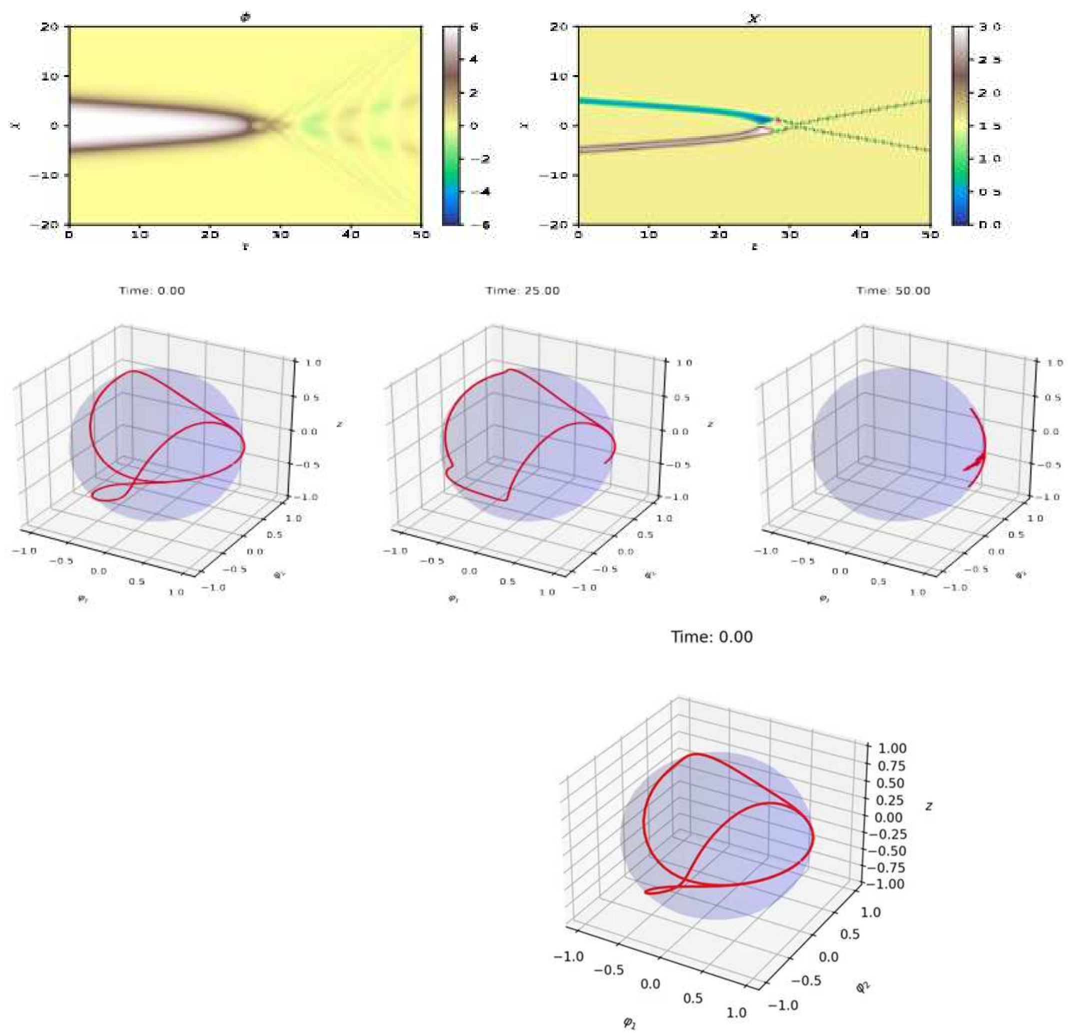
Kink-antikink and lump-antilump scattering

$$
\begin{aligned}
\chi\left(x,0,x_0,v\right) &= \chi_L\left(\gamma\left(x+x_0-vt\right)\right)+\chi_{\bar{L}}\left(\gamma\left(x-x_0+vt\right)\right)+\frac{\pi}{2},\\
\dot{\chi}\left(x,0,x_0,v\right) &= \dot{\chi}_L\left(\gamma\left(x+x_0-vt\right)\right)+\dot{\chi}_{\bar{L}}\left(\gamma\left(x-x_0+vt\right)\right),
\end{aligned}
$$

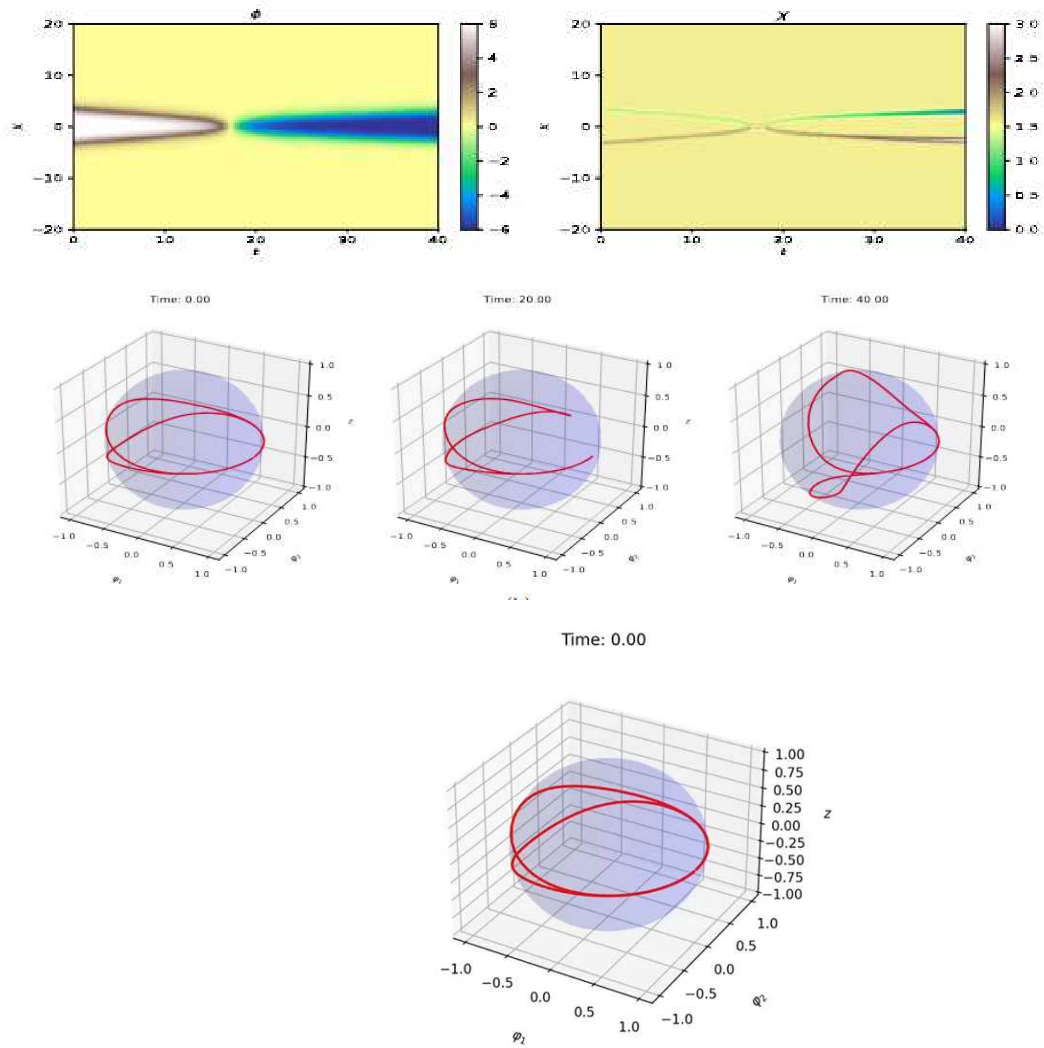
$$
\begin{aligned}
\phi\left(x,0,x_0,v\right) &= \phi_K\left(\gamma\left(x+x_0-vt\right)\right)+\phi_{\bar{K}}\left(\gamma\left(x-x_0+vt\right)\right)-2\pi,\\
\dot{\phi}\left(x,0,x_0,v\right) &= \dot{\phi}_K\left(\gamma\left(x+x_0-vt\right)\right)+\dot{\phi}_{\bar{K}}\left(\gamma\left(x-x_0+vt\right)\right),
\end{aligned}
$$

Kink-antikink and lump-antilump scattering

$v = 0.10, C = 0.5$



$v = 0.20, C = 3$



Conclusion

- The loops can develop interconnections in a complex pattern.
- We did not observe the production of separated loops.
- The numerical solution for kink-antikink and lump-lump scattering for small C suffer from numerical instability. This is shown to be the result of instability for $\chi = 0$.
- the limit $|C| \to \infty$ corresponds to the sine-Gordon solution for φ, with $\chi = \pi/2$. In this limit it is expected the scattering between the defects to be elastic, as occurs with the integrable (1, 1) sine-Gordon model
- our numerical investigation show how a nonintegrable defect constructed with two fields behaves due to scattering as one approaches the conditions for integrability.