Non-linear Superradiance of Charged Black Holes: Beyond Spherical Symmetry

José Ferreira[1,2,*], Miguel Zilhão[1,2], Carlos Herdeiro[1,2]

[1] Center for R&D in Mathematics and Applications
[2] University of Aveiro, Portugal

* jpmferreira@ua.pt

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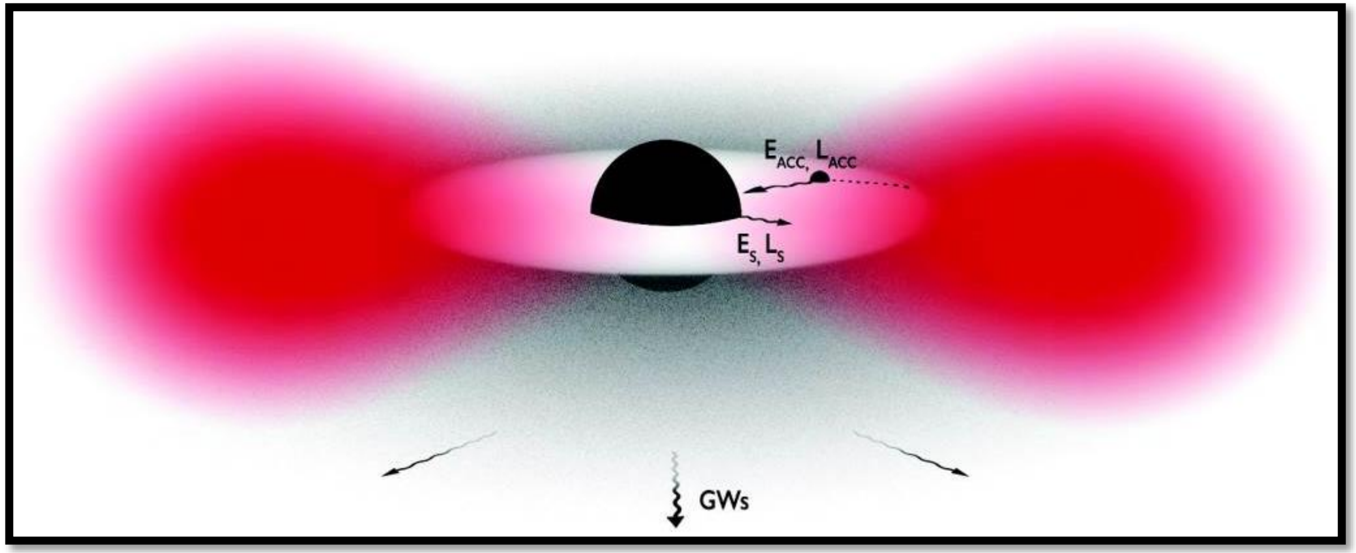
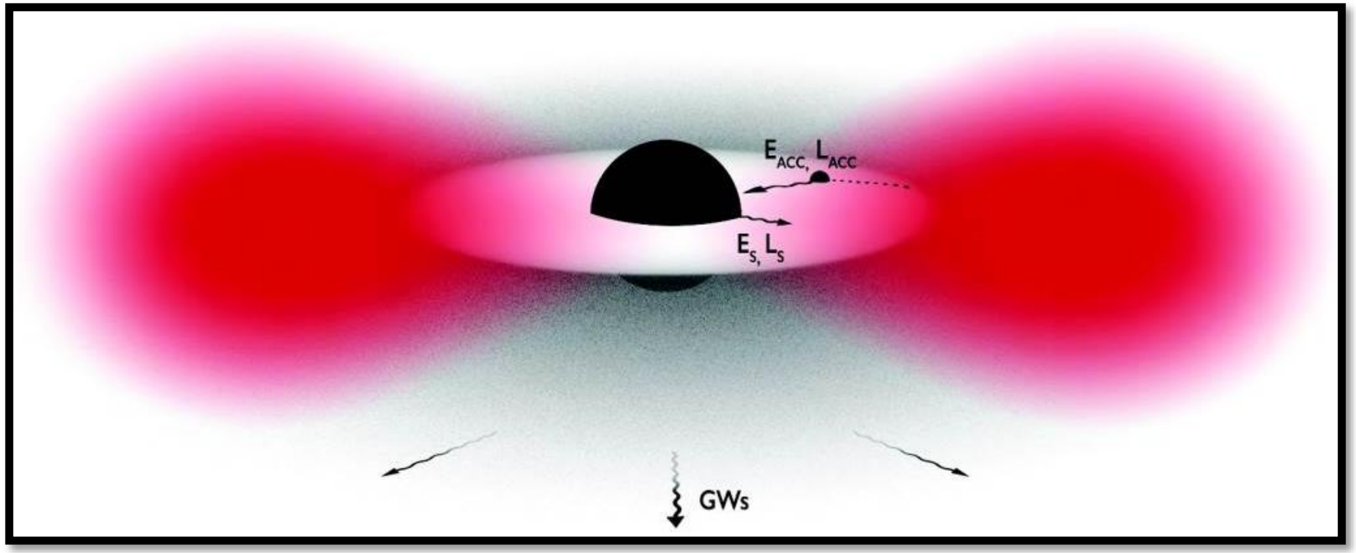


R. Brito, V. Cardoso, P. Pani – Superradiance (Springer, 2020)



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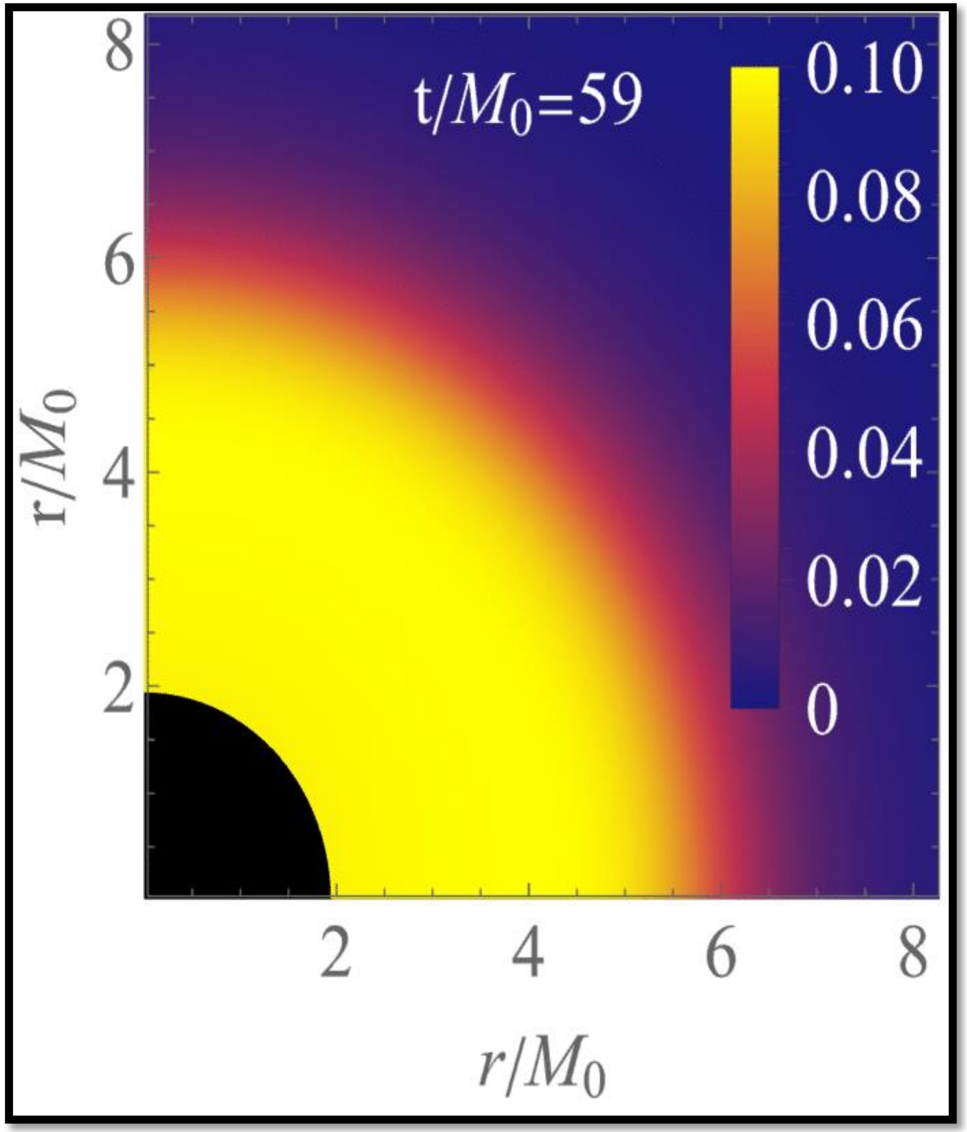
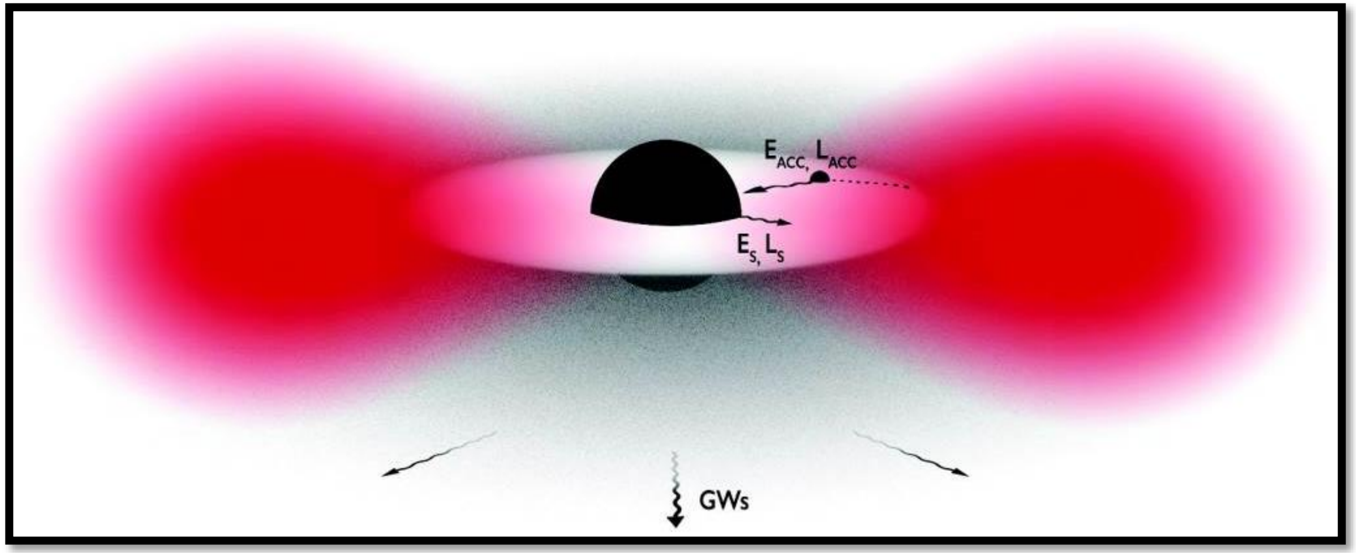


Zhang et al “Nonlinear self-interaction induced black hole bomb”. In: Phys. Rev. D 110 (4 Aug. 2024), p. L041505.



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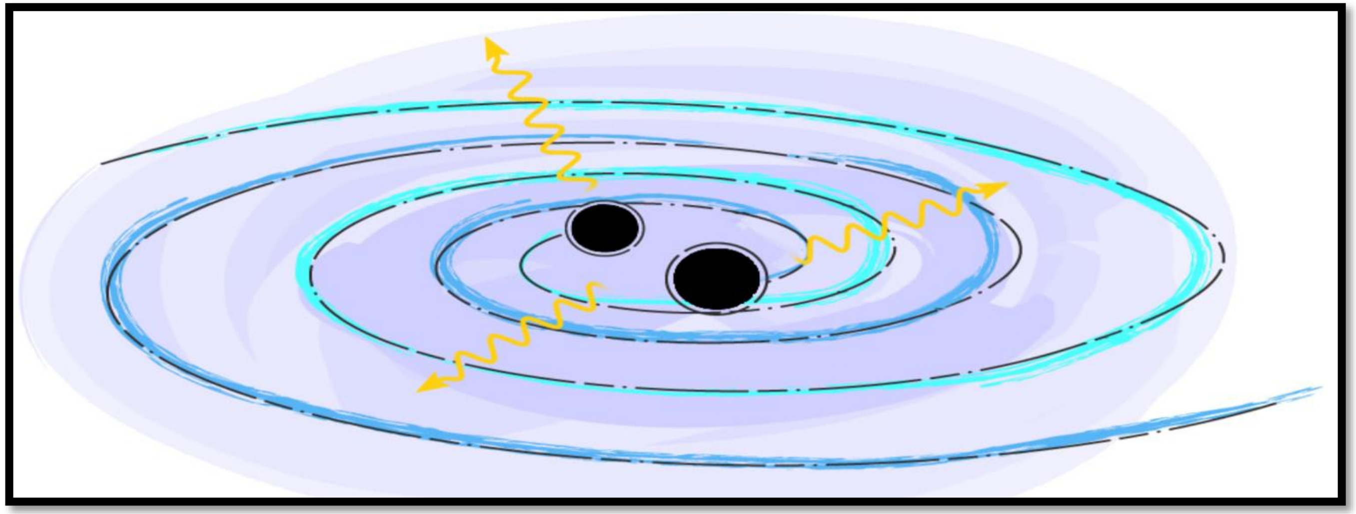

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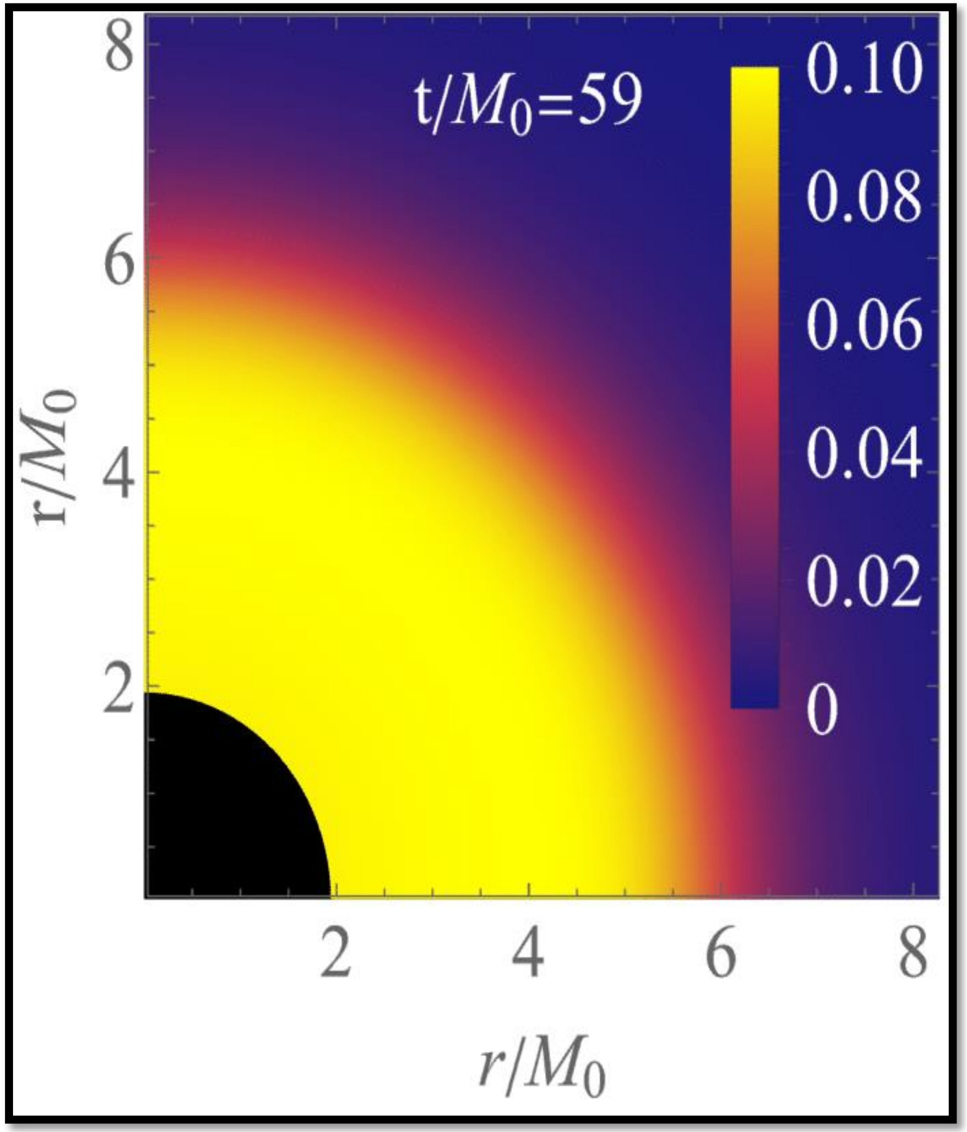

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Einstein-Maxwell-Scalar Model

$$S = \int \sqrt{-g} \left(\frac{R}{16\pi} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - (\widetilde{\nabla}_\alpha \phi)^* \widetilde{\nabla}^\alpha \phi - V(|\phi|^2) \right) d^4x$$

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$$\widetilde{\nabla}_\mu \equiv \nabla_\mu - i\mathbf{q}A_\mu$$

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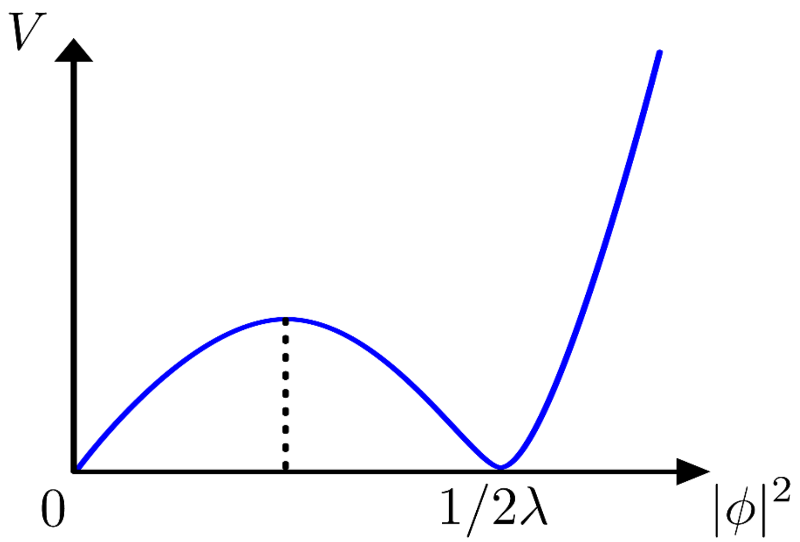
$$\tilde{\nabla}_\mu \equiv \nabla_\mu - i\mathbf{q} A_\mu$$

- $$\begin{cases} V(|\phi|^2) = \mu^2 |\phi|^2 (1 - 2\lambda |\phi|^2)^2 \\ \mu = 1, \ \lambda = 2500 \end{cases}$$



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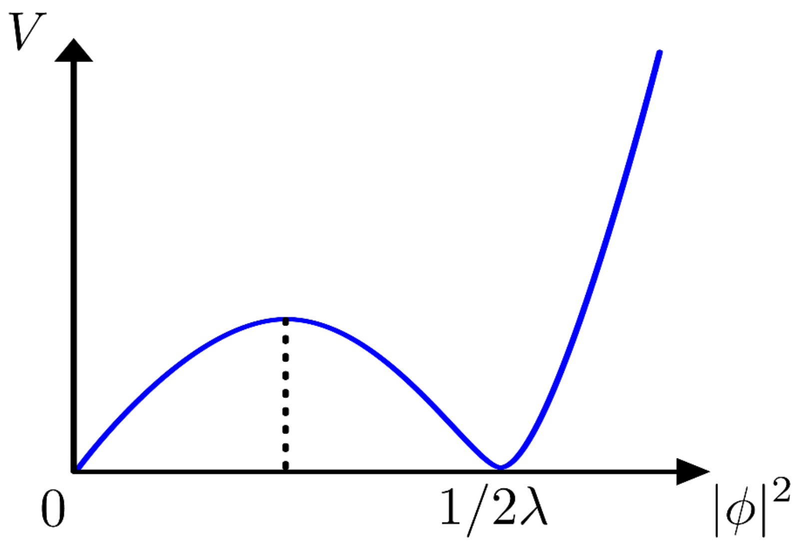
- $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}$



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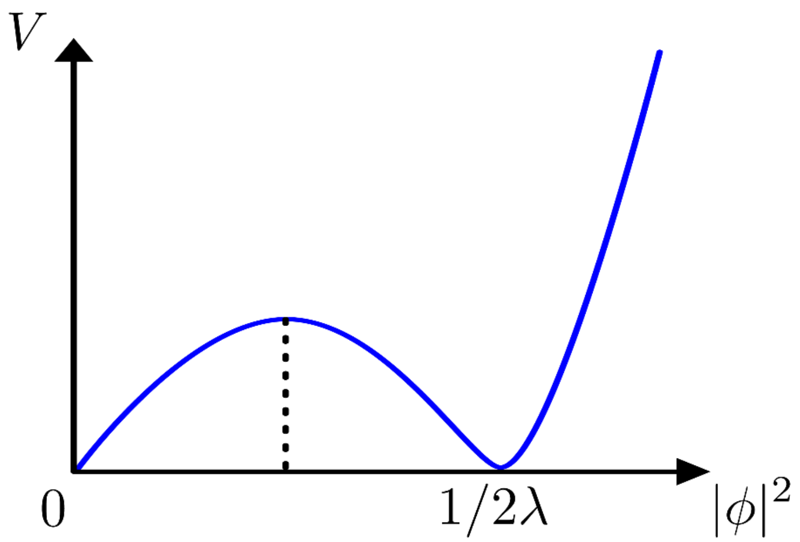
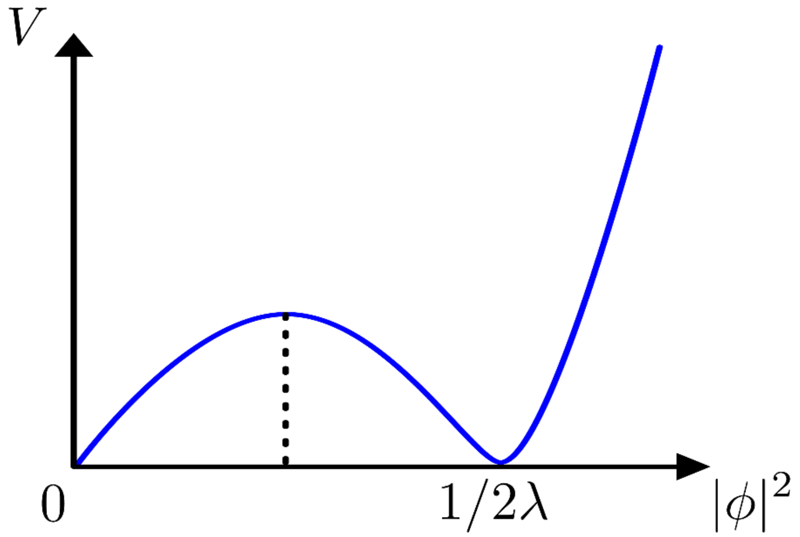
- $\nabla_\alpha F^{\mu\alpha} = 4\pi j^\mu, \quad j^\mu \equiv \frac{iq}{4\pi}\left[(\tilde{\nabla}^\mu\phi)^*\phi - \phi^*(\tilde{\nabla}^\mu\phi)\right]$



Einstein-Maxwell-Scalar Model

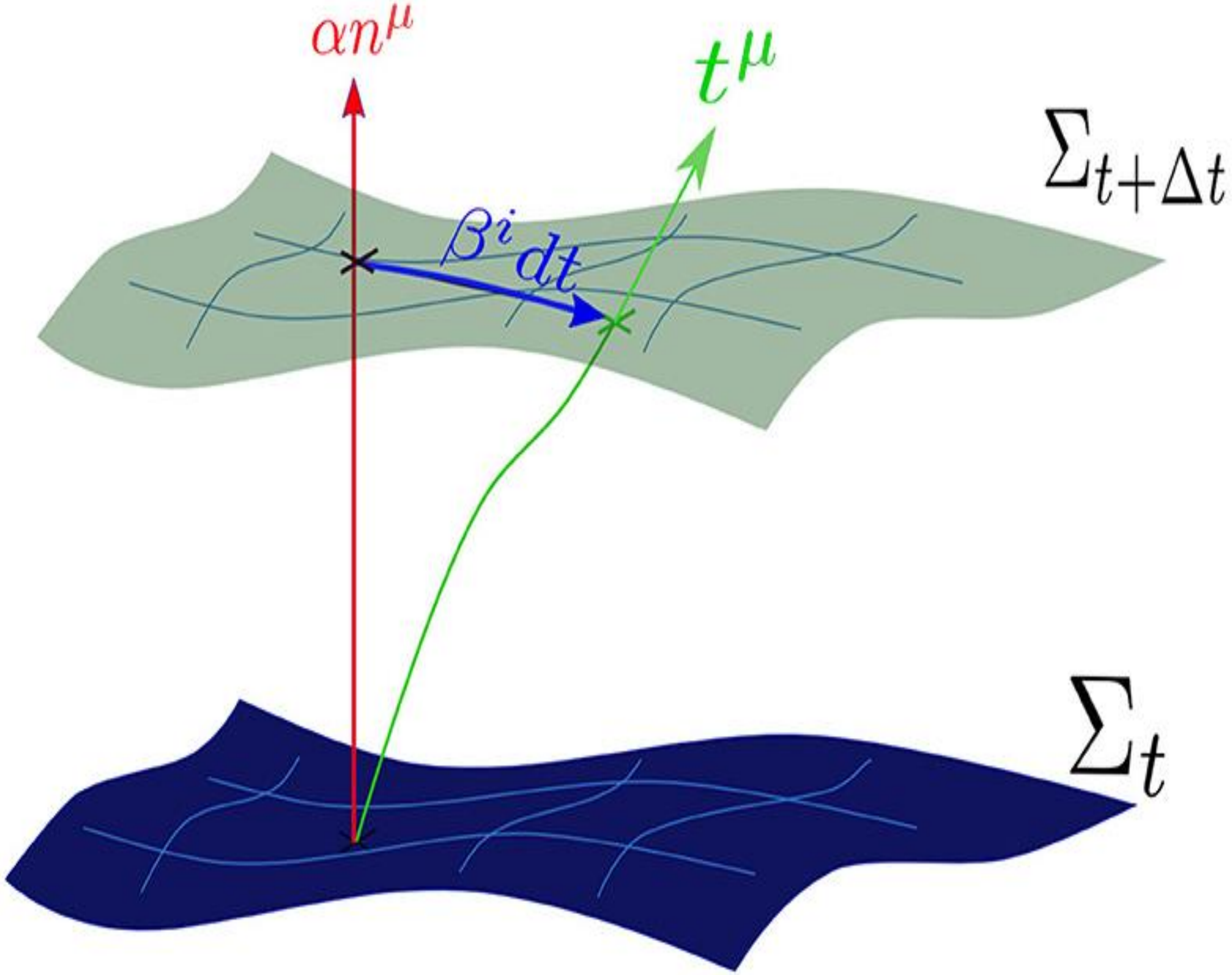
$$S = \int \sqrt{-g} \left(\frac{R}{16\pi} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - (\tilde{\nabla}_\alpha \phi)^* \tilde{\nabla}^\alpha \phi - V(|\phi|^2) \right) d^4x \qquad \boxed{\tilde{\nabla}_\mu \equiv \nabla_\mu - i\mathbf{q}A_\mu}$$

- $$\begin{cases} V(|\phi|^2) = \mu^2 |\phi|^2 (1 - 2\lambda |\phi|^2)^2 \\ \mu = 1,\ \lambda = 2500 \end{cases}$$



- $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$
- $$\nabla_\alpha F^{\mu\alpha} = 4\pi j^\mu, \quad j^\mu \equiv \frac{iq}{4\pi} \left[(\tilde{\nabla}^\mu \phi)^* \phi - \phi^* (\tilde{\nabla}^\mu \phi) \right]$$
- $$\tilde{\nabla}_\alpha \tilde{\nabla}^\alpha \phi = \frac{dV}{d\,|\phi|^2} \phi$$

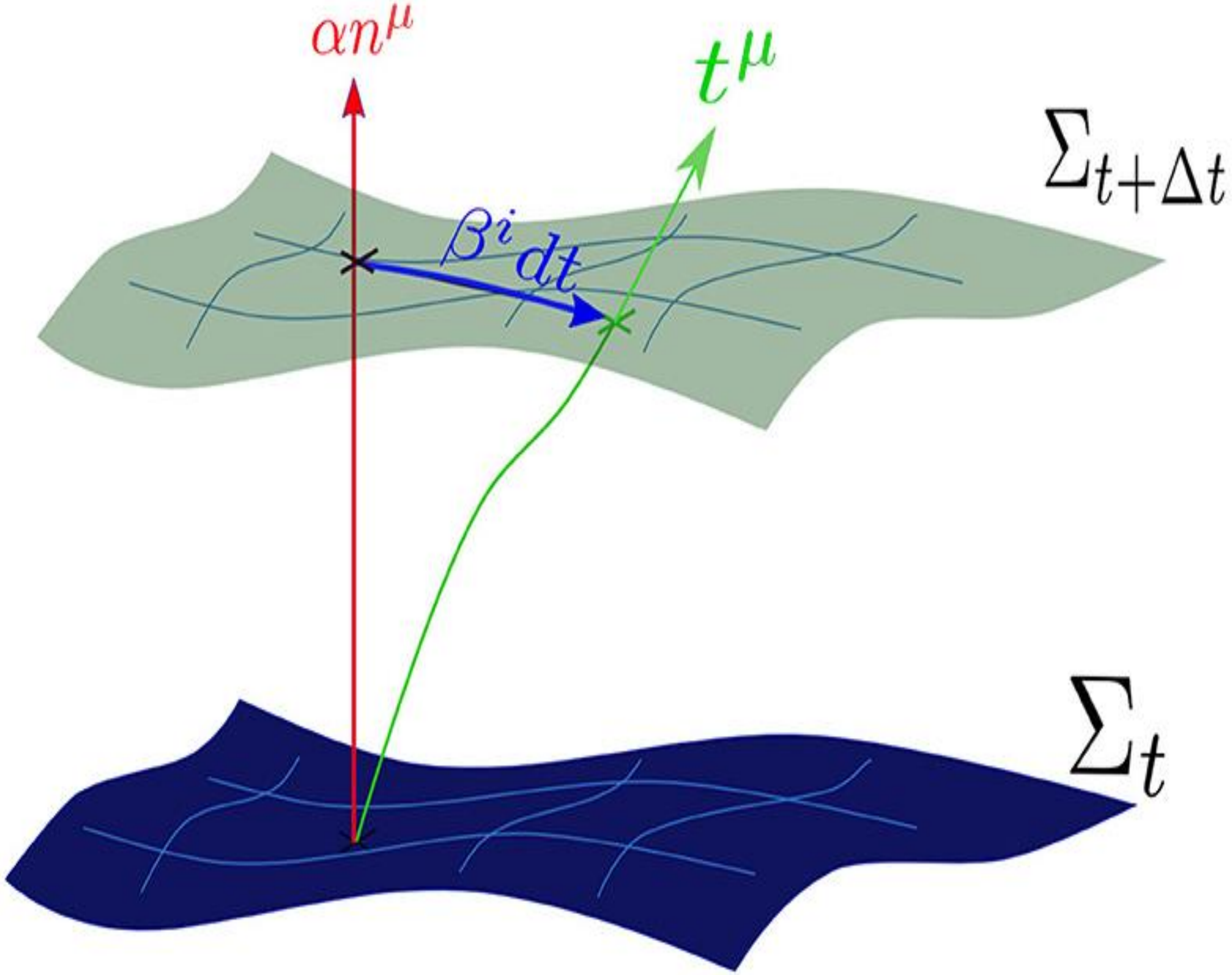
3+1 Decomposition



Palenzuela C (2020) Introduction to Numerical Relativity. *Front. Astron. Space Sci.* 7:58

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

3+1 Decomposition



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Lapse: $\alpha \equiv \alpha(x^i)$

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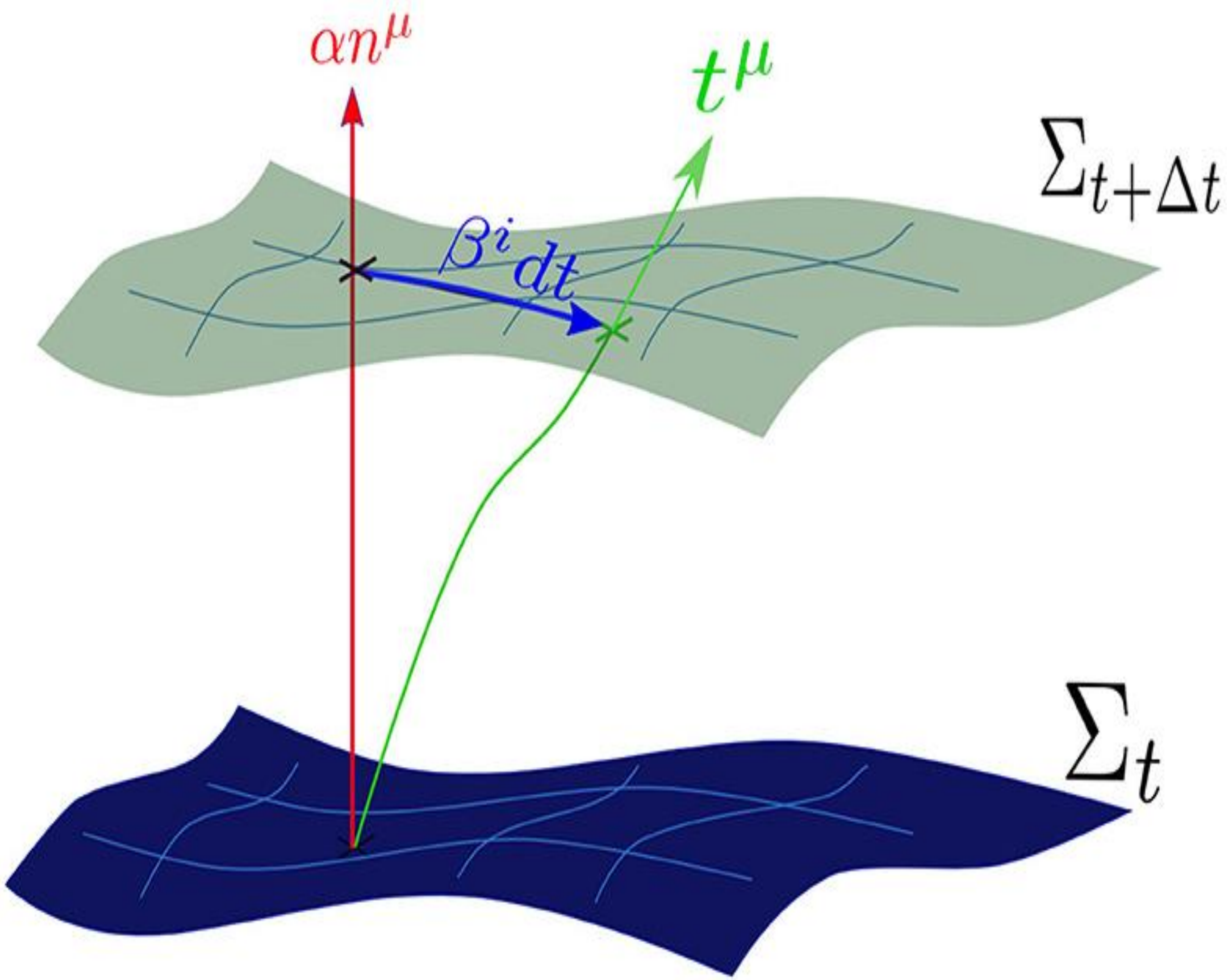

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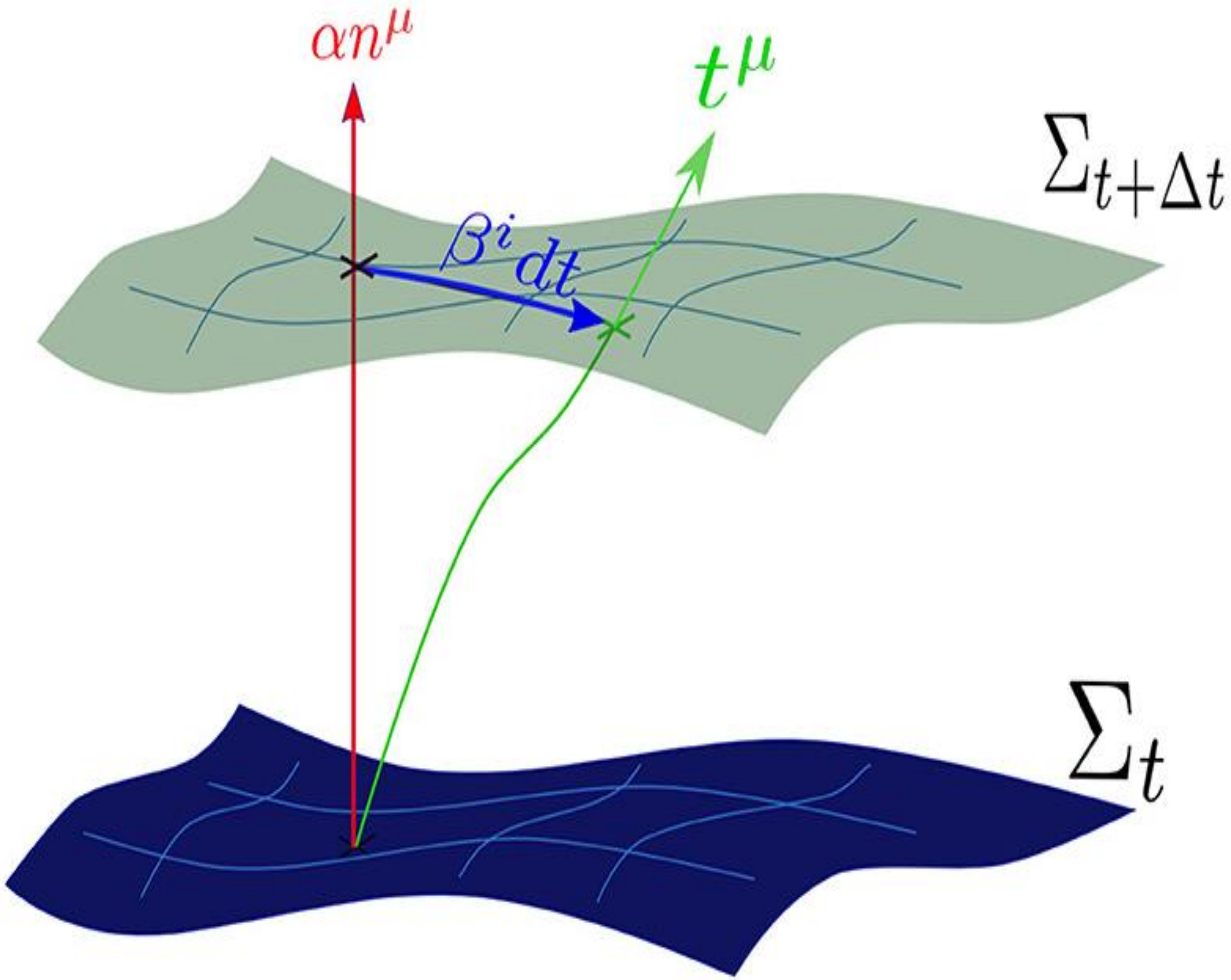
Lapse: $\alpha \equiv \alpha(x^i)$

Shift: $\beta^i \equiv \beta^i(x^i)$

3+1 Decomposition



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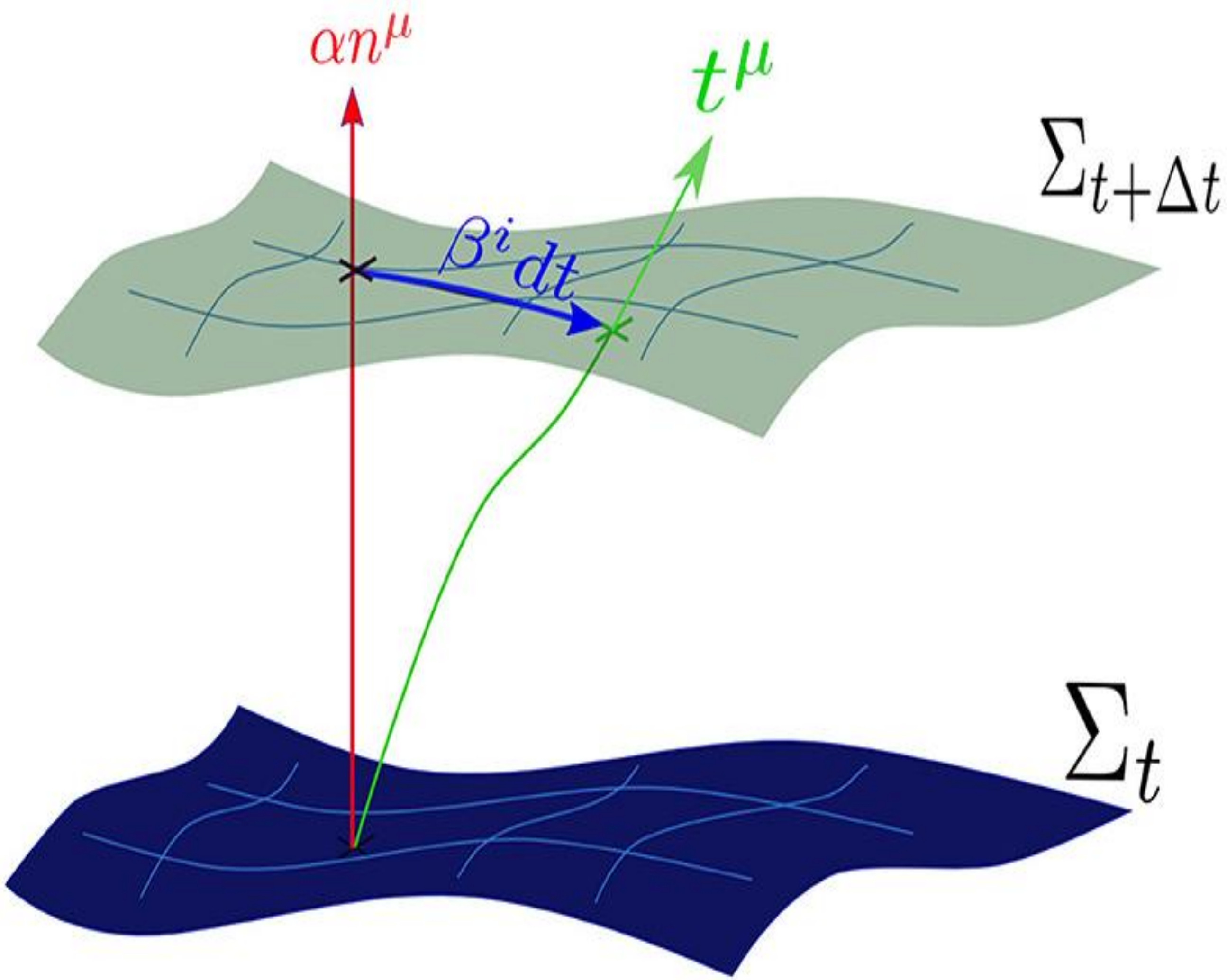
3D Metric: $\gamma_{ij} = g_{ij}$

3+1 Decomposition



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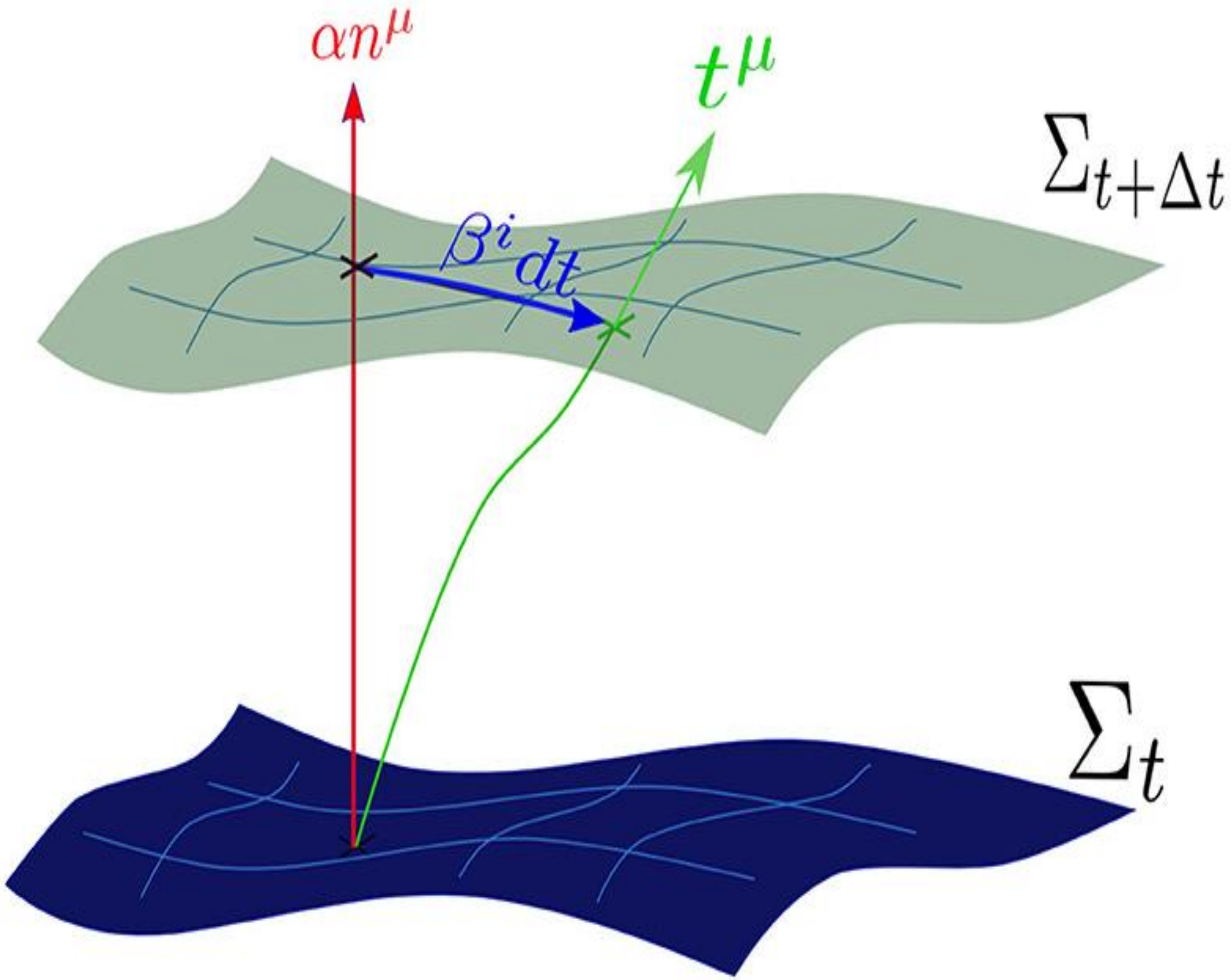
3+1 Decomposition



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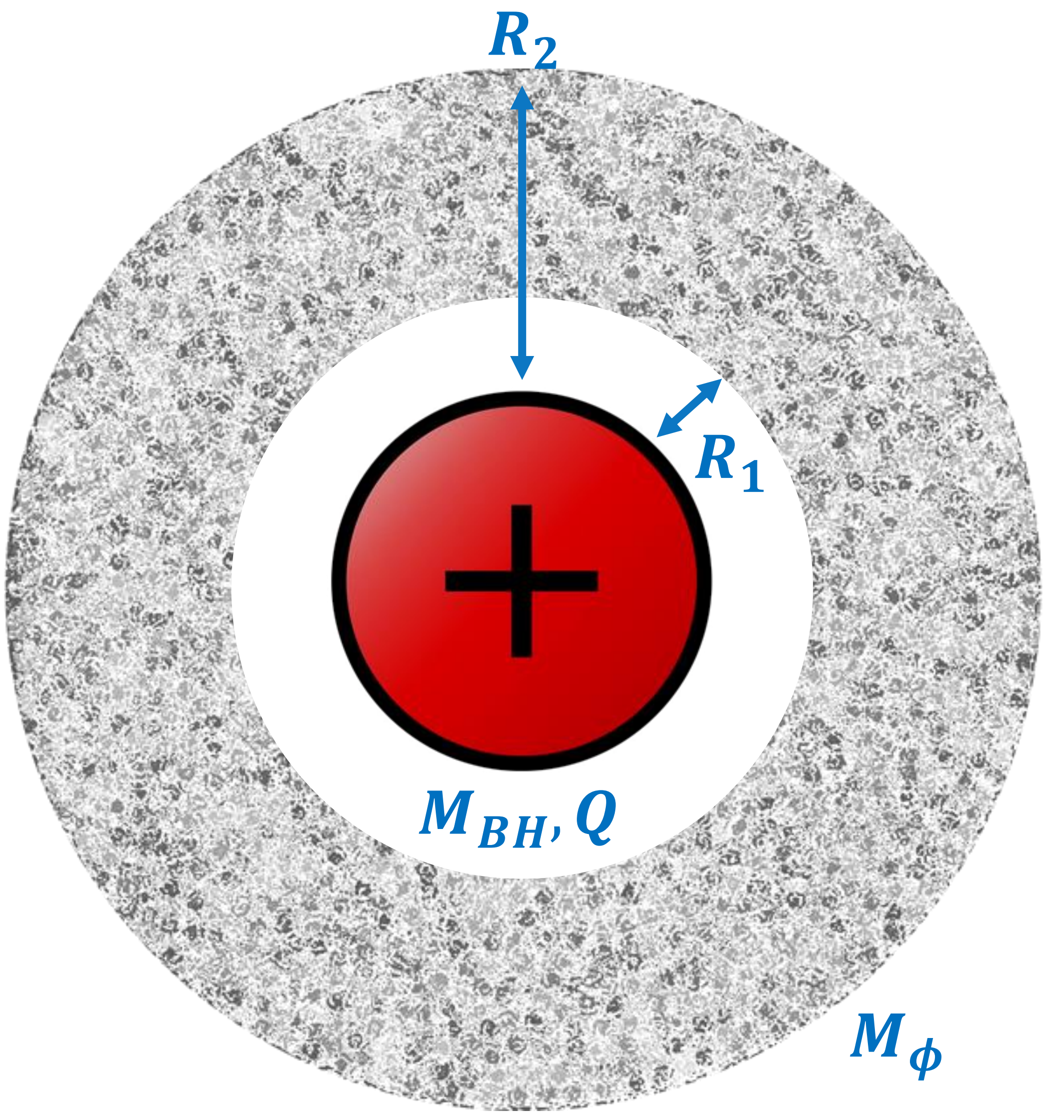
Lapse: $\alpha \equiv \alpha(x^i)$

Shift: $\beta^i \equiv \beta^i(x^i)$

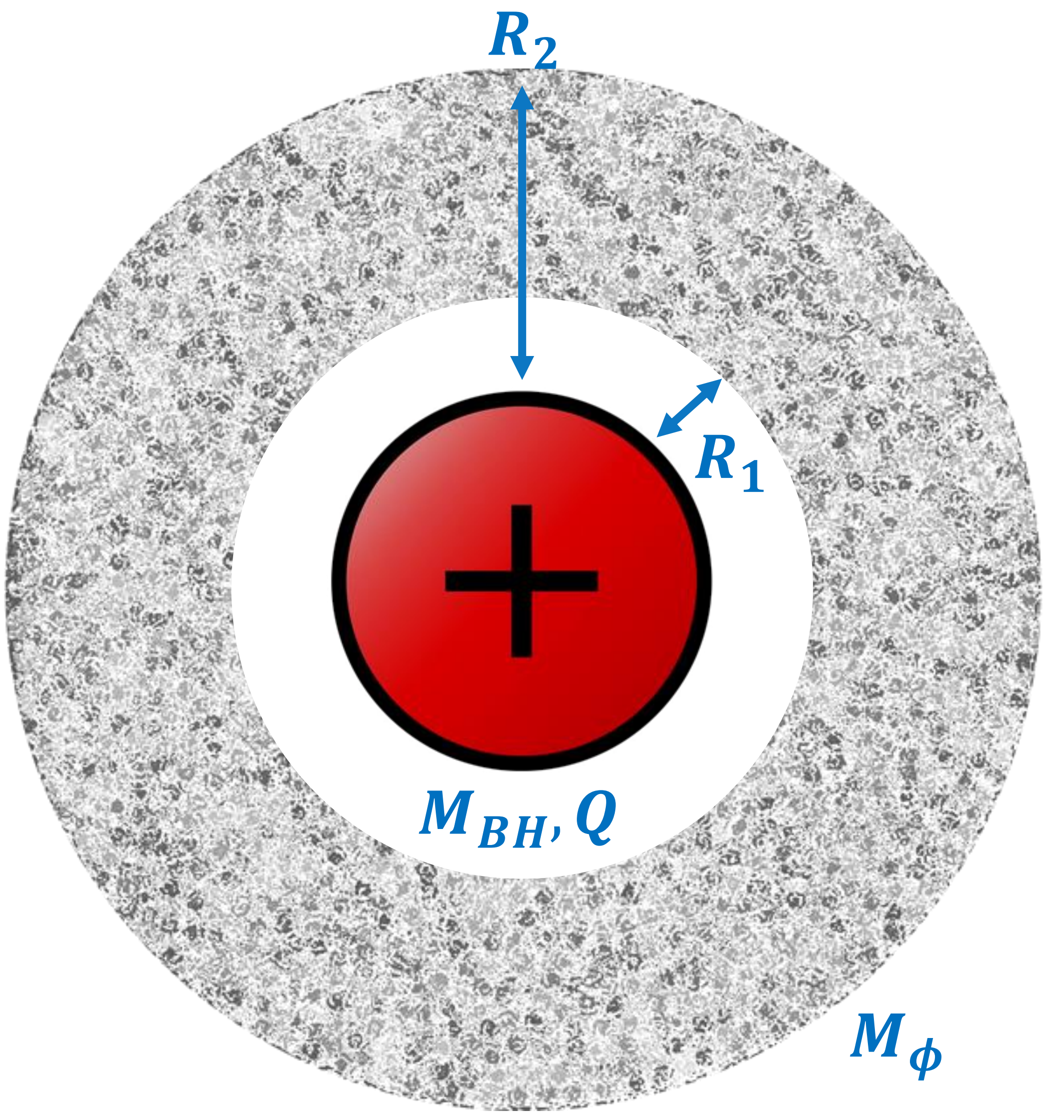
3D Metric: $\gamma_{ij} = g_{ij}$

Normal vector: $n^\mu = -(\alpha, 0, 0, 0)$

Time vector: $t^\mu \equiv \alpha n^\mu + \beta^\mu$

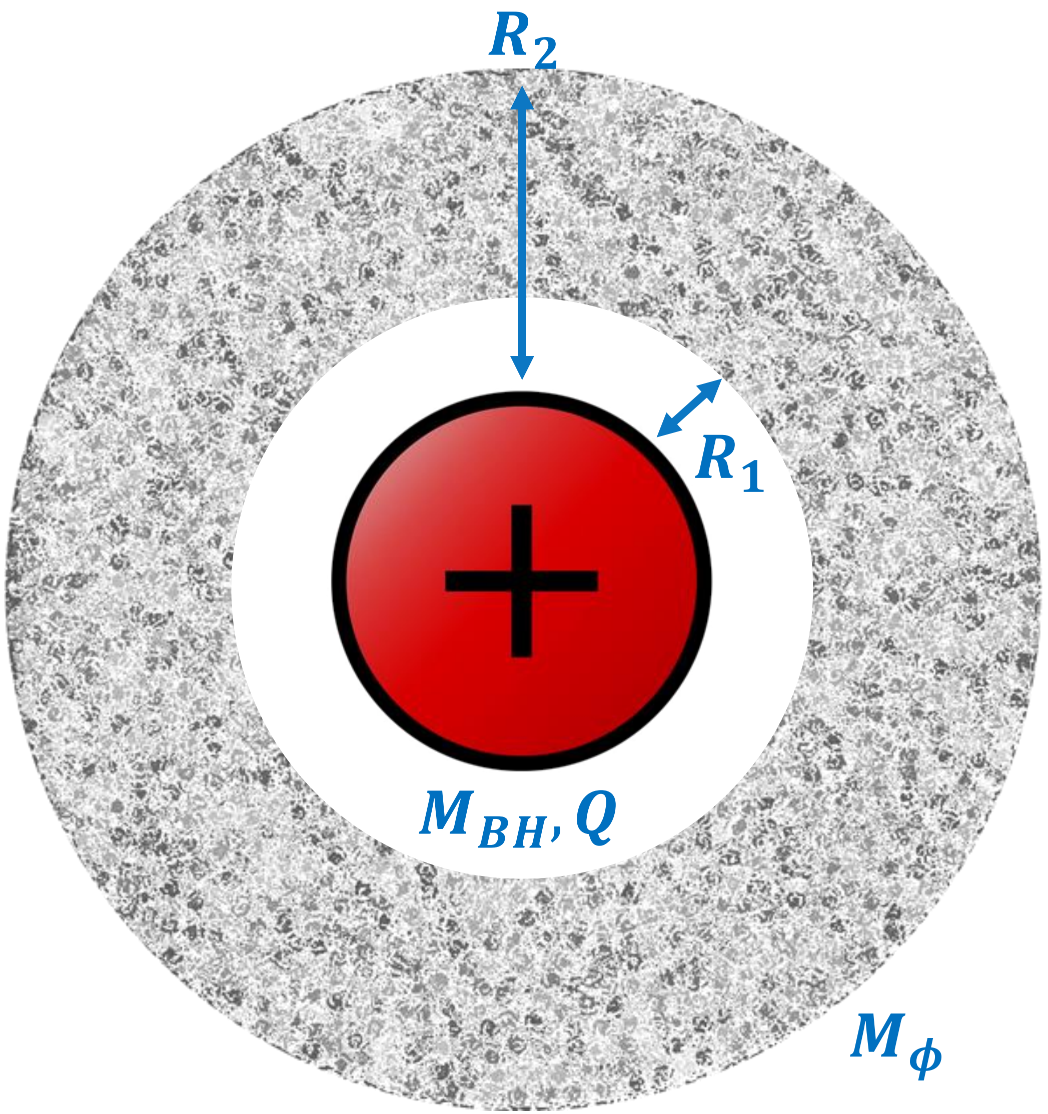


Parameters: $M_{\rm BH}\,, Q\,, M_\phi$



Parameters: $M_{\rm BH}\,, Q\,, M_\phi$

- $\phi \in [R_1, R_2]$, $\mathrm{Im}(\phi) = 0$



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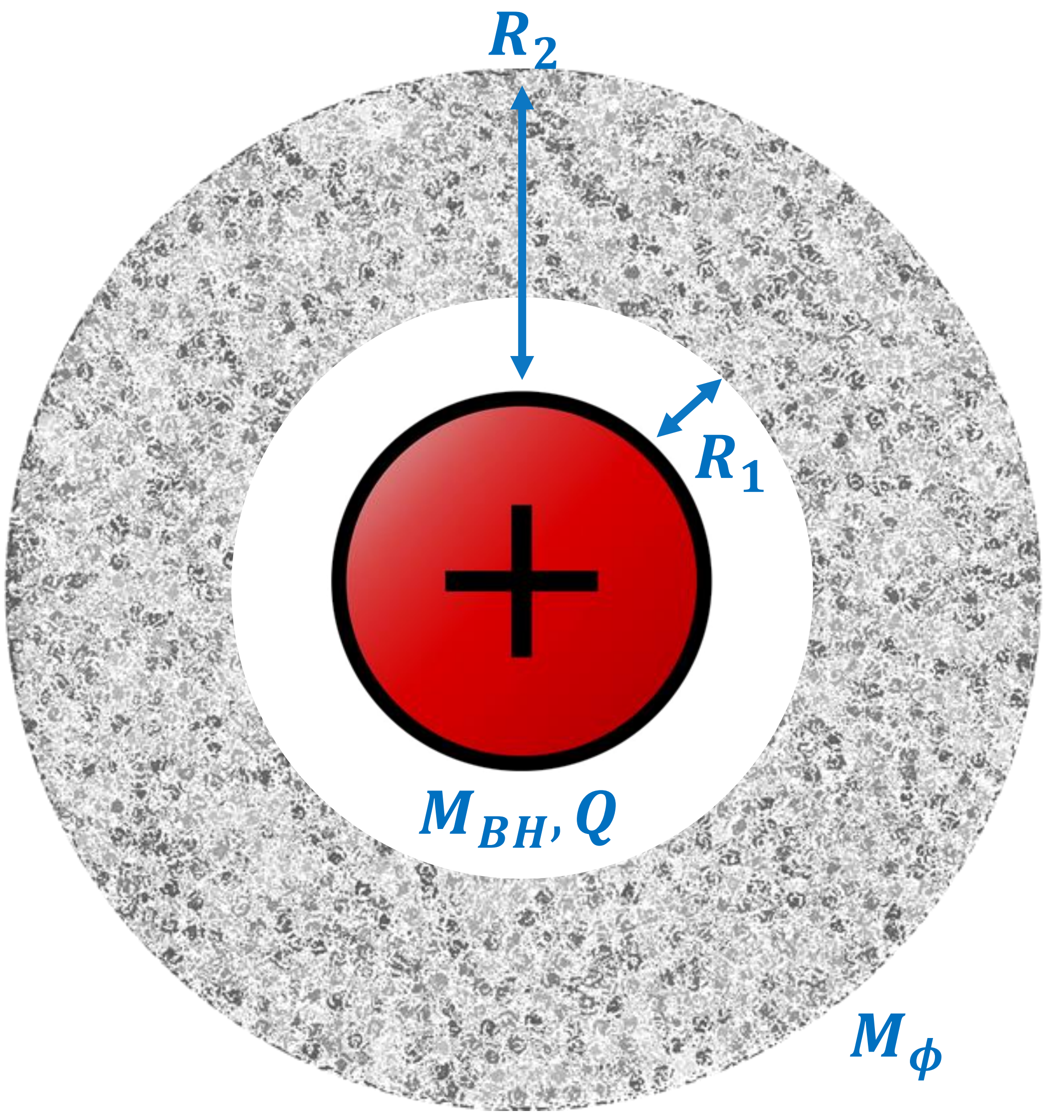
- $K_\phi = 0$



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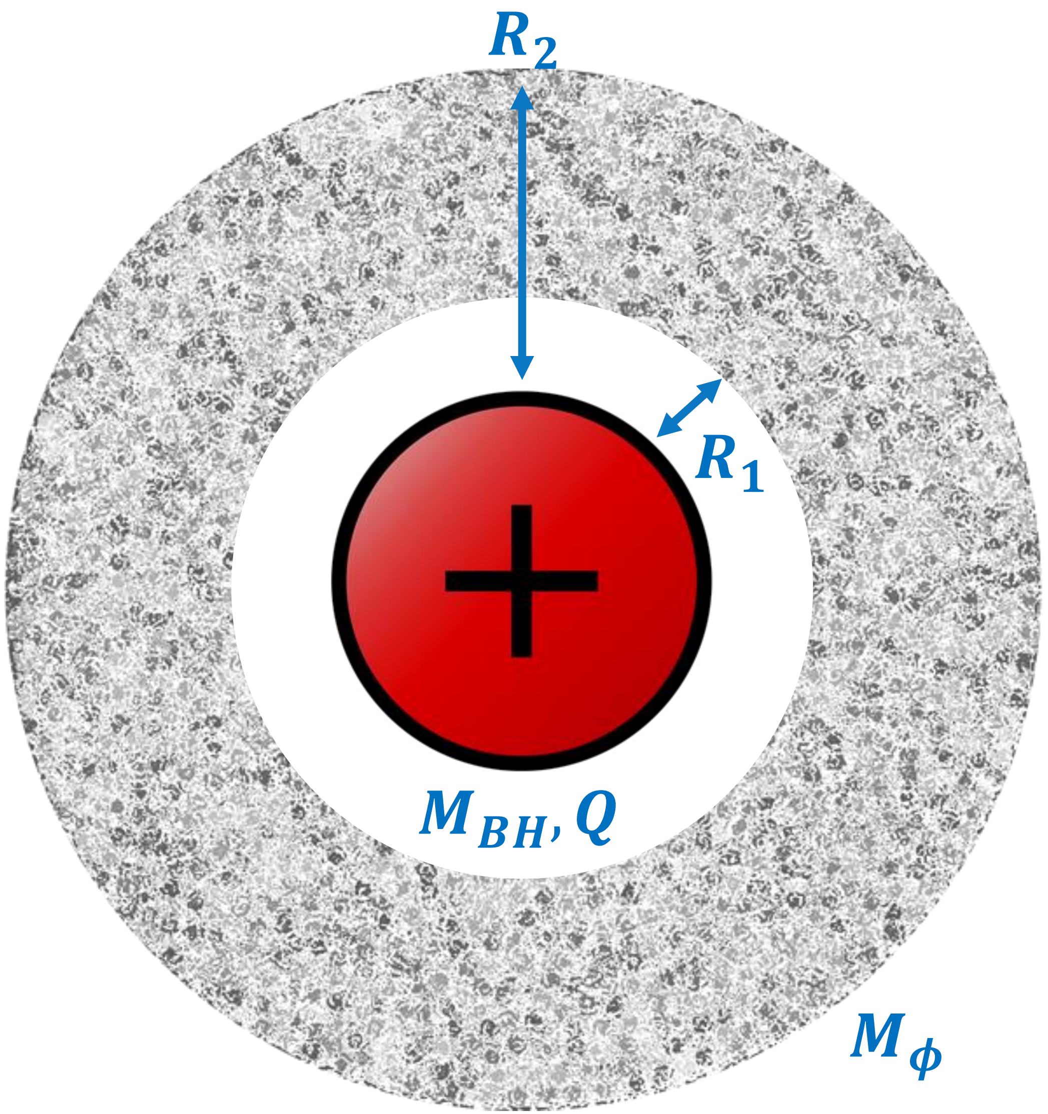
- $\gamma_{ij} = \psi^4 \eta_{ij}$



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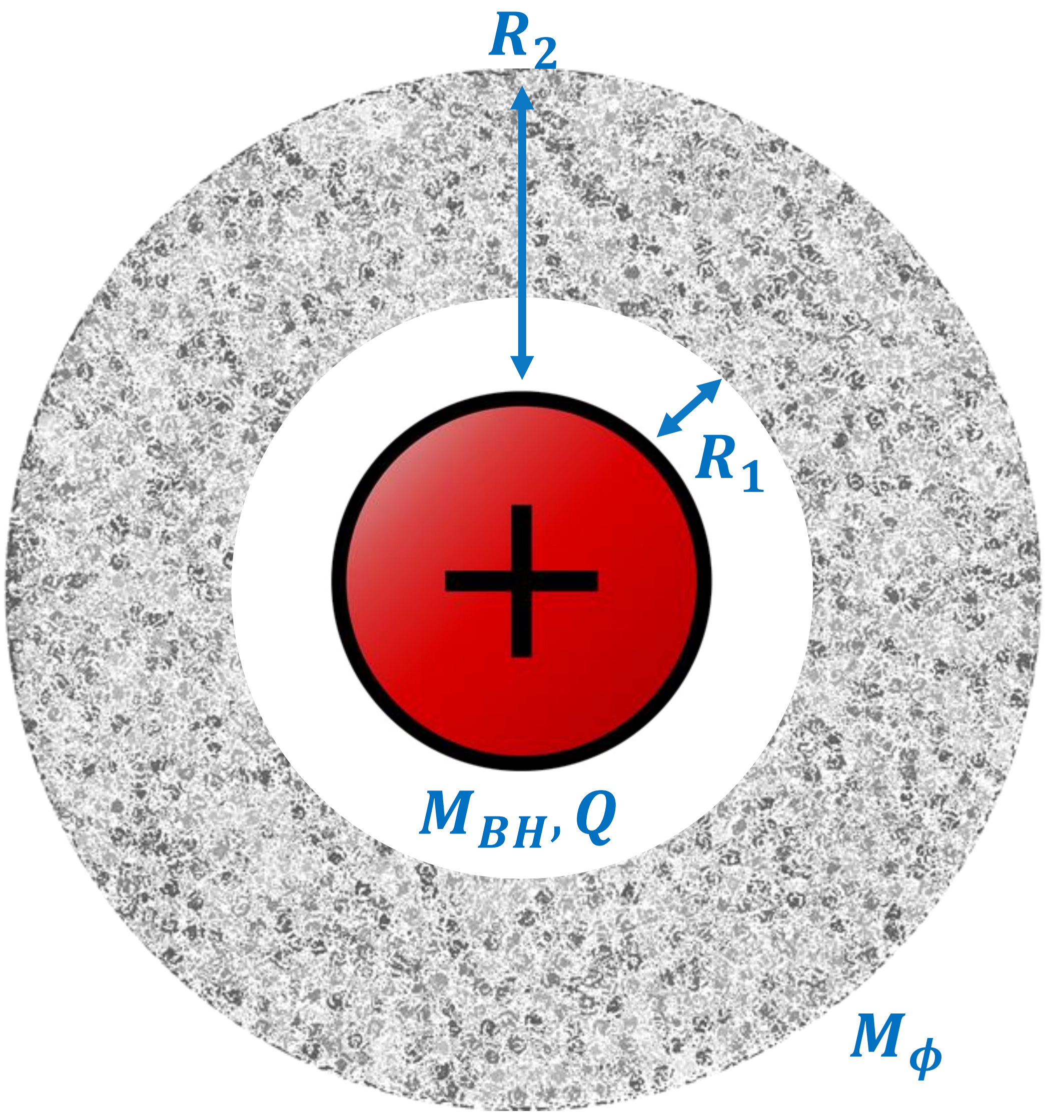
- $E^i = \dfrac{Q}{\sqrt{4\pi} R^3} \dfrac{x^i}{\psi^6}$



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$$\frac{1}{R}\partial_R^2(R\psi) = -\psi^{-3}\frac{Q^2}{4R^4} - 2\pi\psi(\partial_R\phi)^2 + 2\pi\psi^5 V(|\phi|)$$

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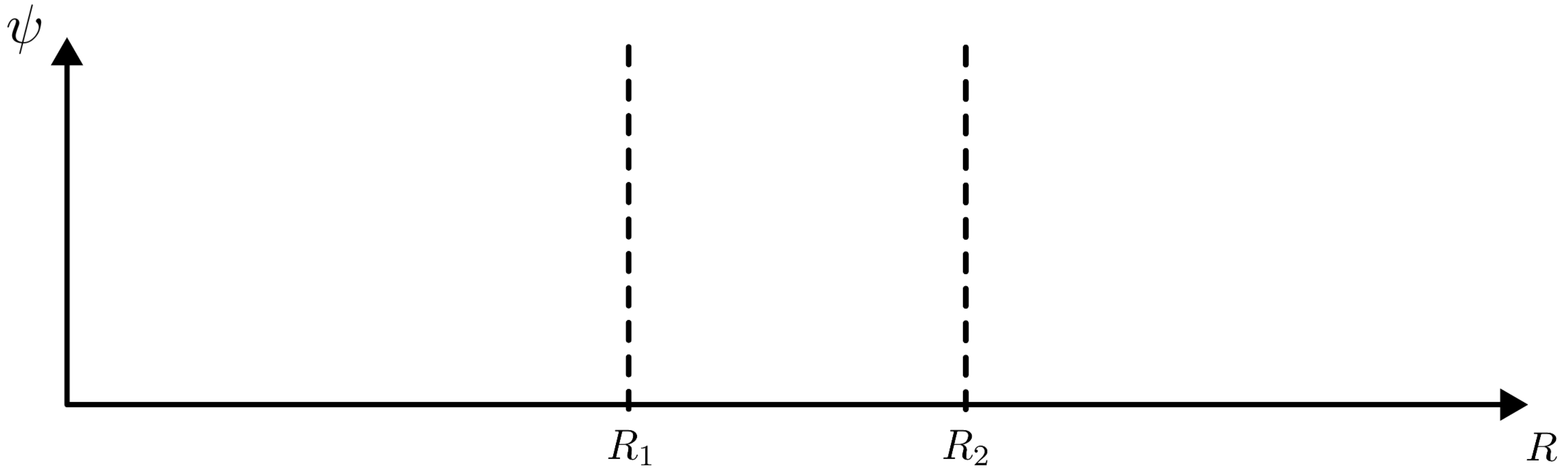

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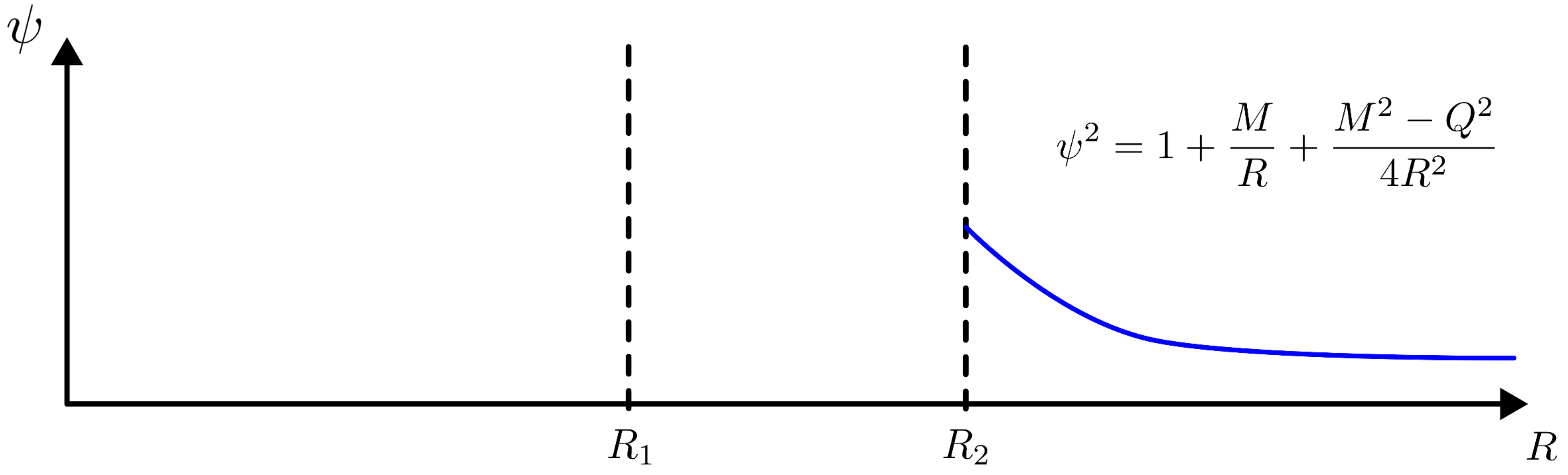

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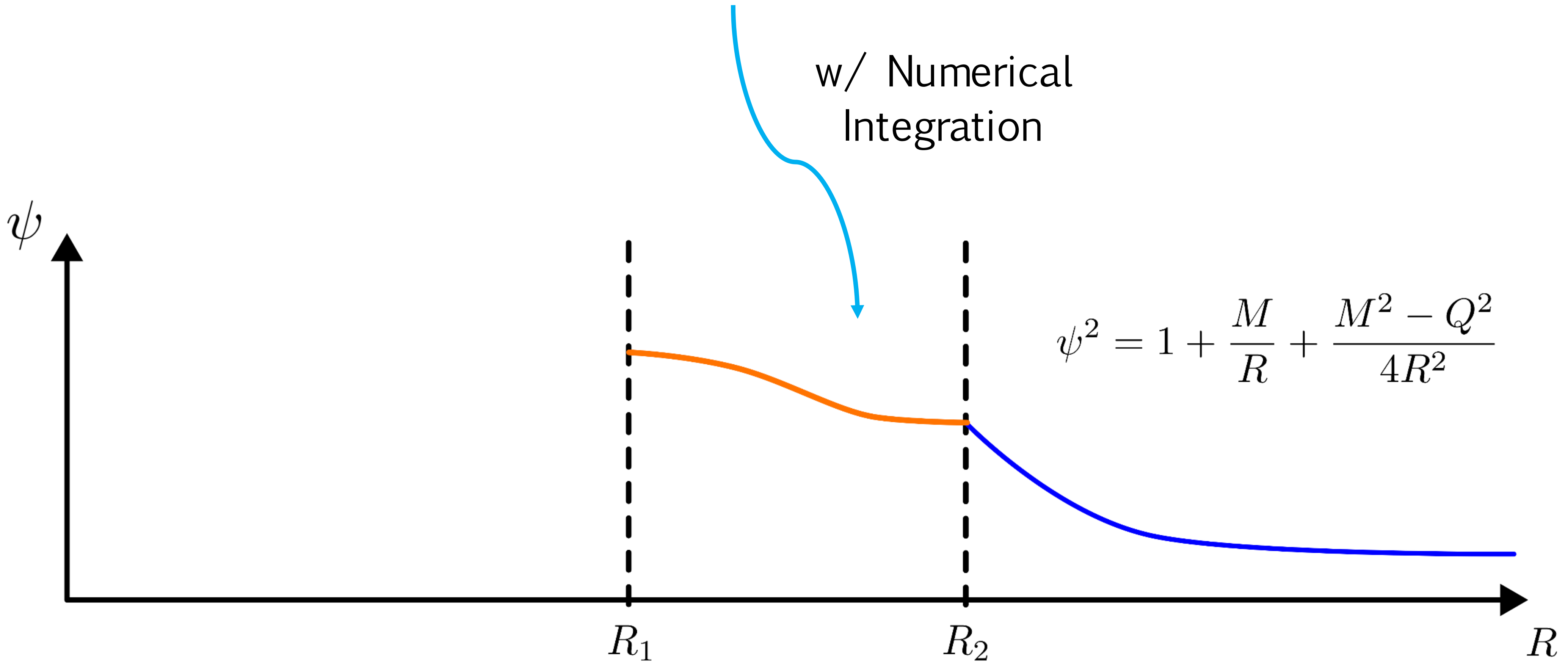

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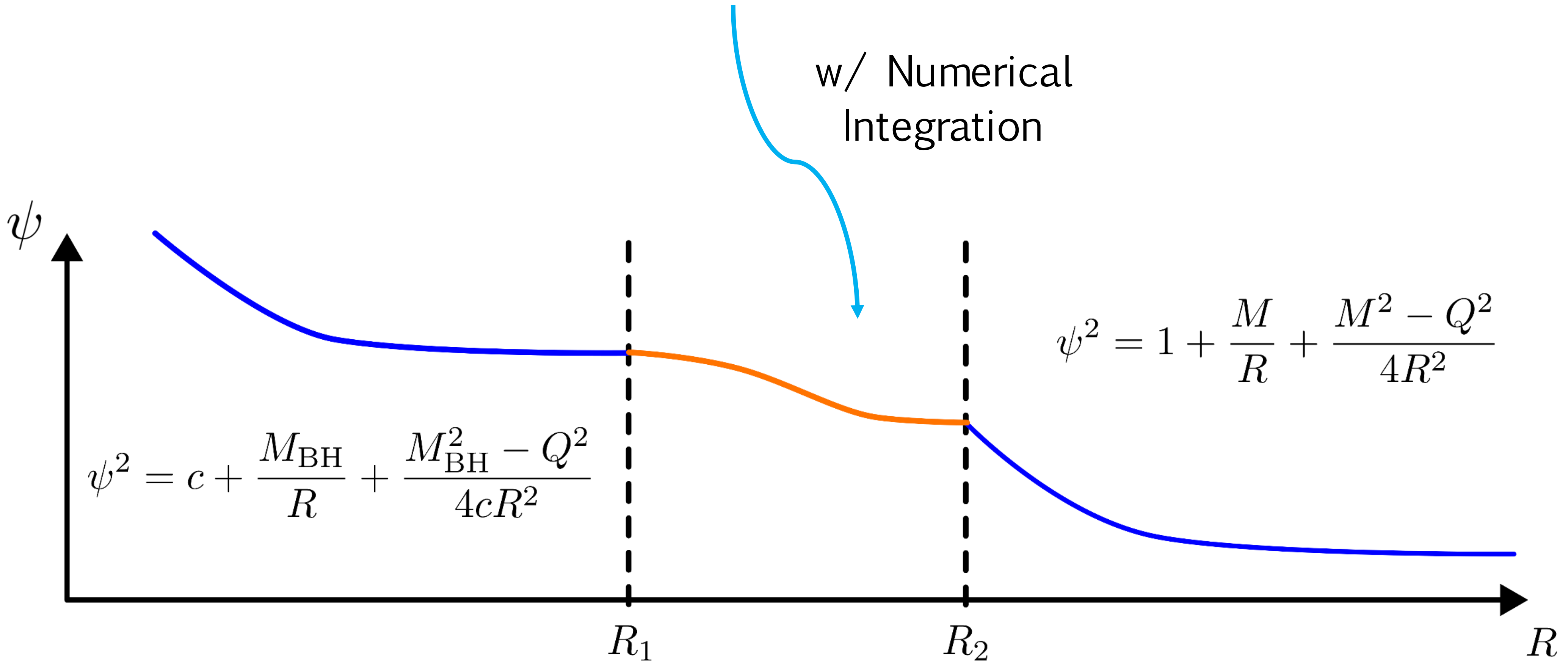
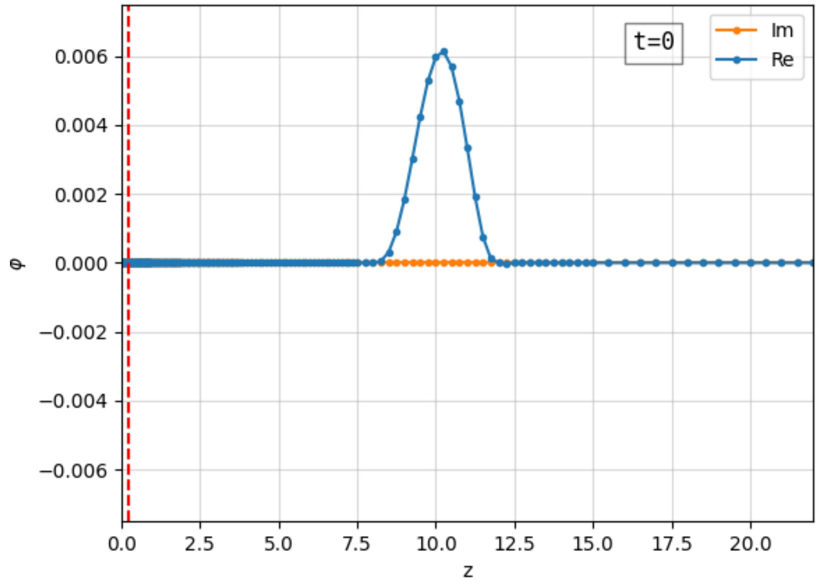


3. Results

- $M_{BH} = 0.85$
- $Q = 0.75$
- $M_{\phi} = 0.15$

$q = 20$

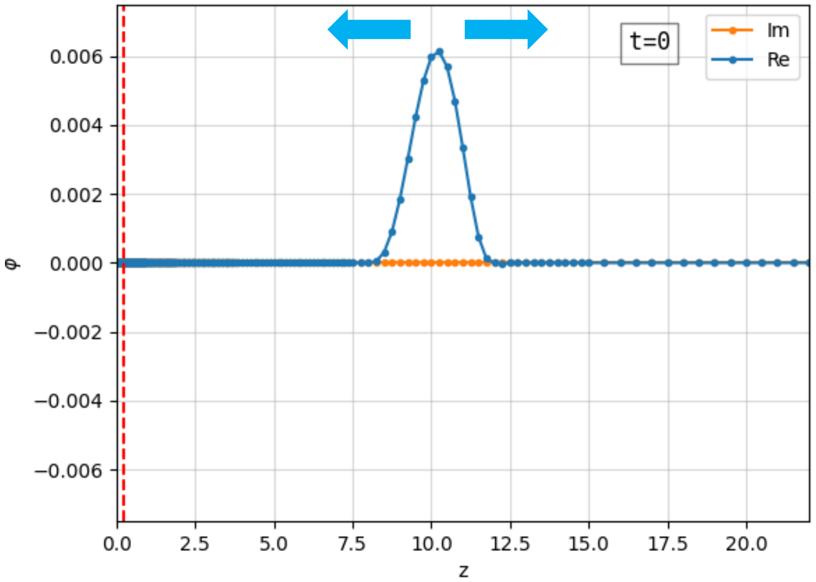
$t = 0$



t=0
Im
Re
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.006
φ
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
z

$q = 20$

$t = 0$


t=0
Im
Re
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.006
φ
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
z

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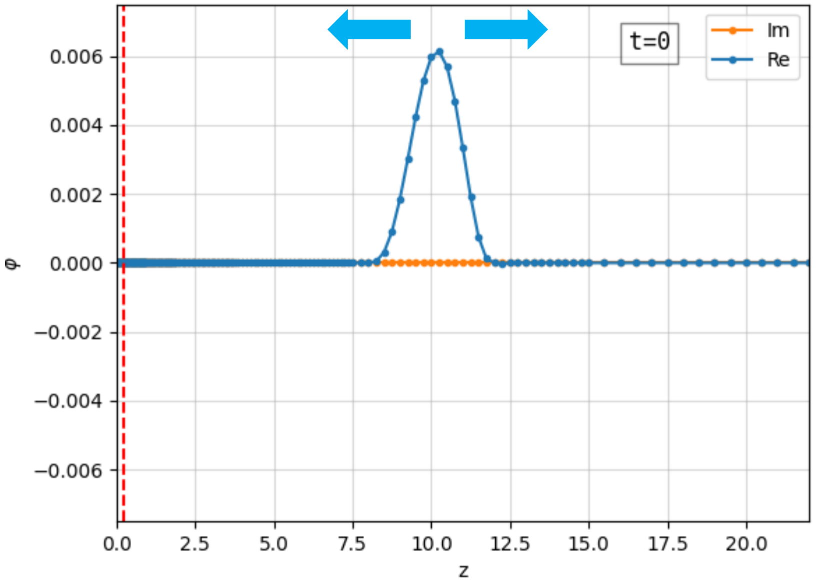
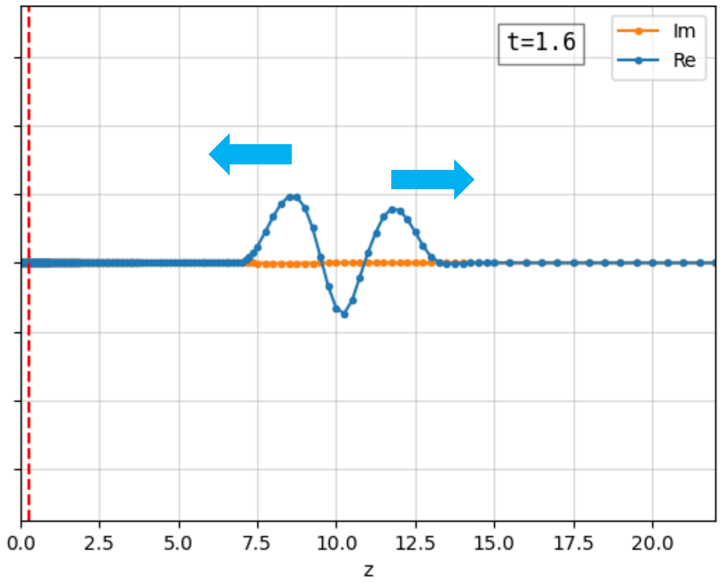
$t = 0$

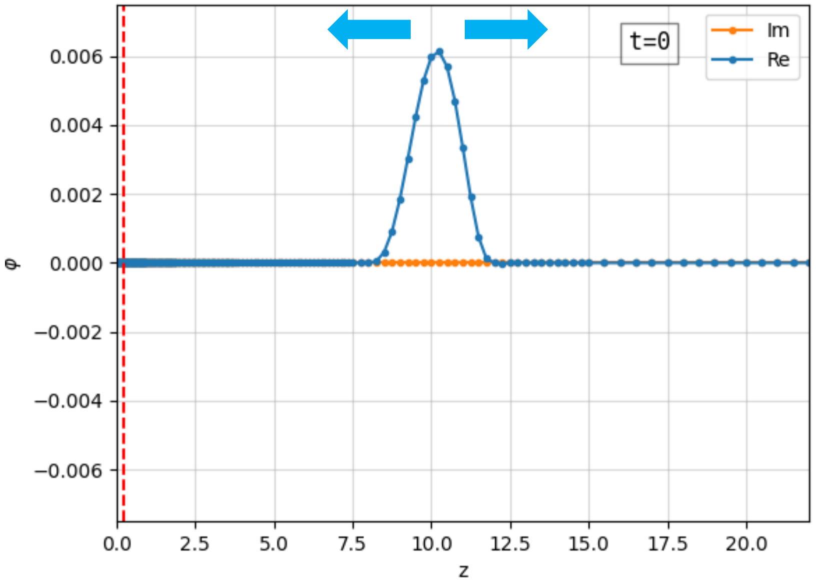

t=0
Im
Re
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.006
φ
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
z

$t = 1.6$

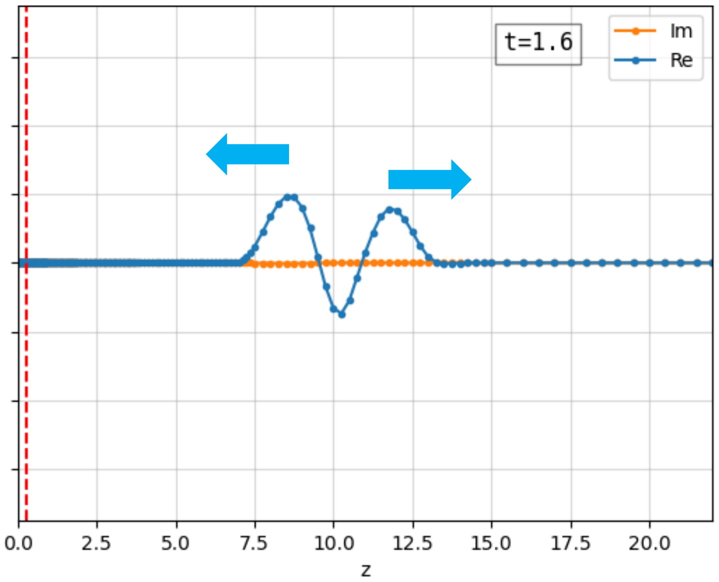

t=1.6
Im
Re
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
z

$q = 20$

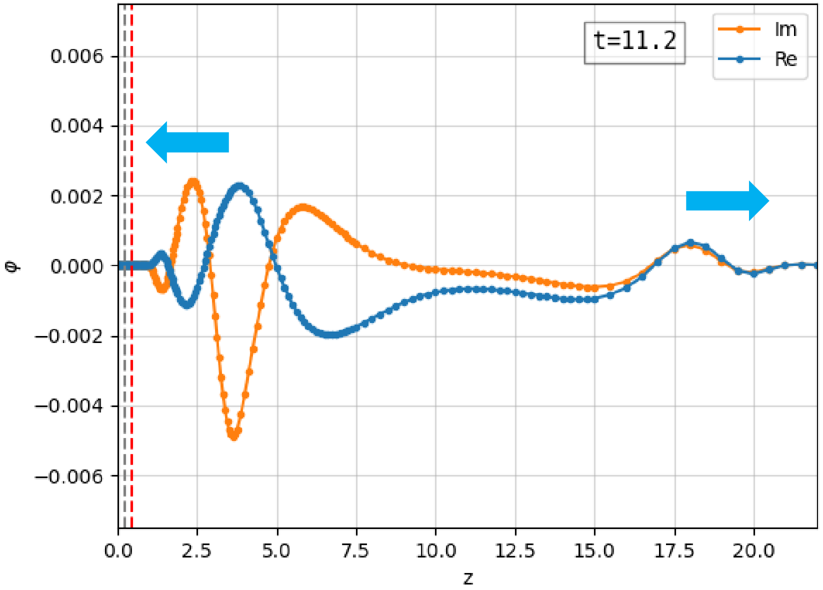
$t = 0$



$t = 1.6$

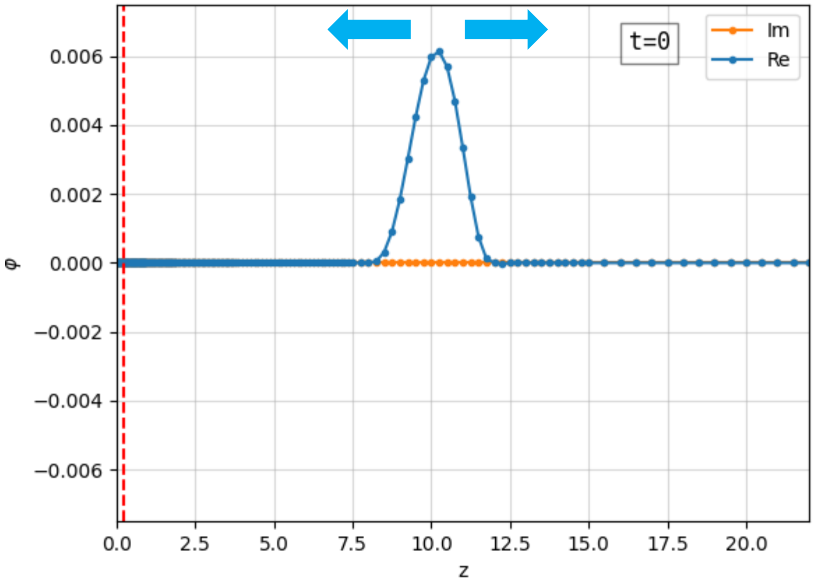


$t = 11.2$

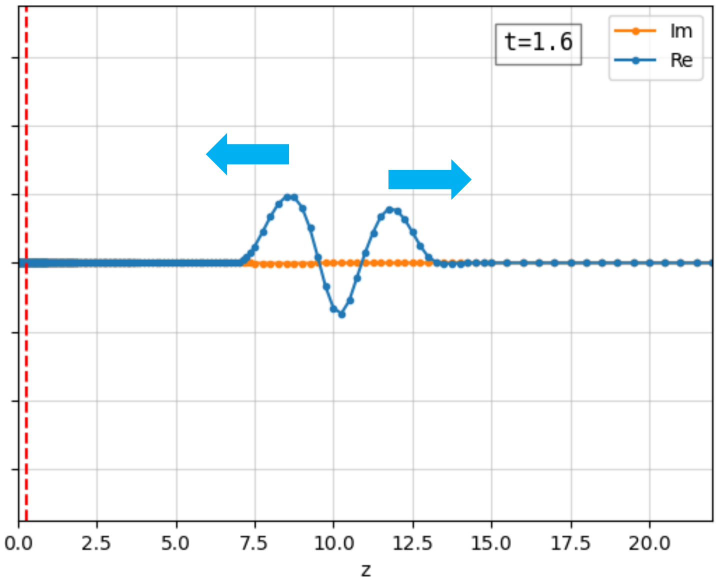


$q = 20$

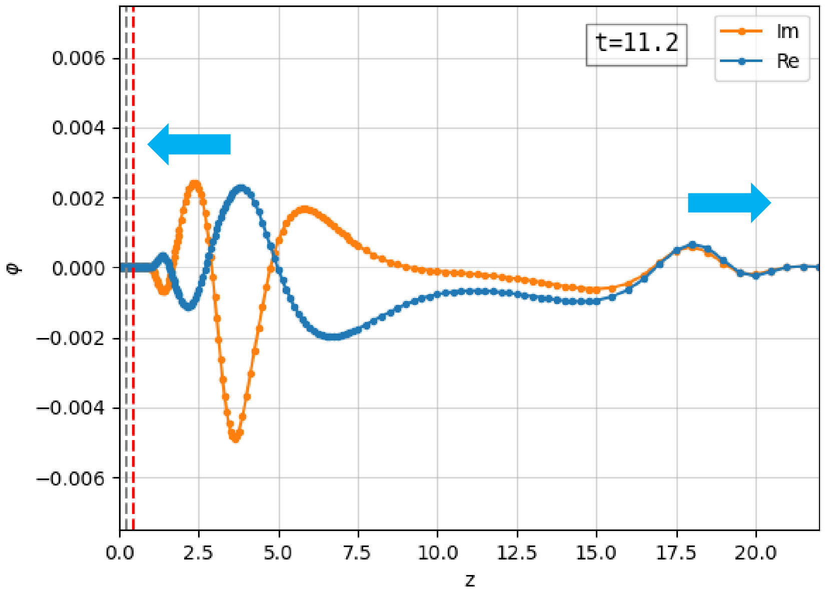
$t = 0$



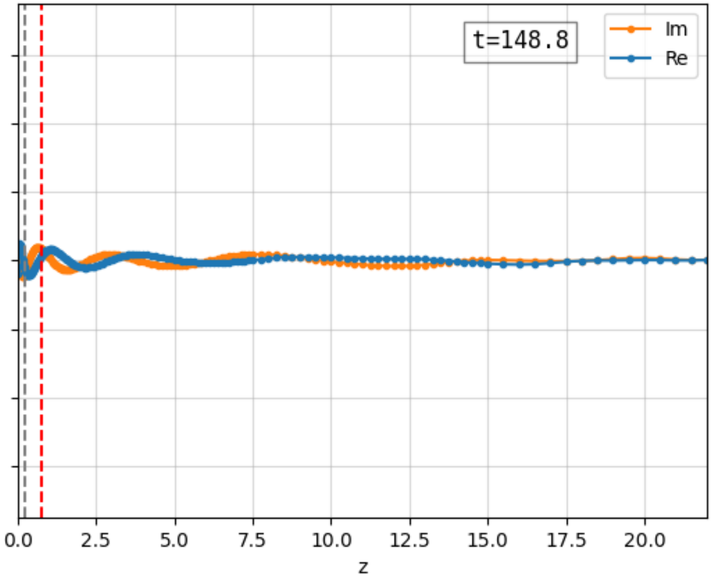
$t = 1.6$



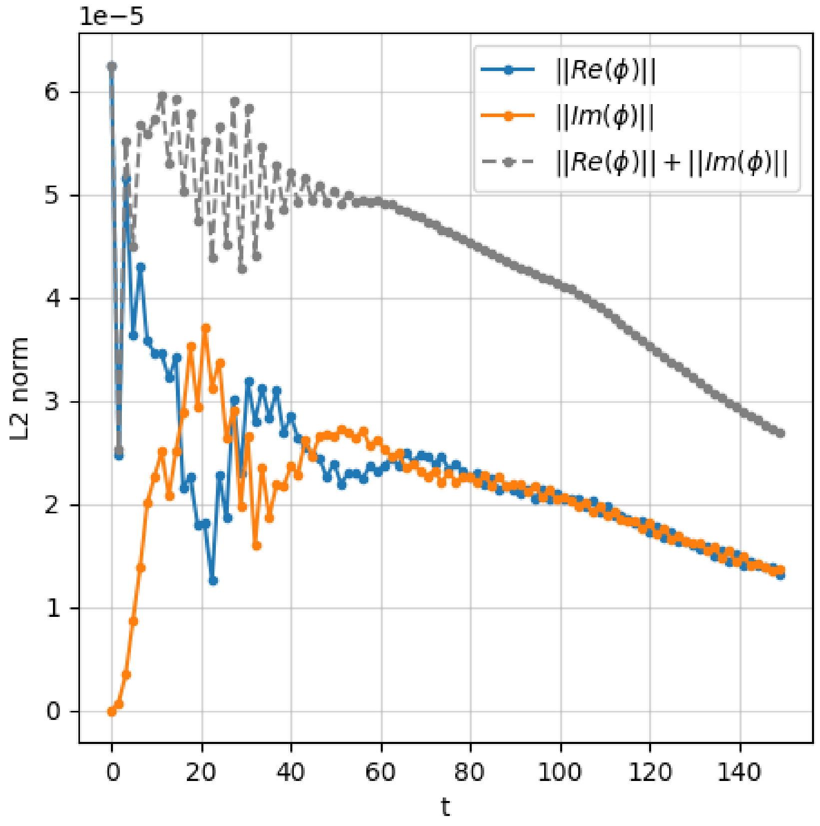
$t = 11.2$



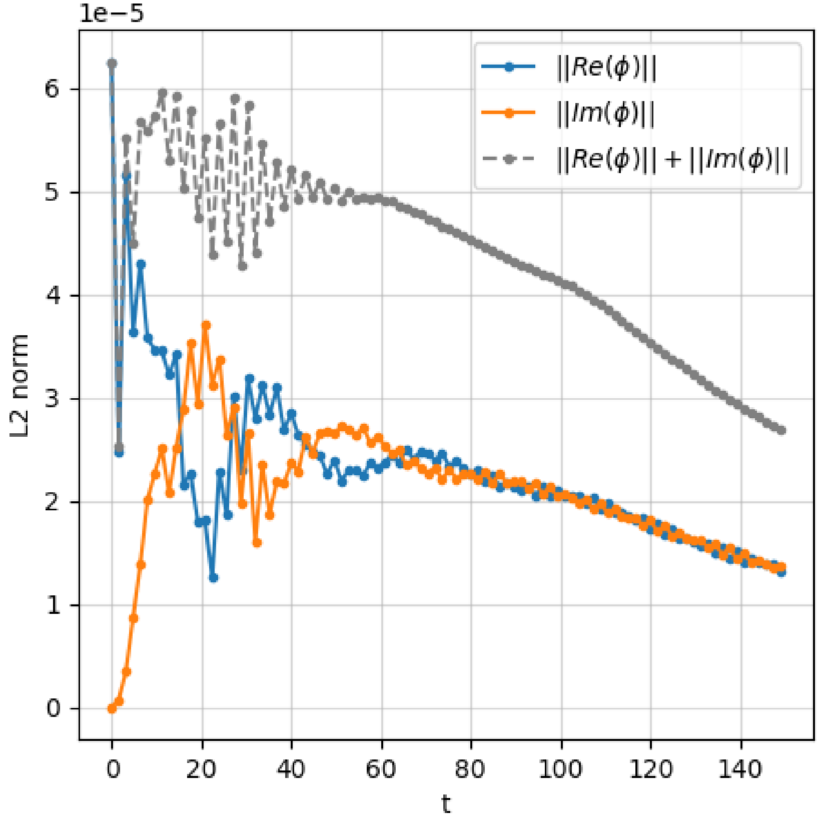
$t = 148.8$



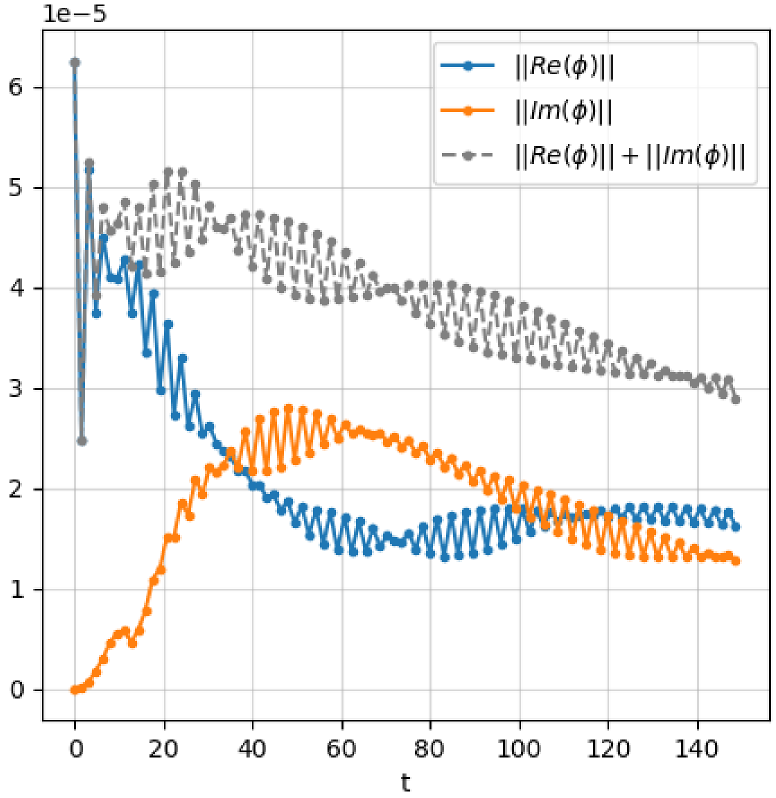
$q = 20$



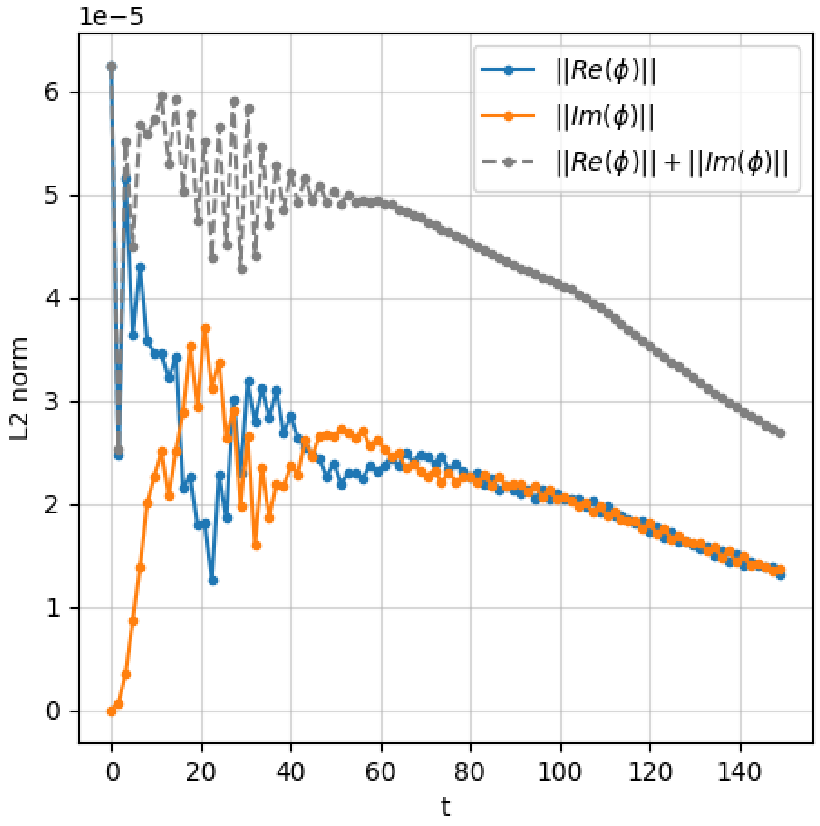
$q = 20$


1e−5
	Re(ϕ)	
	Im(ϕ)	
|Re(ϕ)|| + ||Im(ϕ)|
L2 norm
t
0
1
2
3
4
5
6
0
20
40
60
80
100
120
140

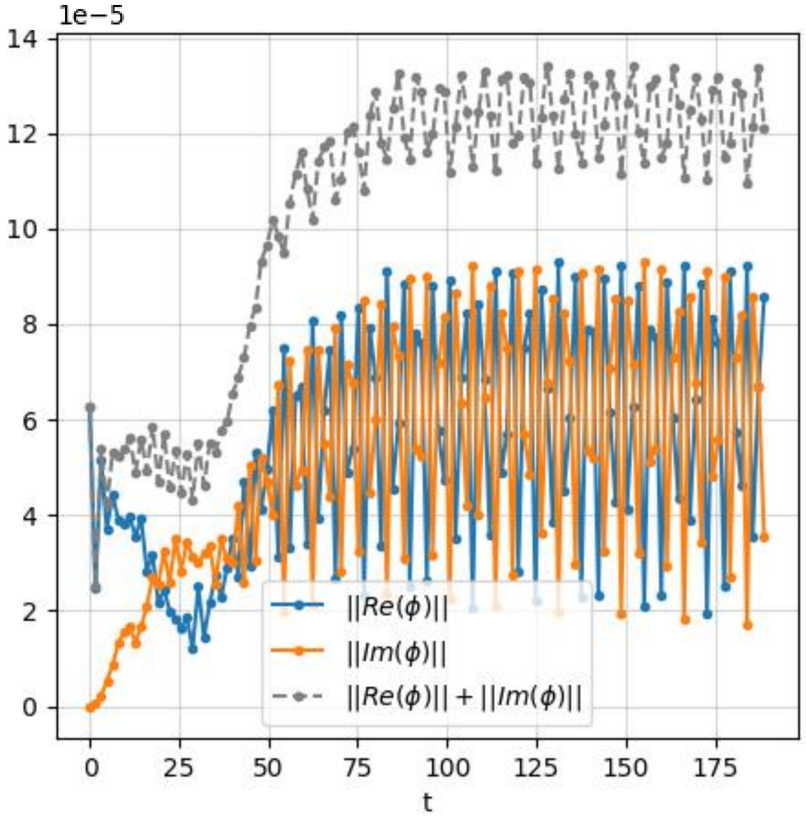
$q = 4$


1e−5
	Re(ϕ)	
	Im(ϕ)	
|Re(ϕ)|| + ||Im(ϕ)|
t
0
1
2
3
4
5
6
0
20
40
60
80
100
120
140

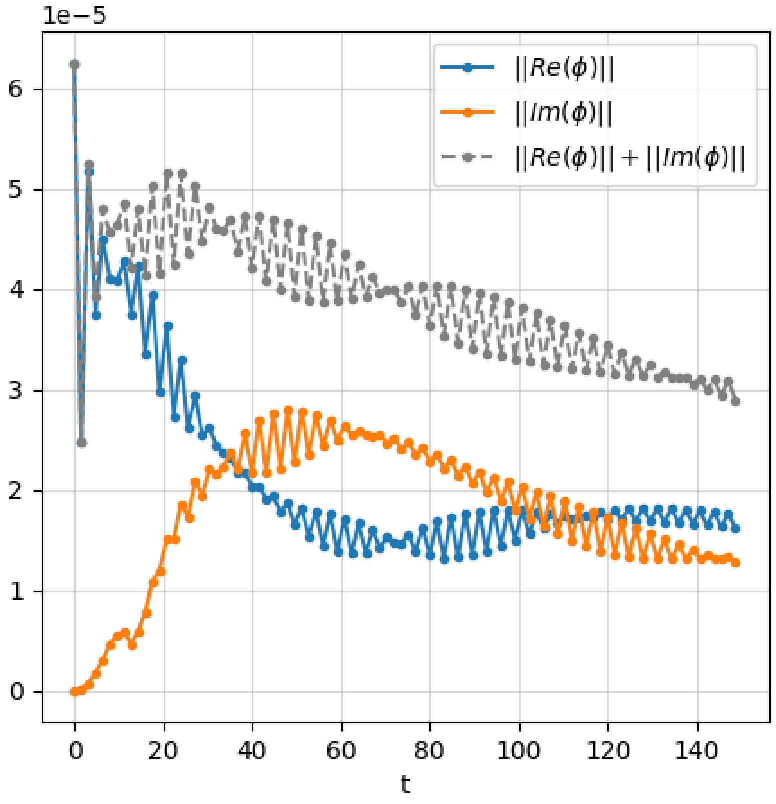
$q = 20$


1e−5
L2 norm
t
	Re(ϕ)	
	Im(ϕ)	
|Re(ϕ)|| + ||Im(ϕ)|

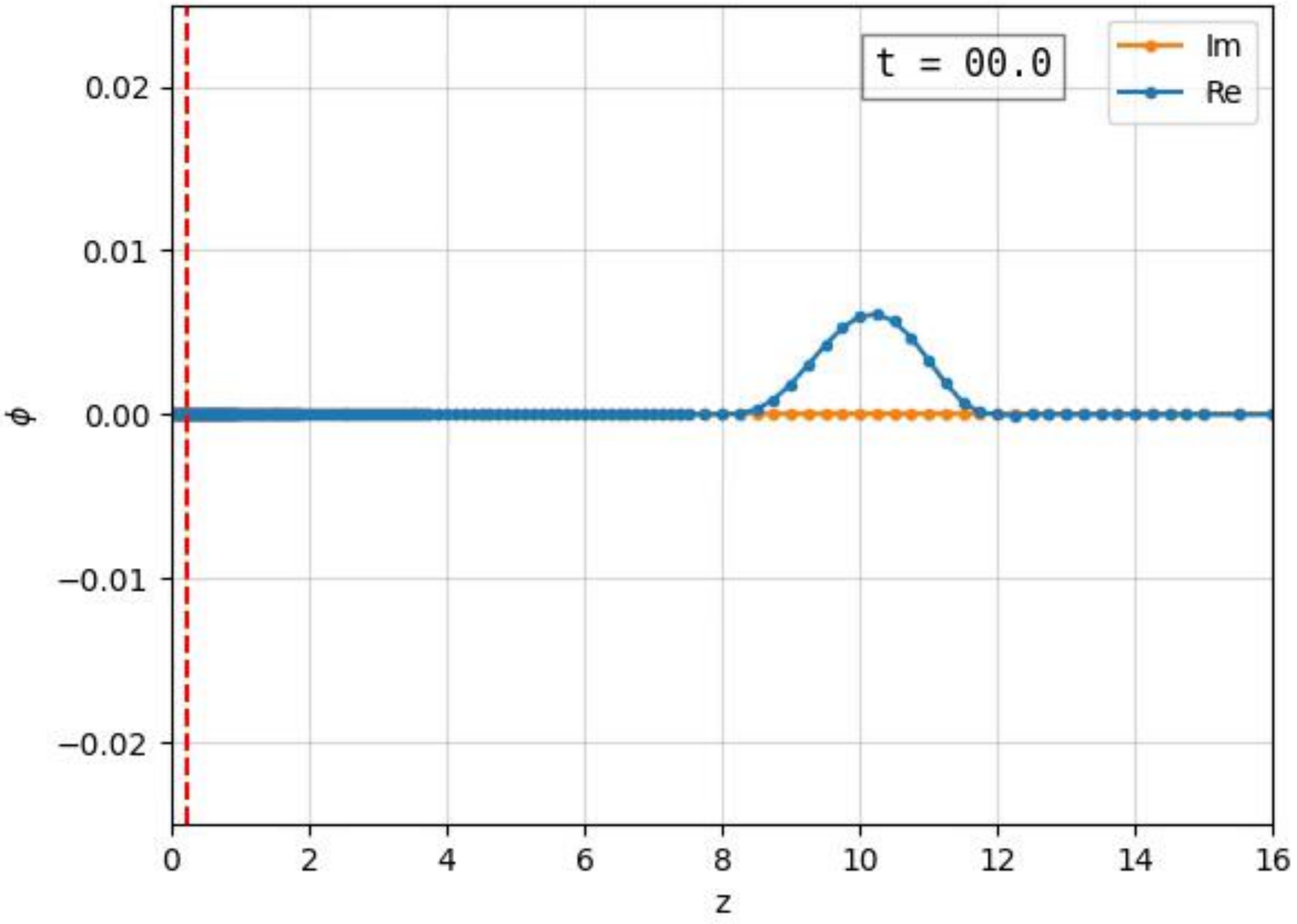
$q = 12$


1e−5
t
	Re(ϕ)	
	Im(ϕ)	
|Re(ϕ)|| + ||Im(ϕ)|

$q = 4$


1e−5
t
	Re(ϕ)	
	Im(ϕ)	
|Re(ϕ)|| + ||Im(ϕ)|

$q = 12$



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Thank you

Questions?