Exploring the motion of photons in spacetimes sourced by nonlinear electrodynamics

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Marco de Paula, Haroldo Junior, Pedro Cunha, and Luís Crispino
PhD Student – PPGF/UFPA

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The action associated with Maxwell's theory can be written as

$$S = \frac{1}{16\pi}\int F d^4x, \qquad (1)$$

$F \rightarrow \mathcal{L}(F)$

from which we can obtain the **Maxwell's equations**, namely

$$\vec{\nabla}\cdot\vec{E} = 0, \qquad (2a)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}, \qquad (2b)$$

$$\vec{\nabla}\cdot\vec{B} = 0, \qquad (2c)$$

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t}. \qquad (2d)$$

In **absence** of **charges** and **currents**!

In this case,

$$S = \frac{1}{16\pi} \int \mathcal{L}(F) d^4x, \tag{3}$$

and we can obtain the **nonlinear Maxwell's equations**:

$$\vec{\nabla} \cdot \vec{E} = j^0_{\text{NED}}, \tag{3a}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}, \tag{3b}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \tag{3c}$$

$$\vec{\nabla} \times \vec{B} = \vec{j}_{\text{NED}} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}. \tag{3d}$$

According to NED, photons obey

$$p_\mu p_\nu \bar{\eta}^{\mu\nu} = 0, \tag{4}$$

Effective metric

where

$$\bar{\eta}^{\mu\nu} = \mathcal{L}_F \eta^{\mu\nu} - 4\mathcal{L}_{FF} F^{\mu}{}_{\sigma} F^{\sigma\nu}. \tag{5}$$

Standard metric

The spacetime (Dymnikova, 2004):

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \quad (5)$$

where

$$f(r) = 1 - \frac{4M}{\pi r}\left[\arctan\left(\frac{r}{z}\right) - \frac{rz}{r^2+z^2}\right], \quad (6)$$

with z given by

$$z \equiv \frac{\pi Q^2}{8M}. \quad (7)$$

Considering a **local static observer**, we can show that

$$g^{\mu\nu}p_\mu p_\nu \leq 0. \quad (8)$$

Let us place the static observer at a finite radial coordinate, the shadow radius can be written as

$$r_s = r_0 \sin\beta_l, \tag{9}$$

where

Observer's position

$$\sin\beta_l = \frac{b_l\sqrt{r_0 + z^2}\sqrt{f_0}}{\sqrt{r_0\left(r_0 + z^2\right) + 3b_l^2 z^2 f_0}}. \tag{10}$$

If we place the observer at infinity, we obtain

$$r_s^{\mathrm{obs}} = \lim_{r_0\to\infty} r_0 \sin\beta_l = b_l. \tag{11}$$

Critical impact parameter

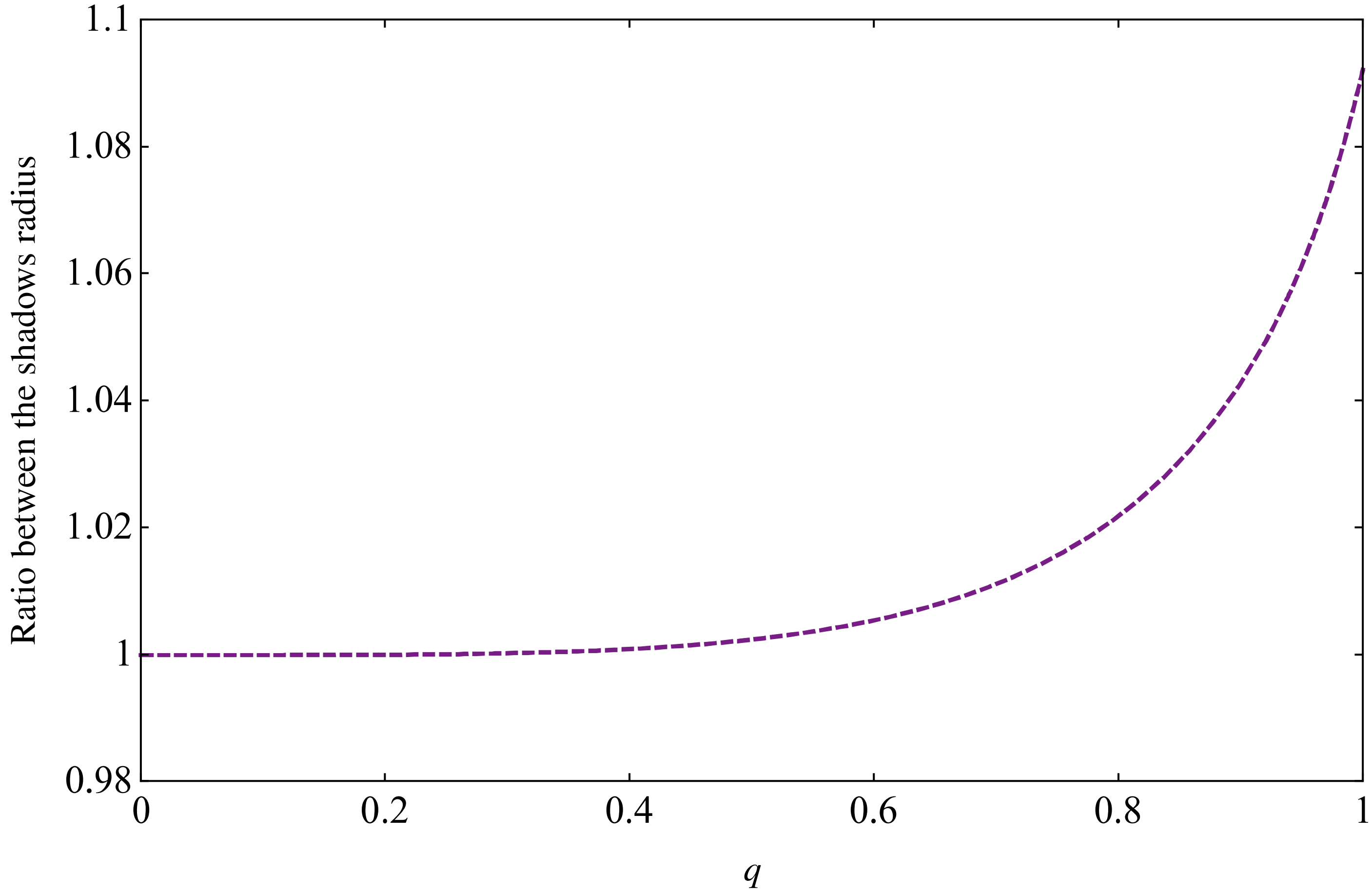


Fig. 1: Ratio between the shadow radius of effective and standard geometries. For simplicity, we have defined $q \equiv Q/Q_{\text{ext}}$, where $Q_{\text{ext}} = 1.07305M$.

The geodesic equations for the effective geometry:

$$\ddot{x} + \bar{\Gamma}^{\mu}{}_{\nu\alpha}\dot{x}^{\nu}\dot{x}^{\alpha} = 0, \tag{12}$$

where

$$\bar{\Gamma}^{\mu}{}_{\nu\alpha} \equiv \frac{1}{2}\bar{g}^{\mu\beta}\left(\partial_{\nu}\bar{g}_{\beta\alpha} + \partial_{\alpha}\bar{g}_{\nu\beta} - \partial_{\beta}\bar{g}_{\nu\alpha}\right). \tag{13}$$

However,

$$\bar{g}^{\mu\nu} = \mathcal{L}_F g^{\mu\nu} - 4\mathcal{L}_{FF}F^{\mu}{}_{\sigma}F^{\sigma\nu}. \tag{14}$$

Therefore, for a static observer in the standard geometry:

$$\ddot{x} + \Gamma^{\mu}{}_{\nu\alpha}\dot{x}^{\nu}\dot{x}^{\alpha} = \mathcal{F}^{\mu}{}_{\nu\alpha}. \tag{15}$$

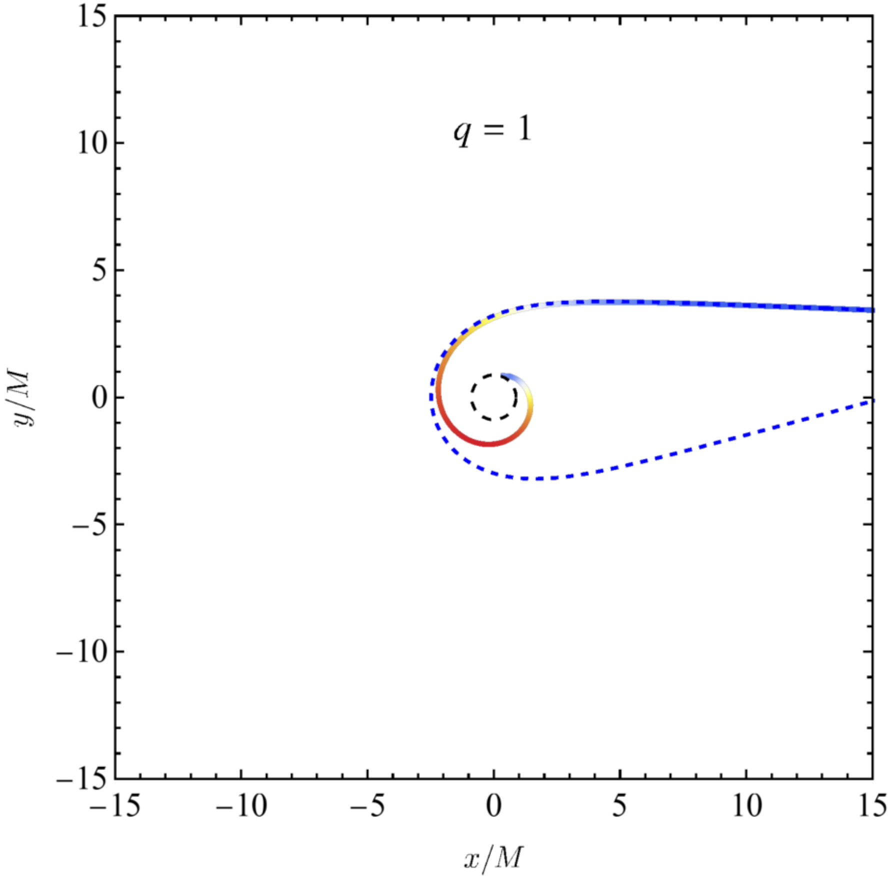
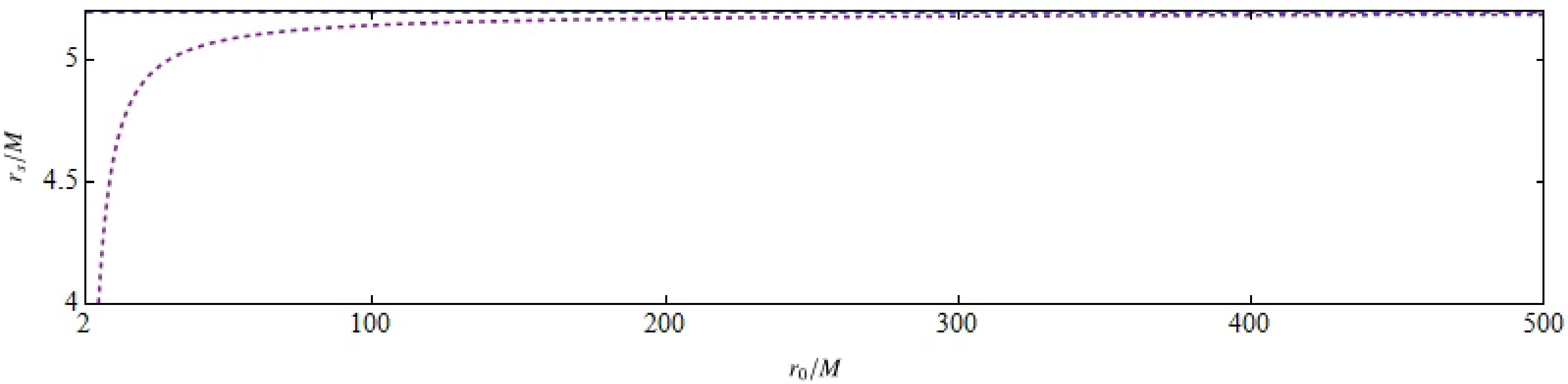


Fig. 2: Comparison between the trajectories of photons (continuous line) and massless particles (dashed line). Here we set $q = 1$, $\beta = 0.04$, and $r_0 = 100M$.

The norm of the four-momentum is non-negative:

$$g^{\mu\nu}p_\mu p_\nu \leq 0.$$

There is a non-trivial contribution to the shadow radius:



A new interpretation for the effective geometry:

$$\ddot{x} + \Gamma^{\mu}{}_{\nu\alpha}\dot{x}^{\nu}\dot{x}^{\alpha} = \mathcal{F}^{\mu}{}_{\nu\alpha}.$$

THANK YOU ALL!
ANY QUESTIONS OR COMMENTS?

Grav@Zon
Gravity @ Amazonia
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