Same as Ever:
Looking for (In)variants in the Black Holes Landscape

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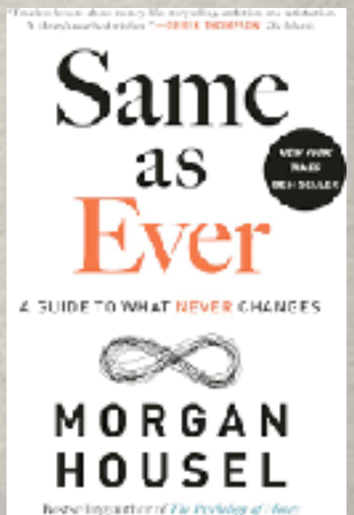
"Black Holes Inside and Out 2024: visions for the future of black hole physics"
arXiv:2410.14414

Prologue

In science one is constantly looking for the next breakthrough,
the next game changer.
It then becomes easy to overlook the basic, which is what is
the same as ever.

The same applies in business.
The Amazon founder Jeff Bezos made the following remark:
I very frequently get the question:
"What's going to change in the next 10 years?"
And that is a very interesting question; it's a very common one. I almost never get the question:
"What's not going to change in the next 10 years?"
And I submit to you that that second question is actually the more important of the two -- because you can build a business strategy around the things that are stable in time.

So what kind of question looks for
``the same as ever”
in our scientific field?



What is general for (equilibrium, single) black holes ?
(and what is model dependent?)

There are paradigmatic black hole (BH) features,
learned from the canonical electro-vacuum General Relativity (GR)
solutions (i.e. Kerr-Newman), e.g.:

- BHs have a spherical horizon topology;
- Spinning BHs have an ergo-region;
- BHs have light rings (and other bound photon orbits); -> Today;
- BHs have no hair. -> Today;

Many of these (and other) features have phenomenological impact.

It is natural to question the generality of such features
in broader classes of models.

Plan of the talk:

balance between generality (desirable) and concreteness (often enlightening)

1) General statements;
- light rings;
- "hair".

2) Selected family of examples;

3) Final comments and outlook.

Comment: In this talk the main focus will be models without (spin 1) gauge invariant fields. Introducing these fields opens many possibilities of non-Kerr(-Newman) BHs.

1) Generic statements

1) Do all black holes have light rings (LRs)?

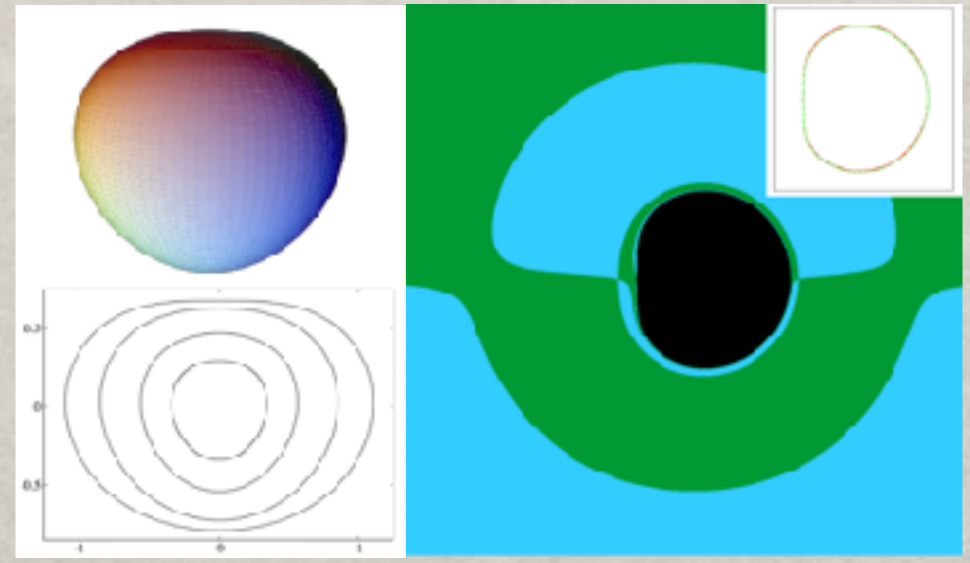
(i.e. “planar”, bound photon orbits?)

Generic statement (theorem):

A stationary, axisymmetric, asymptotically flat BH in D=4, with a non-extremal, topologically spherical Killing horizon admits, at least, one standard LR outside the horizon for each rotation sense. Cunha, CH, PRL 124 (2020) 181101

No field equations imposed.

No Z2 symmetry required.



Imaging a KK BH without Z2 symmetry

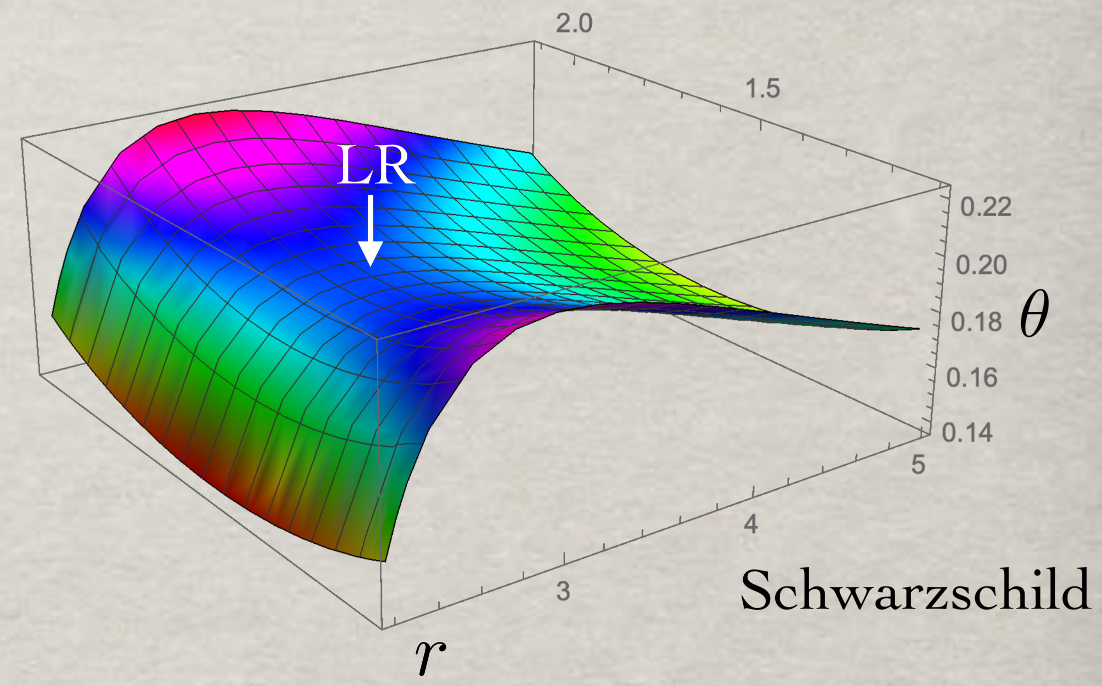
Cunha, CH, Novo, PRD 109 (2024) 064050

The LRs theorem exemplifies a field equations independent statement. It is topological and associates to LRs a topological charge.

Cunha, Berti, CH, PRL 119 (2017) 251102

Central idea:
LRs are *critical points* of two 2D potentials

$$\nabla H_{\pm}\Big|_{\mathrm{LR}} = 0$$



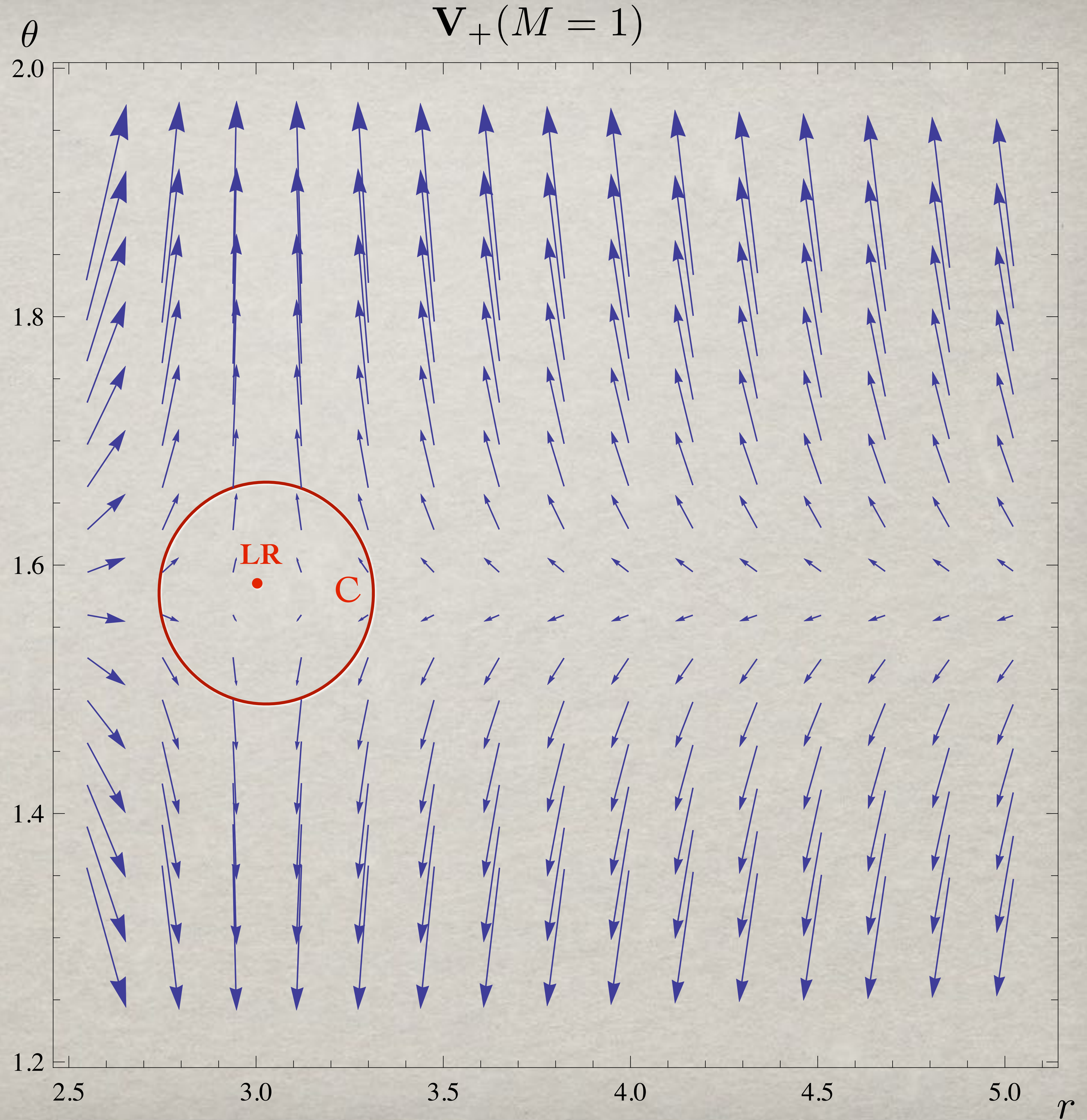
These potentials define vector fields on this 2D space as their gradients:

$$\mathbf{V}_{\pm} = \nabla H_{\pm}$$

Circulating a closed contour in the 2D space, the winding of these vector fields defines an integer **topological charge** w:

$$\oint_C d\Omega = 2\pi w$$

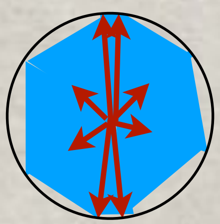
angle of the vector field with some reference direction

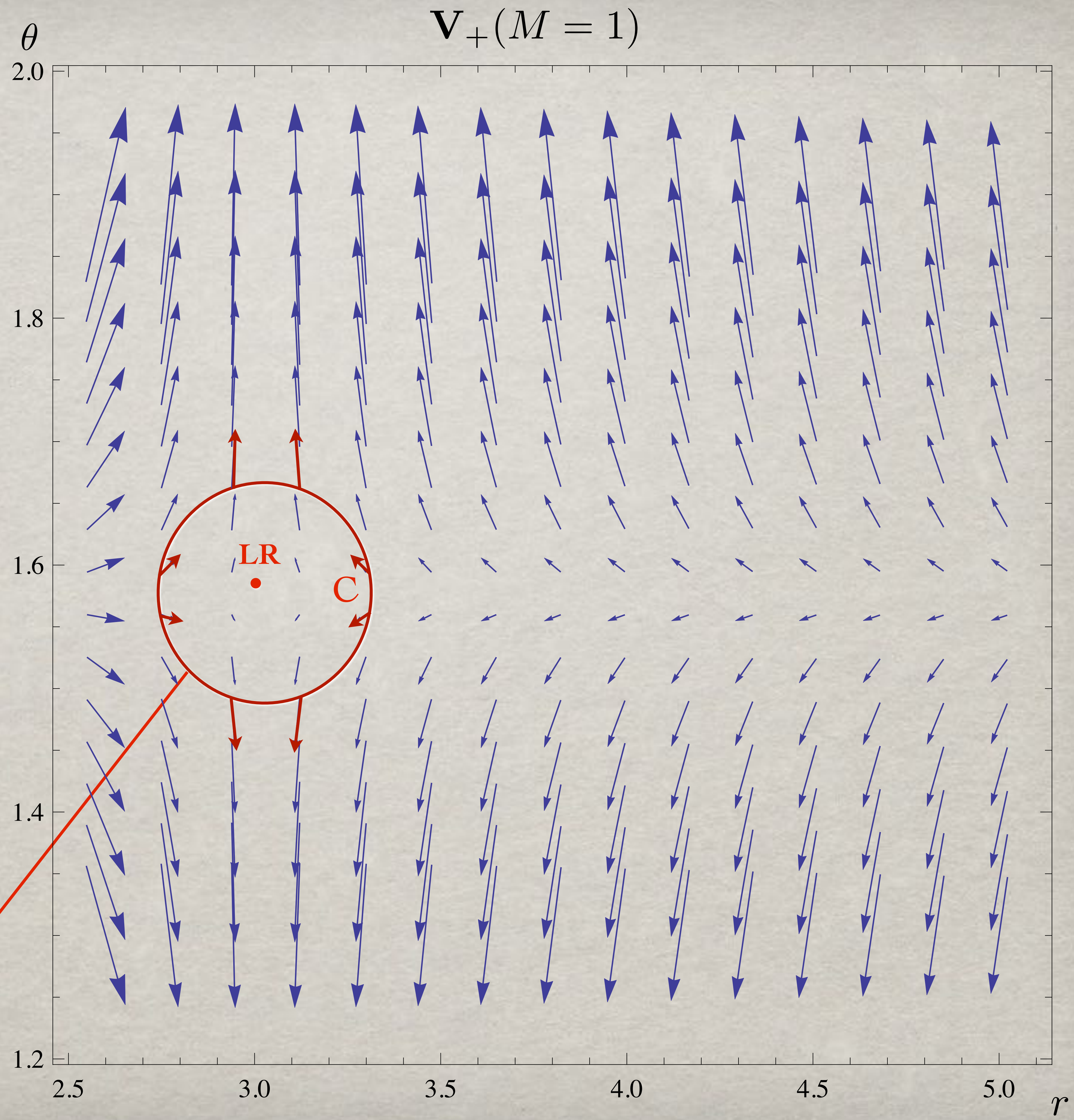


Circulating a closed contour in the 2D space, the winding of these vector fields defines an integer **topological charge** w:

$$\oint_C d\Omega = 2\pi w$$

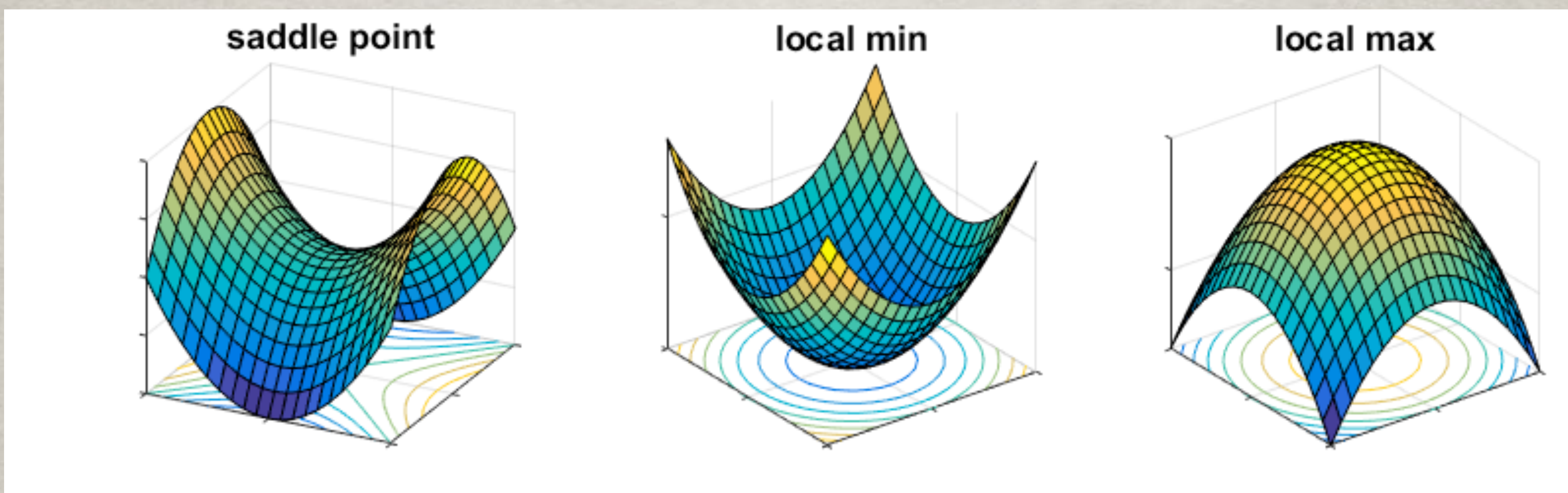
Winding of the vector field





Because the winding opposes the circulation the topological charge is w= -1

But not all possible LRs have w=-1:



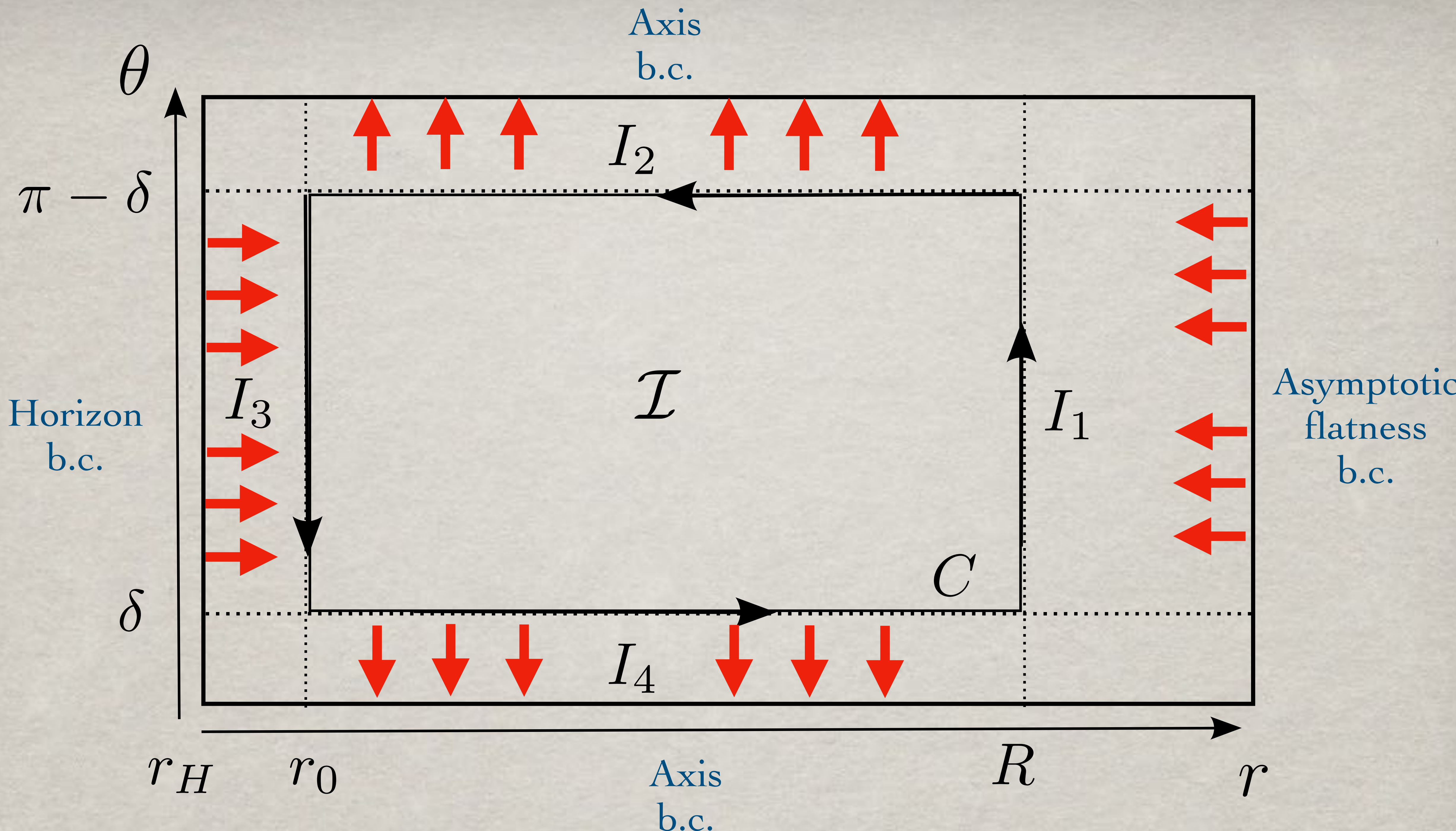
$w = -1$ $\qquad$ $w = +1$ $\qquad$ $w = +1$

Standard LRs
(Kerr-like)

Exotic LRs
(Kerr-unlike)

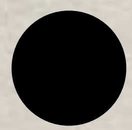
The topological charge is additive:

$$\oint_C d\Omega = 2\pi \sum_i w_i \ , \qquad w_i = -1, 1 \ .$$

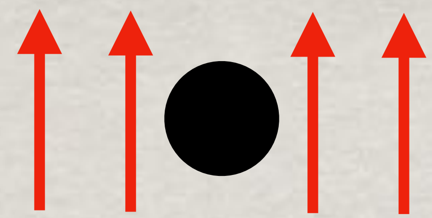


$$w = \lim_{R\to+\infty} \lim_{r_0 \to r_H} \left(\lim_{\delta\to 0} \oint_C d\Omega \right) = -1$$

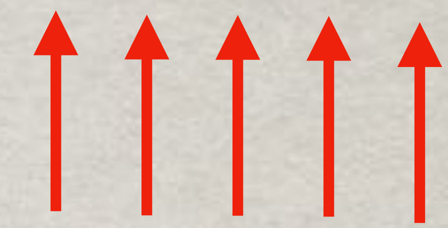
There *are* BHs without LRs.



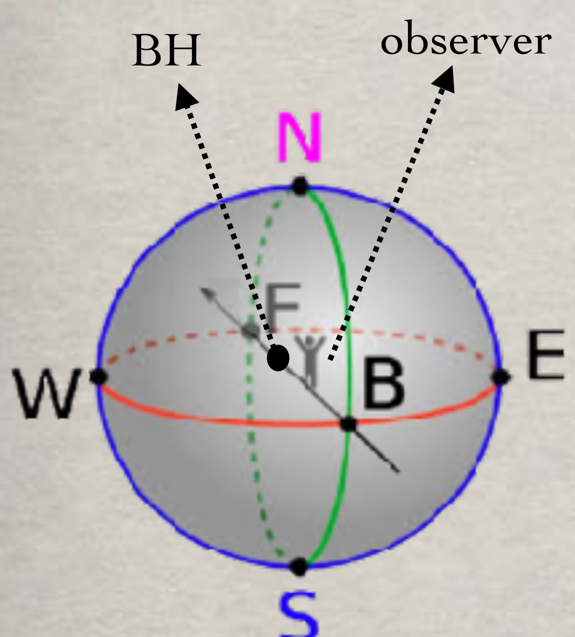
Schwarzschild (mass M)
has w=-1



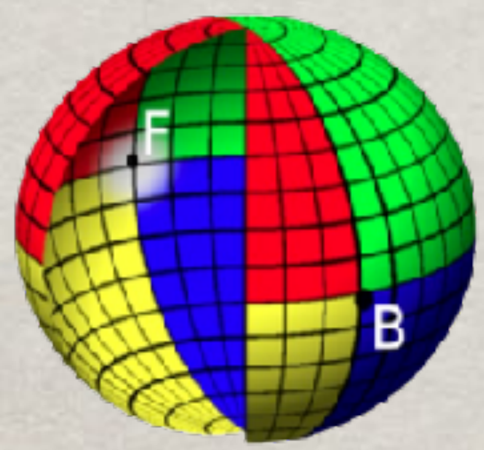
Black Hole in a Melvin Universe has w=0.
For BM large, no LRs.
Júnior et al., PRD 104 (2021) 044018



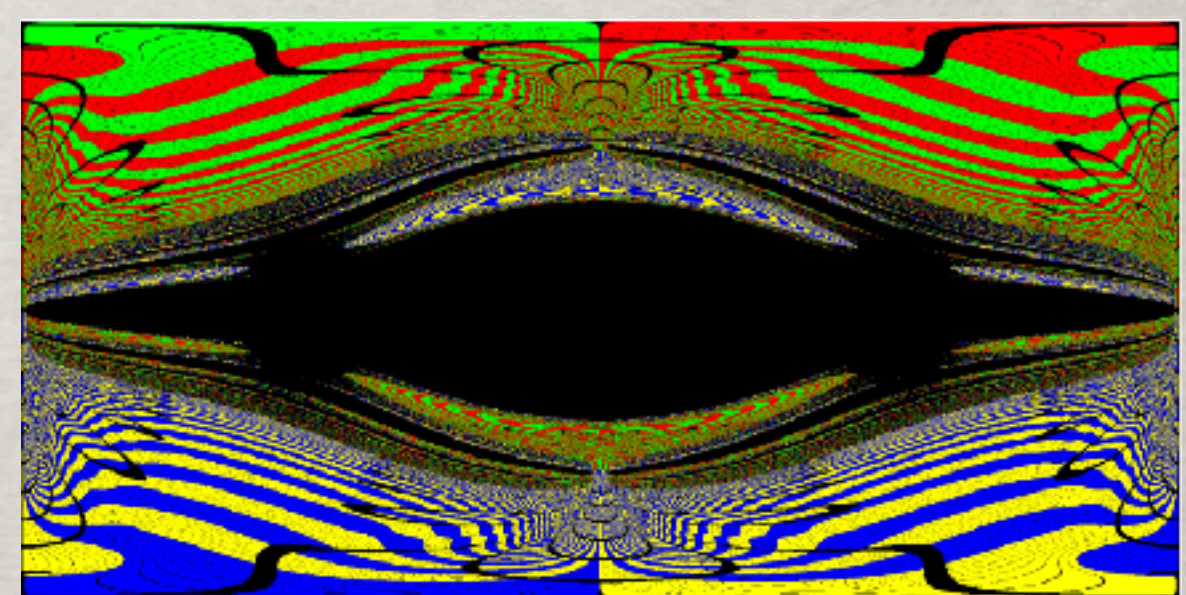
Melvin Universe (magnetic strength B)
has w=+1



An equatorial observer
looking at the BH region…



…within a colourful
celestial sphere…



… observes a panoramic "shadow"
Júnior et al., PRD 105 (2022) 064070

The analysis of LRs, and their non-planar generalisations, *fundamental photon orbits* Cunha, C.H., Radu, PRD 96 (2017) 024039, provides a simple (albeit not precision) diagnosis of how non-Kerr the phenomenology of an alternative model may be.

An informative technique. E.g.:

1) w=-1 also for BHs in different asymptotics (AdS and dS, static BHs) Wei, PRD 102 (2020) 064039; (swirling universes) Moreira, CH, Crispino, PRD 109 (2024) 104020; w=-1 also for traversable wormholes Xavier, CH, Crispino, PRD 109 (2024) 124065

2) w=0 for topologically trivial, asymptotically flat, smooth, horizonless compact objects Cunha, Berti, CH, PRL 119 (2017) 251102; Di Filippo 2404.07357

3) w=0 for asymptotically Melvin BHs in electrovacuum; transition to w=-1 in Einstein-Maxwell-dilaton at KK value Júnior et al., PRD 105 (2022) 064070

4) D=4 stationary, axisymmetric, asymptotically flat with ergoregion must have at least one light ring outside the ergoregion Ghosh, Sarkar, PRD 104 (2021) 044019

5) Generalisation to higher dimensional BHs Tavlayan, Tekin, PRD 107 (2023) 024016

6) Generalisation to non-spherical horizons and multi BHs Cunha, CH, Novo, PRD 109 (2024) 064050

7) Generalisation to models where photons do not propagate along null geodesics Murk, Soranidis 2406.07957

8) Technique inspired "Topology for timelike circular orbits" Wei, Liu, PRD 107 (2023) 064006; Cunha, CH, Novo, CQG 39 (2022) 225007

9) Technique inspired "Topology of black hole thermodynamics" Wei, Liu, PRD 105 (2022) 104003, …

…

2) Do all (asymptotically flat) black holes have no "hair"?

(i.e. are BHs uniquely determined by M,J,Q - asymptotically measured quantities subject to a Gauss law - and no other independent characteristics ("hair")? Ruffini, Wheeler, Physics Today, January 1971, 30

<u>Generic statements (theorems):</u>

- BH uniqueness theorems (electro-vacuum)

Israel, PRD 164 (1967) 1776; Carter, PRL 26 (1971) 331;
Robinson, PRL 34 (1975) 905 (1975); Rácz and Wald, CQG 13 (1996) 53;…
Review: Chrusciel, Costa, Heusler, LRR 15 (2012) 7

- "No hair" (no-go) theorems (beyond electro-vacuum): many different theorems/arguments applying to different models and under different assumptions. Reviews: Bekenstein, gr-qc/9605059; CH, Radu, IJMPD24 (2015) 09, 1542014; Volkov 1601.08230;…

E.g. for models with scalar fields and no gauge fields:
Review: CH, Radu, IJMPD24 (2015) 09, 1542014

Theory Lagrangian density $\mathcal{L}$	No-hair theorem
Scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi$	Chase[22]
Massive-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{2}\mu^2\Phi^2$	Bekenstein[11]
Massive-complex-scalar-vacuum $\frac{1}{4}R - \nabla_\mu\Phi^*\nabla^\mu\Phi - \mu^2\Phi^*\Phi$	Pena– –Sudarsky[61]
Conformal-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}R\Phi^2$	Xanthopoulos– –Zannias[32] Zannias[33]
V-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)$	Heusler[46, 47, 50] Bekenstein[26] Sudarsky[51]
P-scalar-vacuum $\frac{1}{4}R + P(\Phi, X)$	Graham– –Jha[62]
Einstein-Skyrme $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi^a\nabla^\mu\Phi^a$ $-\kappa\lvert\nabla_{[\mu}\Phi^a\nabla_{\nu]}\Phi^b\rvert^2$	
Scalar-tensor theories $\varphi\hat{R} - \frac{\omega(\varphi)}{\varphi}\hat{\nabla}_\mu\varphi\hat{\nabla}^\mu\varphi - U(\varphi)$	Hawking[27] Saa[34, 35] Sotiriou– –Faraoni[31]
Horndeski/Galileon theories Full $\mathcal{L}$ in eq. (41)	Hui– –Nicolis[45]

A simple and paradigmatic no-hair theorem is due to Bekenstein, that uses only the scalar field equation (not the Einstein equations).

Bekenstein, PRL 28 (1972) 452; PRD 5 (1972) 1239; PRD 5 (1972) 2403; PRD 51 (1995) 6608

Consider a stationary, axi-symmetric, asymptotically flat BH spacetime. Write the spacetime metric in coordinates adapted to these symmetries (t, r, θ, ϕ), so that the two Killing vector fields read k = ∂/∂t, m = ∂/∂ϕ.

Assumption 1: **consider a canonical and minimally coupled scalar field to Einstein's gravity.** Allowing the possibility of a potential, V (Φ), the action is:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)\right)$$

Thus the scalar field obeys the (possibly non-linear) Klein-Gordon equation: $\nabla_\mu\nabla^\mu\Phi - V'(\Phi) = 0$

Assumption 2: **the scalar field inherits the spacetime symmetries.** In particular for the coordinates chosen above this means that:

$$\partial_t\Phi = 0 = \partial_\phi\Phi$$

Then, multiply the Klein-Gordon equation by Φ and integrate over the BH exterior space-time:

$$\int d^4x\sqrt{-g}\,[\Phi\nabla_\mu\nabla^\mu\Phi - \Phi V'] = 0$$

Integrating the first term by parts:

$$\int d^4x\sqrt{-g}\,[-\nabla_\mu\Phi\nabla^\mu\Phi - \Phi V'] + \cancel{\int_{\mathcal{H}} d^3\sigma n^\mu \Phi\nabla_\mu\Phi} = 0$$

where the boundary term is computed on the horizon and vanishes.

Assumption 3 (v1.0): the potential V obeys: $\Phi V' \geq 0$
everywhere, and the equality holds only for (possibly) some discrete values of the field. This holds for a non-self-interacting massive scalar field.

Since the gradient of the field is orthogonal to both Killing vectors and thus must be spacelike or zero, the first term in the integrand is non-positive. Then, the whole integrand is non-positive and the equality holds iff the field vanishes everywhere, which establishes the no-hair theorem. Remarkably, this theorem, did not use the Einstein equations.

The same theorem was sketched, in the same year, in the paper by Hawking on the absence of new black holes in Brans-Dicke theory. Hawking, CMP 25 (1972) 167

Remark:

Assumption 3 (in v1.0) does not seem particularly significant and may be violated for some potentials of interest. Different energy conditions have been invoked for different theorems in the literature within this class of models:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)\right)$$

Some examples:

Assumption 3 (v1.0): $\Phi V' \geq 0$

Bekenstein, PRD 51 (1995) 6608

Assumption 3 (v2.0): (convexity) $V''(\Phi) \geqslant 0$

Sotiriou, Faraoni, PRL 108 (2012) 081103

Assumption 3 (v3.0): $V(\Phi) \geqslant 0$

Heusler and Straumann, CQG 9 (1992) 2177
Heusler, Helv. Phys. Acta 69 (1996) 501

Assumption 3 (v4.0): (Weak EC) $\rho = \frac{1}{2}\partial_\alpha\Phi\partial^\alpha\Phi + V \geqslant 0$

Bekenstein, PRD 51 (1995) 6608
Heusler, JMP 33 (1992) 3497
Sudarsky, CQG 12 (1995) 579

All apply for a simple mass term:

$$V(\Phi) = \frac{1}{2}\mu^2\Phi^2$$

(...)

Each of these theorems gives hints (sometimes confirmed) of how to get hairy BHs

(only scalars, D=4, asymptotically flat)

Theory Lagrangian density $\mathcal{L}$	No-hair theorem	Known scalar hairy BHs with regular geometry on and outside $\mathcal{H}$ (primary or secondary hair; regularity)
Scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi$	Chase[22]	
Massive-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{2}\mu^2\Phi^2$	Bekenstein[11]	
Massive-complex-scalar-vacuum $\frac{1}{4}R - \nabla_\mu\Phi^*\nabla^\mu\Phi - \mu^2\Phi^*\Phi$	Pena– –Sudarsky[61]	Herdeiro–Radu[136, 137] (primary, regular); generalizations:[159]
Conformal-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}R\Phi^2$	Xanthopoulos– –Zannias[32] Zannias[33]	Bocharova–Bronnikov–Melnikov– –Bekenstein (BBMB)[16–18] (secondary, diverges at $\mathcal{H}$); generalizations:[87]
V-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)$	Heusler[46, 47, 50] Bekenstein[26] Sudarsky[51]	Many, with non-positive definite potentials:[71–75, 78–80] (typically secondary, regular)
P-scalar-vacuum $\frac{1}{4}R + P(\Phi, X)$	Graham– –Jha[62]	
Einstein-Skyrme $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi^a\nabla^\mu\Phi^a$ $-\kappa\lvert\nabla_{[\mu}\Phi^a\nabla_{\nu]}\Phi^b\rvert^2$		Droz–Heusler–Straumann[126] (primary but topological; regular); generalizations:[129, 131]
Scalar-tensor theories $\varphi\hat{R} - \frac{\omega(\varphi)}{\varphi}\hat{\nabla}_\mu\varphi\hat{\nabla}^\mu\varphi - U(\varphi)$	Hawking[27] Saa[34, 35] Sotiriou– –Faraoni[31]	
Horndeski/Galileon theories Full $\mathcal{L}$ in eq. (41)	Hui– –Nicolis[45]	Sotiriou-Zhou[43] (secondary; regular) Babichev–Charmousis[88, 90] (secondary[88] or primary,[90] diverges at $\mathcal{H}^+$ or $\mathcal{H}^-$); generalizations:[91–93]

Focus on a family with this simplest model

non-minimally coupled scalars in quadratic gravity: spontaneous scalarisation

Violates:

Assumption 2 (symmetry non-inheritance)

Assumption 1 (non-minimal coupling)

Assumption 3 (violate energy conditions)

Assumption 1 (non-canonical kinetic terms)

Assumption 1,3 (non-minimal couplings; non-canonical kinetic terms)

CH, Radu, IJMPD 24(2015)1542014

2) A family of examples

"Reasonable" non-Kerr black holes/compact objects:

CH, Lect. Notes Phys. 1017 (2023) 315

Theoretical criteria:

1) Appear in a well motivated and consistent physical (effective) theories;

Kerr: General Relativity

Dynamical Robustness

2) Have a dynamical formation mechanism;

Kerr: gravitational collapse, accretion, mergers,…

3) Be (sufficiently) stable.

Kerr: mode stability established (Whiting, JMP 30 (1989) 1301)

Kerr: linear/non-linear stability, ongoing (Dafermos, Holzegel, Rodnianski, Acta Math. 222 (2019) 1, Dafermos et al., 2104.08222 …)

Correct phenomenology:

No clear tension between observations and the Kerr model

1) all electromagnetic observables (X-ray spectrum, shadows, QPOs, star orbits,...);

2) correct Gravitational wave (GW) templates

Precision phenomenology of "reasonable" non-Kerr models is desirable.

We consider one family (and related mechanism) of <u>potentially</u> “reasonable” non-Kerr BHs with bosonic hair.

The family is illustrated (simplest example) by this model:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \nabla_\alpha\Phi^*\nabla^\alpha\Phi - \mu^2|\Phi|^2\right)$$

New scale can single out BHs of a certain size to become "hairy". Ultralight range is interesting for astrophysical BHs.

There are scalar hairy BH solutions
by violating assumption 2 of Bekenstein's (symmetry non-inheritance)
Smolic, CQG 32 (2015) 145010

- **within GR** (not alternative theories of gravity);
- with matter **obeying main energy conditions**;
- which can yield **distinct phenomenology**;

which are:

- asymptotically flat
- regular on and outside the horizon
- continuously connecting to the Kerr solution
- continuously connected to relativistic Bose-Einstein condensates (boson stars)
- with an independent scalar charge (primary hair)

Black Holes with synchronised hair

CH, Radu, PRL 112 (2014) 221101

Existence proof
Chodosh, Shlapentokh-Rothman, CMP 356 (2017) 1155

But are they dynamically robust?

Ansatz:

$$ds^2 = -e^{2F_0(r,\theta)} N dt^2 + e^{2F_1(r,\theta)} \left(\frac{dr^2}{N} + r^2 d\theta^2 \right) + e^{2F_2(r,\theta)} r^2 \sin^2\theta \left(d\varphi - W(r,\theta) dt \right)^2 \qquad N = 1 - \frac{r_H}{r}$$

$$\Phi = \phi(r,\theta) e^{i(m\varphi - wt)}$$

Data available online at:
<u>http://gravitation.web.ua.pt</u>

Fundamental solutions: m=1

<u>"synchronisation" (sync) condition</u> $\Omega_H = \dfrac{w}{m}$

Ansatz:

$$ds^2 = -e^{2F_0(r,\theta)} N dt^2 + e^{2F_1(r,\theta)} \left(\frac{dr^2}{N} + r^2 d\theta^2 \right) + e^{2F_2(r,\theta)} r^2 \sin^2\theta \, (d\varphi - W(r,\theta)dt)^2 \qquad N = 1 - \frac{r_H}{r}$$

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Continuum of hairy BHs, interpolating between...

Kerr BH

Kerr, PRL 11 (1963) 237

Rotating Boson Stars

Yoshida, Eriguchi, PRD 56 (1997) 762; Schunck, Mielke, PLA 249 (1998) 389

Ansatz:

$$ds^2 = -e^{2F_0(r,\theta)} N dt^2 + e^{2F_1(r,\theta)} \left(\frac{dr^2}{N} + r^2 d\theta^2 \right) + e^{2F_2(r,\theta)} r^2 \sin^2\theta \, (d\varphi - W(r,\theta) dt)^2 \qquad N = 1 - \frac{r_H}{r}$$

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Continuum of hairy BHs, interpolating between...

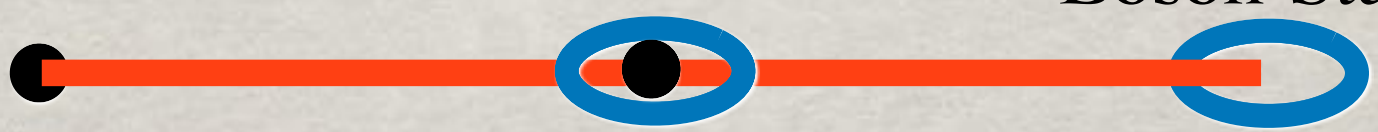
Kerr BH

Vanishing horizon

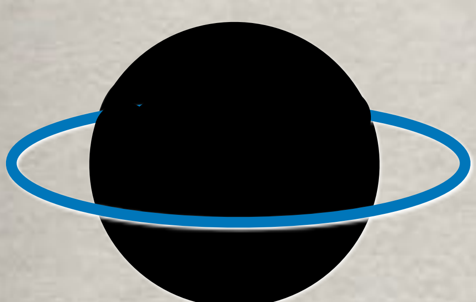
Rotating Boson Stars

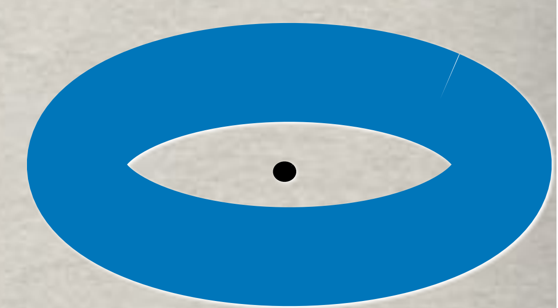
Yoshida, Eriguchi, PRD 56 (1997) 762; Schunck, Mielke, PLA 249 (1998) 389

Kerr, PRL 11 (1963) 237



Vanishing scalar field





Test field approximation → Insights on solutions and their dynamics. ← Test particle (BH) approximation

<u>1) Kerr limit (test scalar field)</u>

How can the scalar field be in equilibrium with a horizon?

Sync means at the threshold of superradiance…

review: Brito, Cardoso, Pani, Lect. Notes Phys. 906 (2015) 1; 971 (2020) 1

… and zero flux through the horizon CH, Radu, CQG 32 (2015) 144001

Linear analysis: Klein-Gordon equation on Kerr

$$\Box\Phi = \mu^2\Phi \qquad\qquad \Phi = e^{-iwt}e^{im\varphi}S_{\ell m}(\theta)R_{\ell m}(r)$$

Radial Teukolsky equation: Teukolsky, PRL 29 (1972) 1114; Brill et al., PRD 5 (1972) 1913

$$\frac{d}{dr}\left(\Delta\frac{dR_{\ell m}}{dr}\right) = \left(a^2w^2 - 2maw + \mu^2r^2 + A_{\ell m} - \frac{K^2}{\Delta}\right)R_{\ell m}$$

$$\Delta \equiv r^2 - 2Mr + a^2$$

$$K \equiv (r^2+a^2)w - am$$

Generically one obtains *quasi*-bound states: $\omega = \omega_R + i\omega_I$

critical frequency

$$w_c = m\Omega_H$$

1) Kerr limit (test scalar field)

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$$\frac{d}{dr}\left(\Delta\frac{dR_{\ell m}}{dr}\right) = \left(a^2w^2 - 2maw + \mu^2r^2 + A_{\ell m} - \frac{K^2}{\Delta}\right)R_{\ell m}$$

$$\Delta \equiv r^2 - 2Mr + a^2$$

$$K \equiv (r^2 + a^2)w - am$$

Generically one obtains *quasi*-bound states: $\omega = \omega_R + i\omega_I$

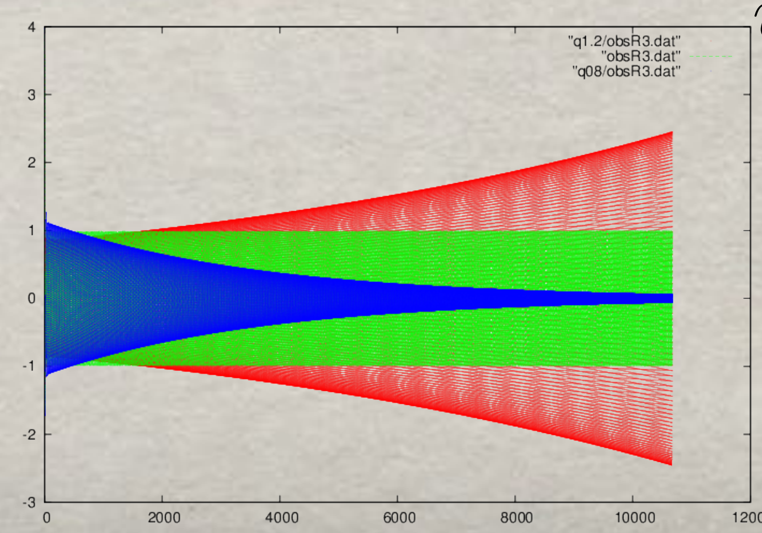
critical frequency $w_c = m\Omega_H$

$w_I < 0$ if $w_R > w_c$ — decay

$w_I = 0$ if $w = w_c$ — true bound states: *stationary clouds*

$w_I > 0$ if $w_R < w_c$ — grow

Press and Teukolsky, Nature 238 (1972) 211



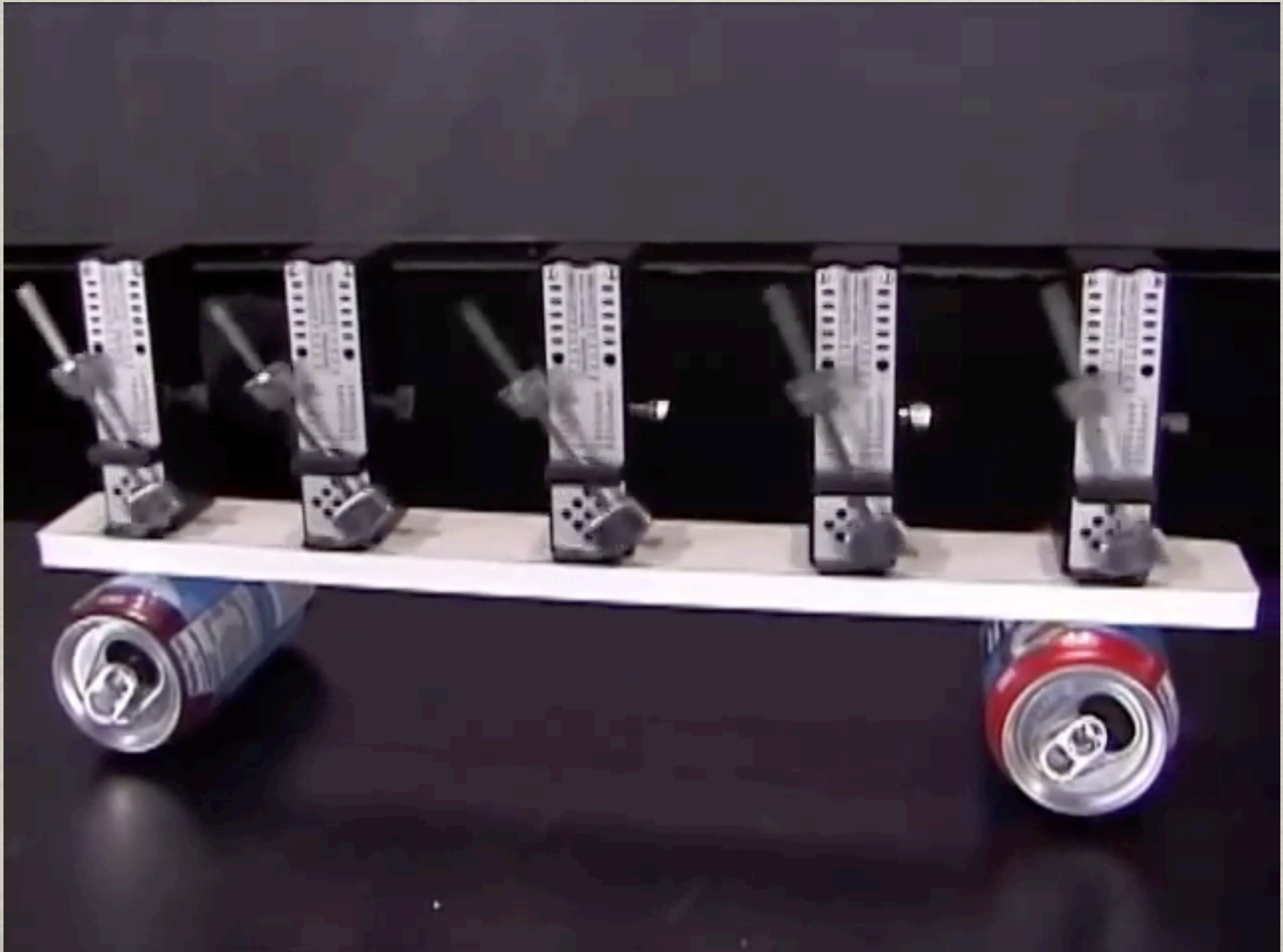
Time evolution of superradiant (red), decaying (blue) and critical (green) modes. Degollado, unpublished

Sync: same as ever

Dynamical synchronisation occurs in physical and biological systems

Harvard Natural Sciences Lecture Demonstrations
https://sciencedemonstrations.fas.harvard.edu/presentations/synchronization-metronomes

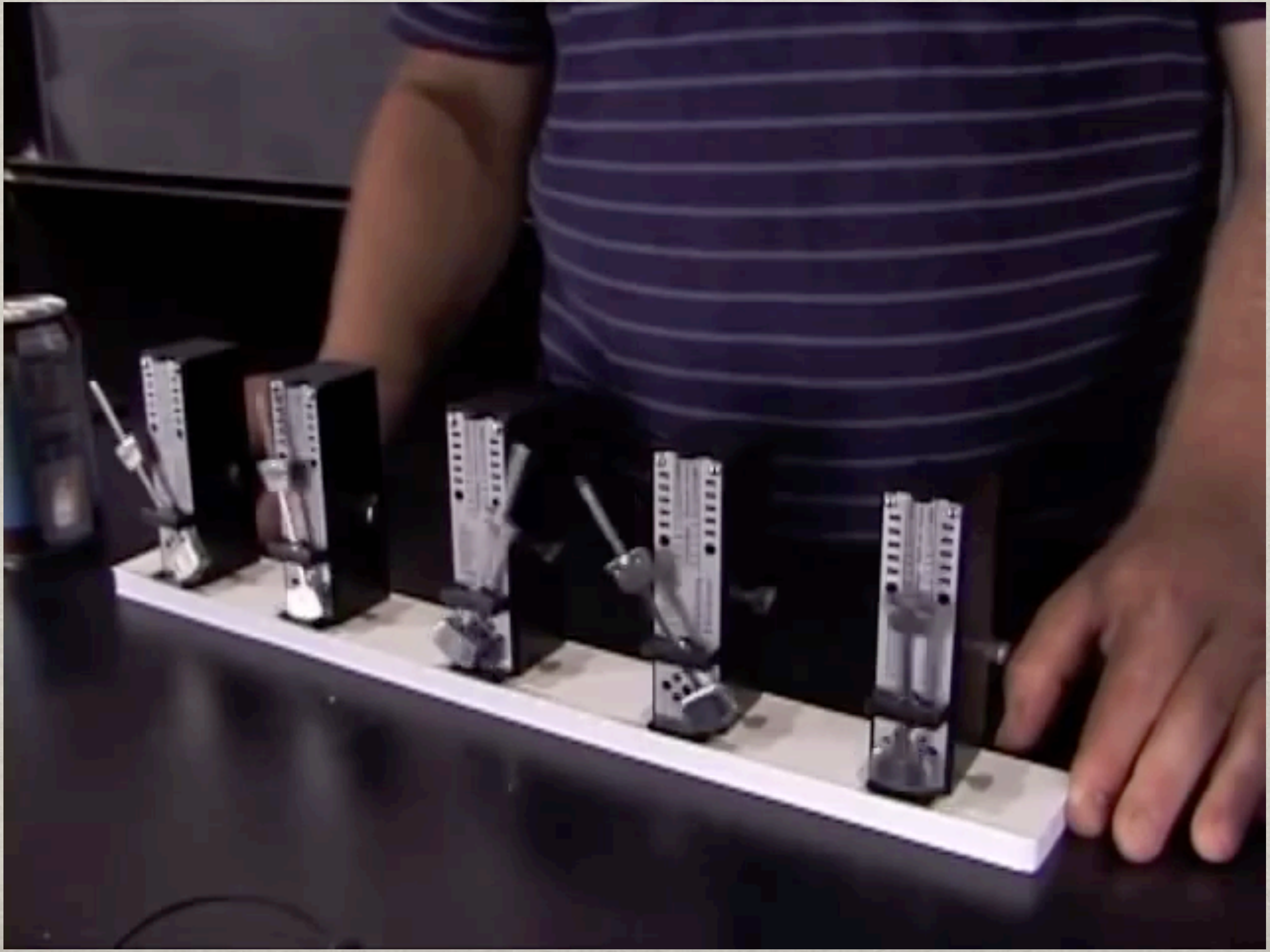
Fine tuning ?



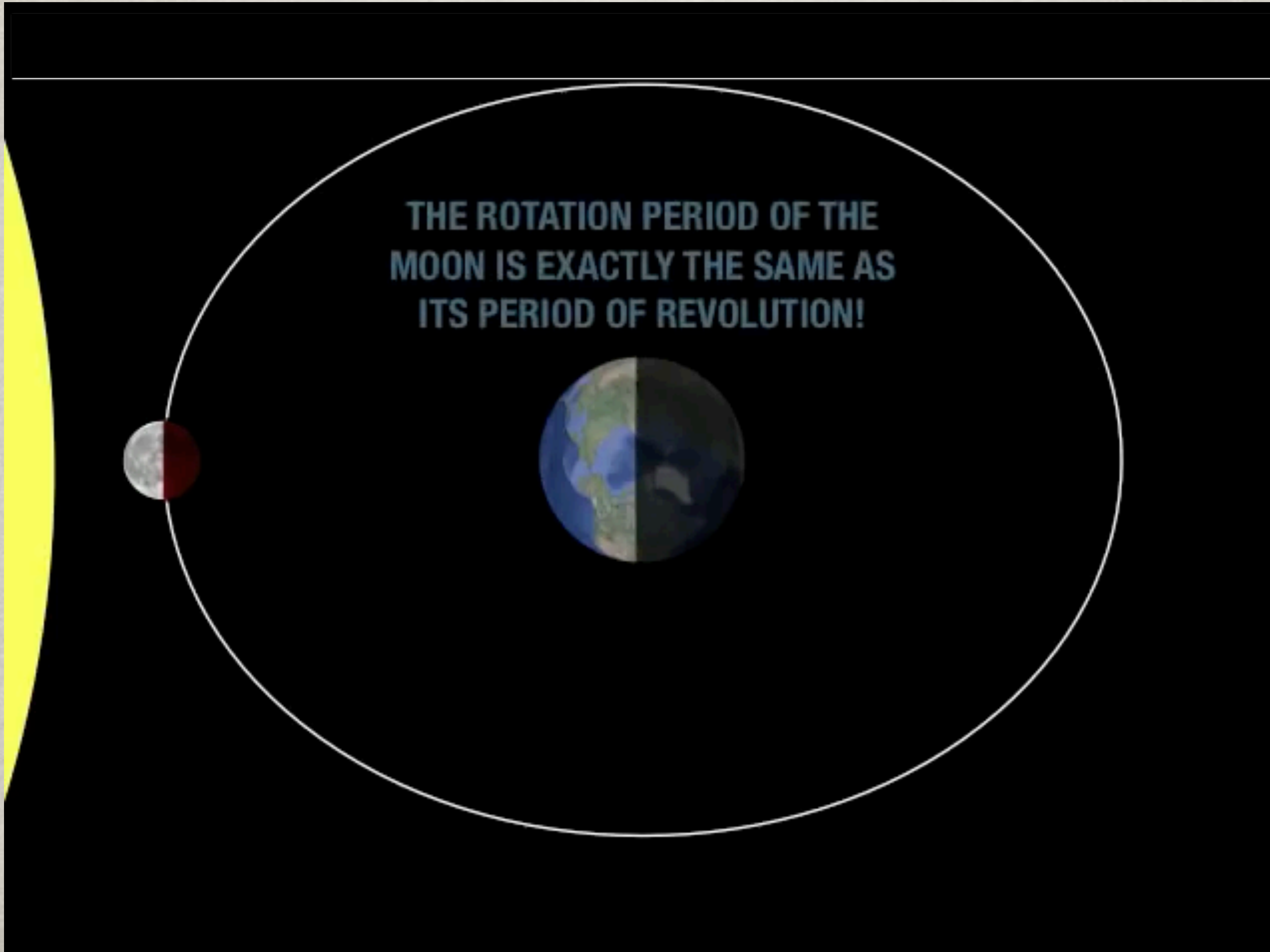
Not really; common in physical and biological systems

Harvard Natural Sciences Lecture Demonstrations

https://sciencedemonstrations.fas.harvard.edu/presentations/synchronization-metronomes



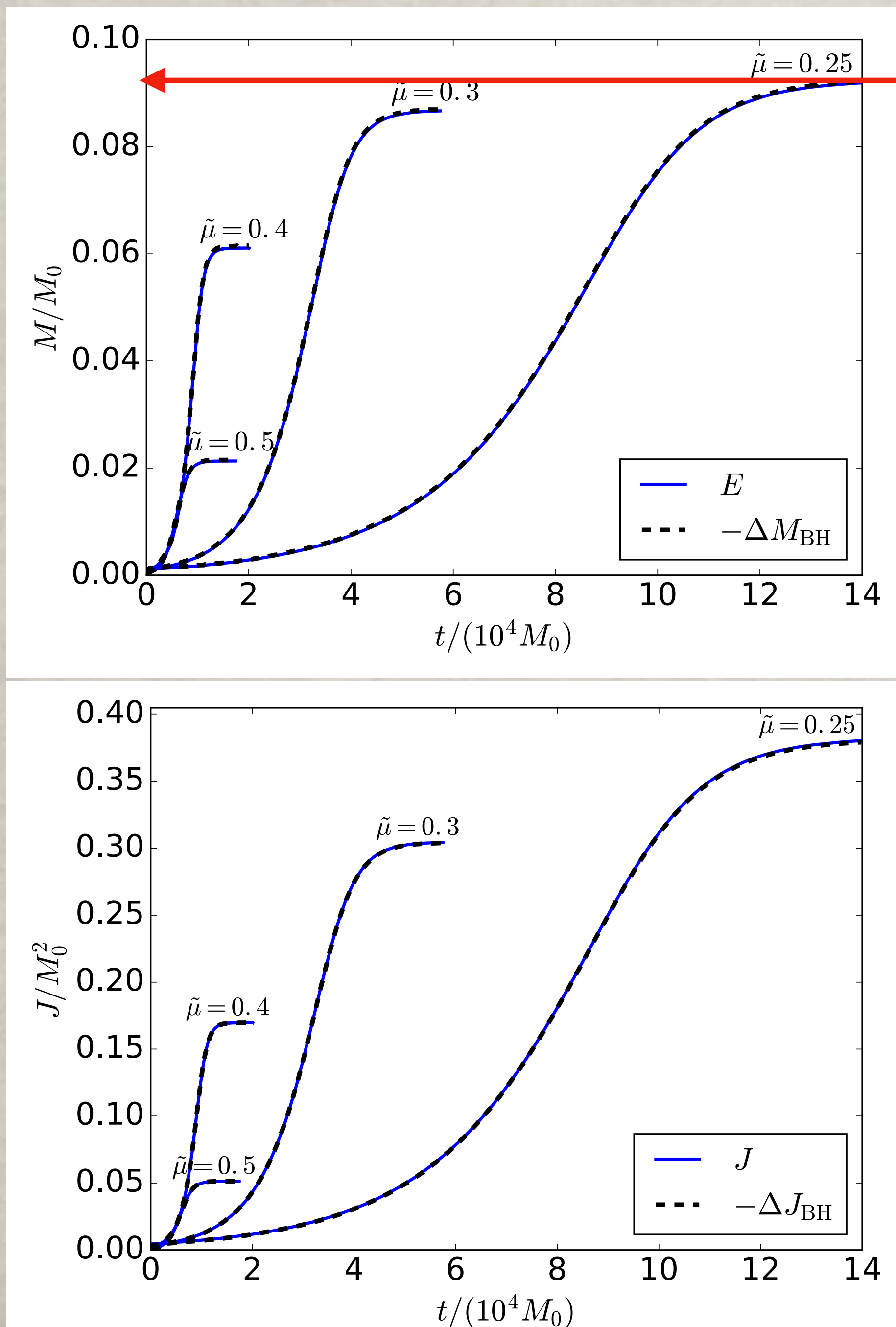
Synchronisation and tidal locking in Newtonian gravity



Dynamical synchronisation (for the Proca model) shows the process reaches an equilibrium state...

East and Pretorius, PRL 119 (2017) 041101

Mass and angular momentum in "hair"

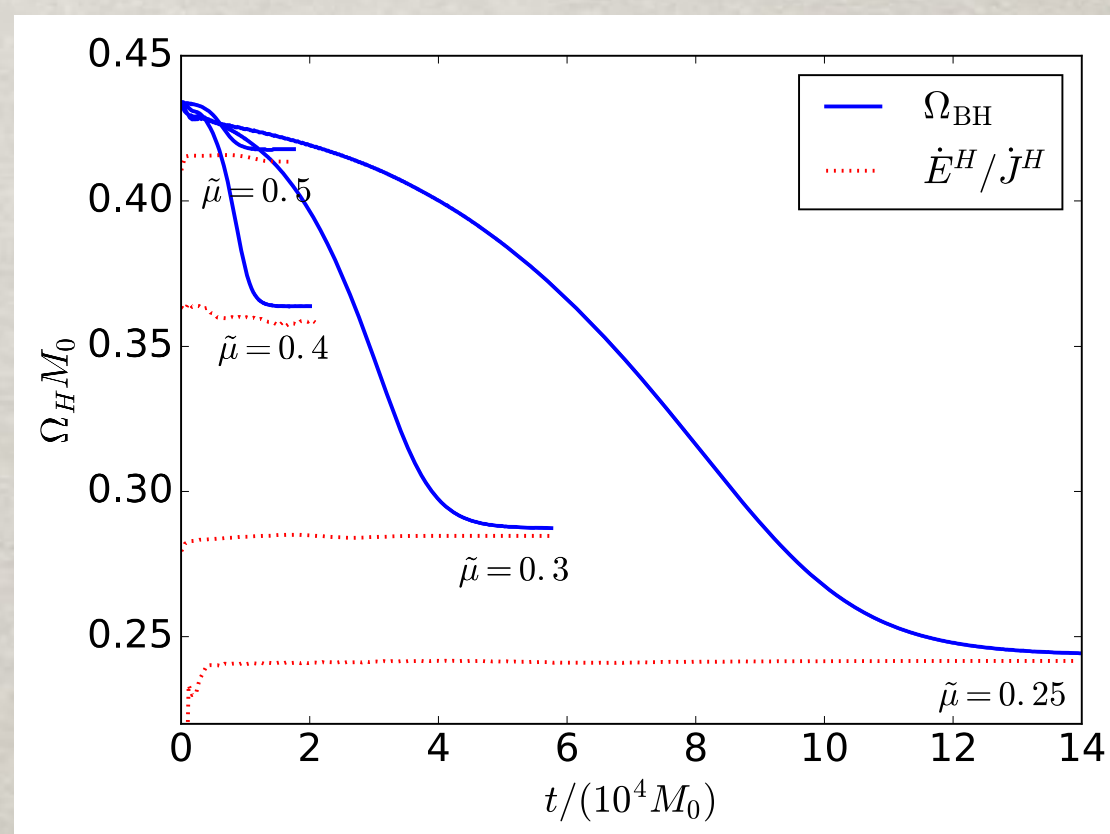


about 9% energy extraction

Energy extraction bound ~10% for both scalar and vector cases (if conservative process)

CH, Radu, Santos, PLB 824 (2022) 136835

Black hole spin down and "synchronisation"



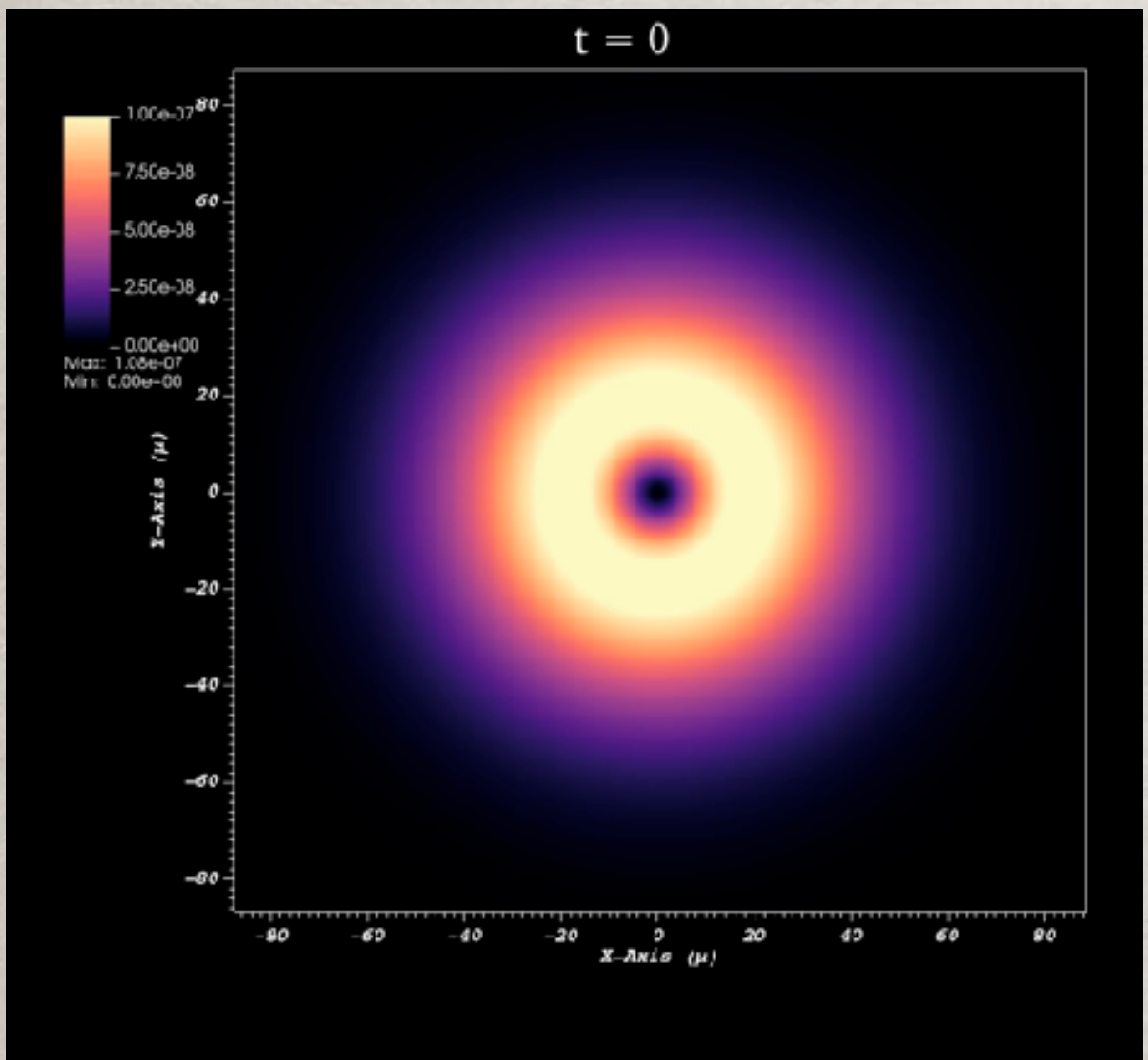
... which is a BH with synchronised Proca hair

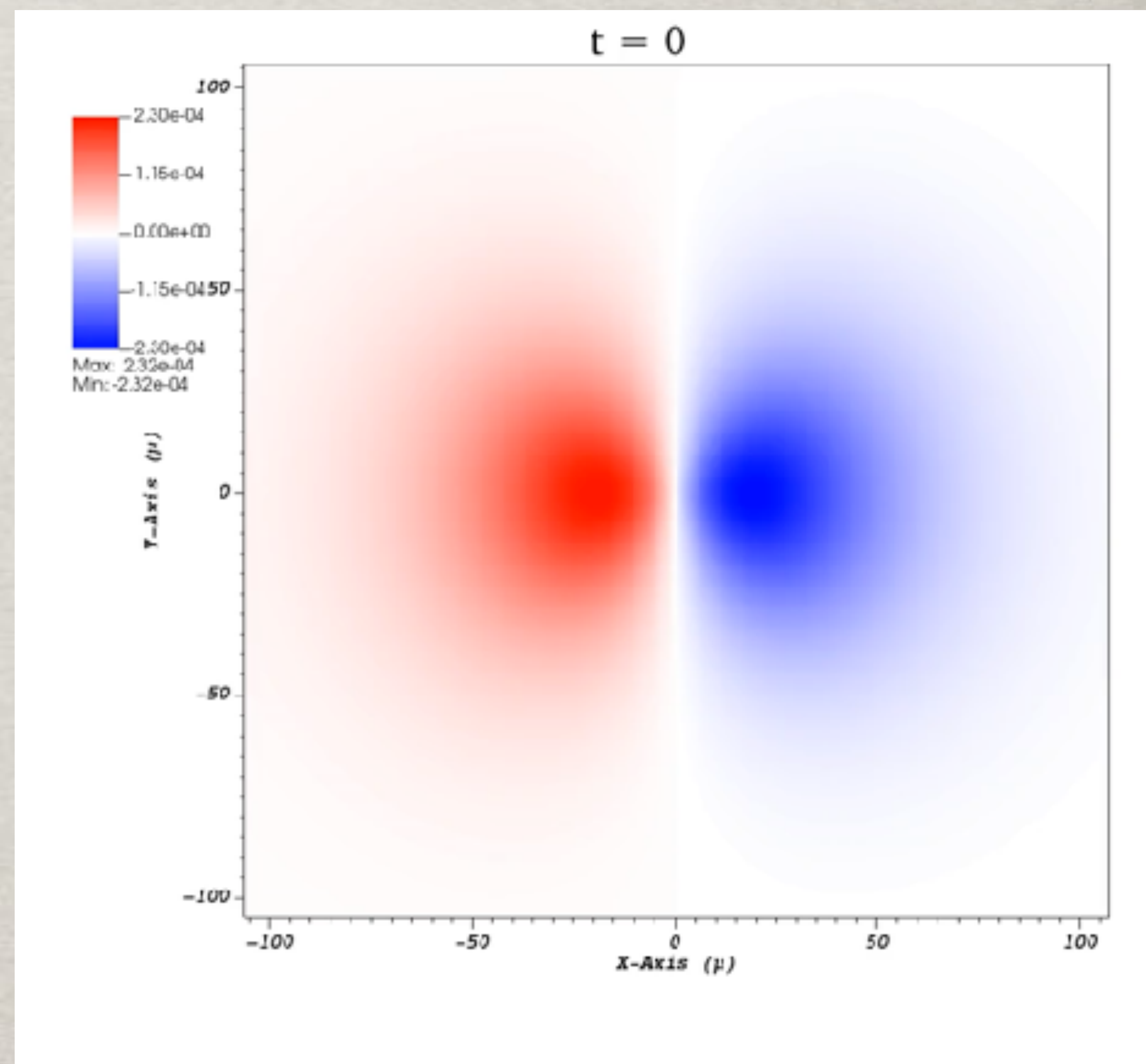
CH, Radu, PRL 119 (2017) 261101

Dolan, Physics10(2017)83

Dynamical evolutions of BHs with synchronised (scalar) hair

[Preliminary results] Simulations: J. Nicoules and M. Zilhão (to appear)





Solution	m	ω	r_H	M	J	M_H/M	J_H/J
"Close" to Kerr	1	0.998200	0.179026	0.127692	0.0346921	0.754980	0.0998357

~25% of total energy in "hair"

In longer timescales, we actually expect something to happen.

These hairy black holes have an ergoregion themselves CH, Radu, PRD 89 (2014) 124018; suggests they are also afflicted by superradiant instabilites of higher "m" modes (when in the superradiant range); confirmed by Ganchev and Santos PRL 120 (2018) 171101

$\mu M \sim 0.2\ ,\quad 10^{10}\ M_\odot$

$$\text{Kerr}\ \xrightarrow[\text{(astrophysical time scale)}]{\Delta t}\ \text{Hairy black hole}\ \xrightarrow[\text{(cosmological time scale)}]{> 1000\Delta t}\ \ldots$$

$\sim 10 \times 10^6$ yrs $\qquad\qquad \sim 10 \times 10^9$ yrs

Is the hairy BH the end of the superradiance instability of Kerr?

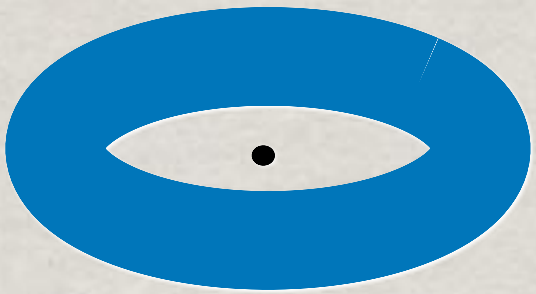
In *theory* no; they are still afflicted by superradiant instabilities of higher *m* modes;

But in *practice* it can be: **dynamical robustness/effective stability**

Degollado, CH, Radu, Phys. Lett. B 781 (2018) 651

2) Boson star limit (test particle approximation)

Is the equilibrium between a small BH and a big boson star stable?

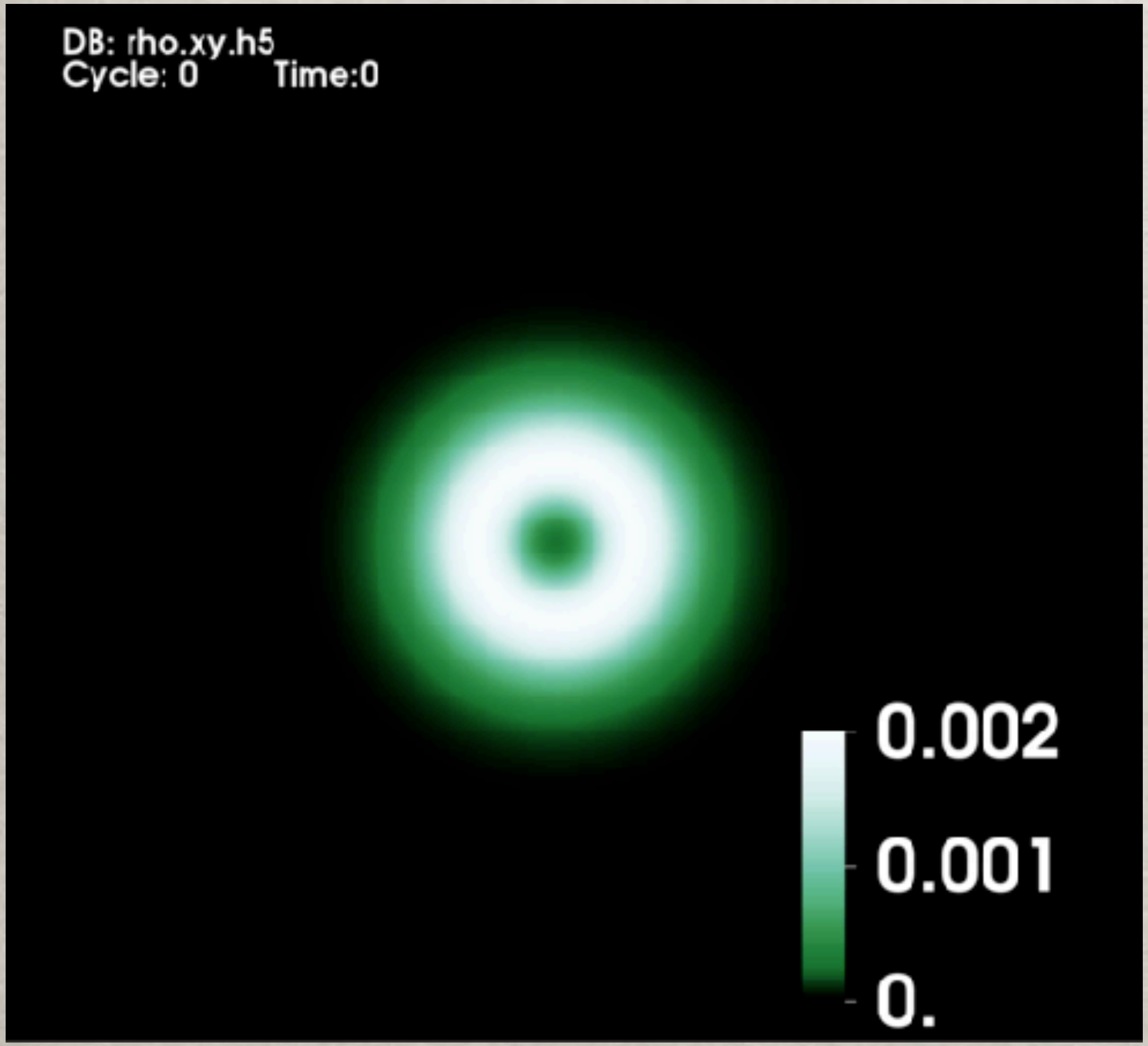


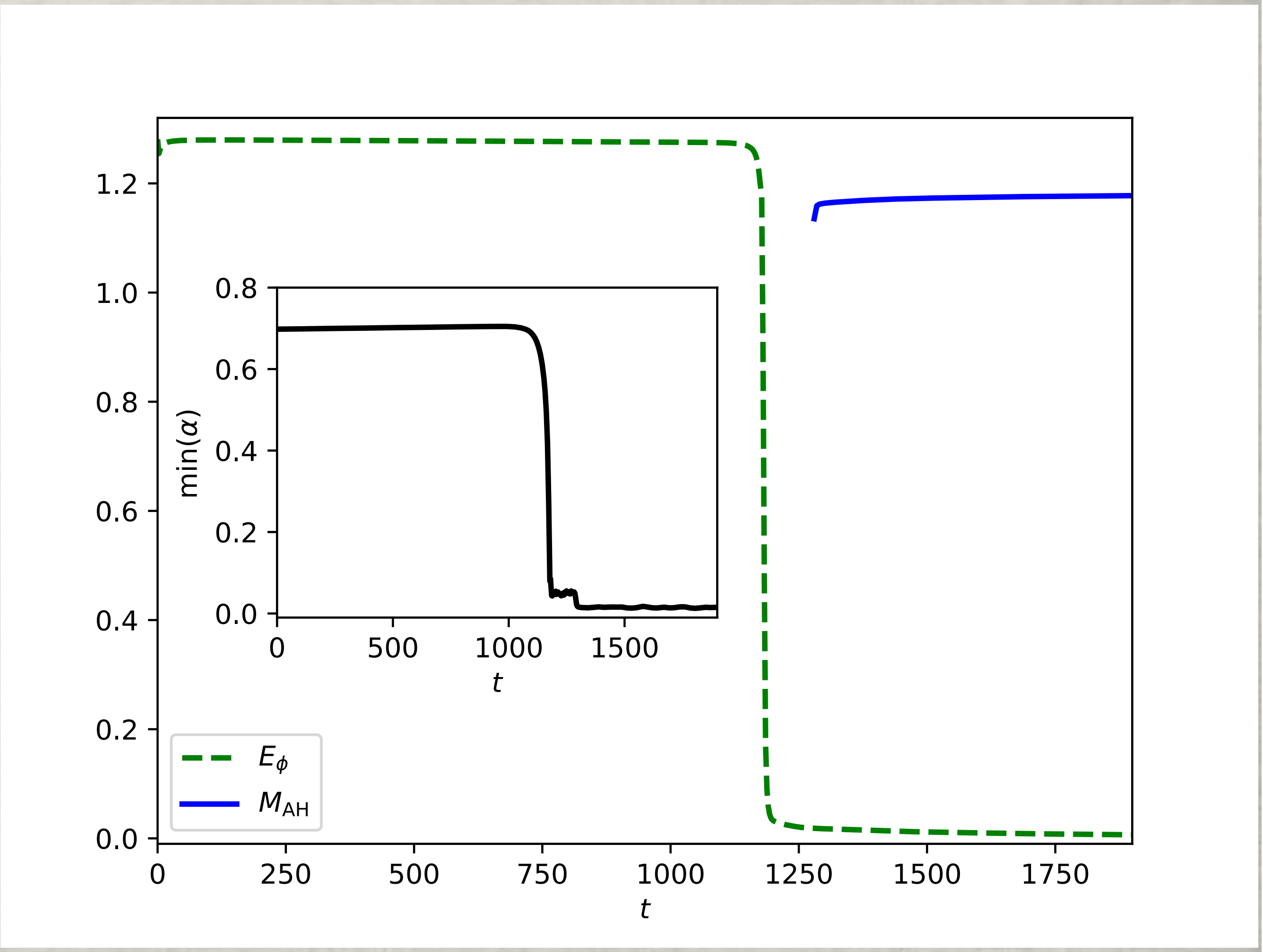
2) Boson star limit (test particle approximation)

Is the equilibrium between a small BH and a big boson star stable?

Spinning scalar boson stars have a non-axisymmetric instability

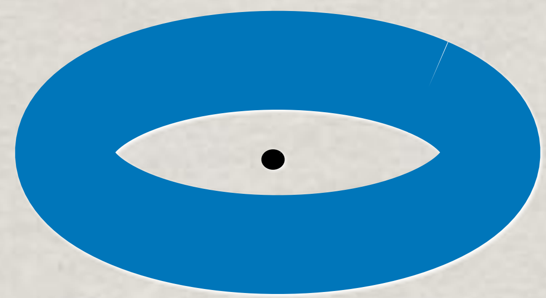
Sanchis-Gual et al., PRL 123 (2019) 221101





2) Boson star limit (test particle approximation)

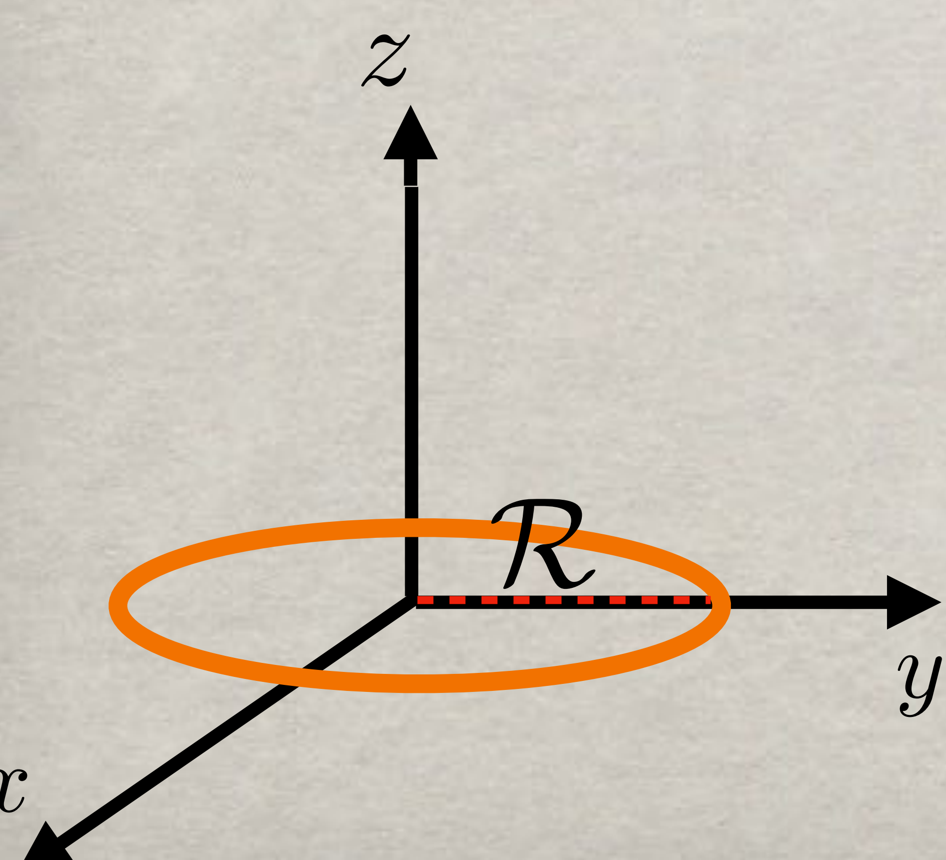
Is the equilibrium between a small BH and a big boson star stable?

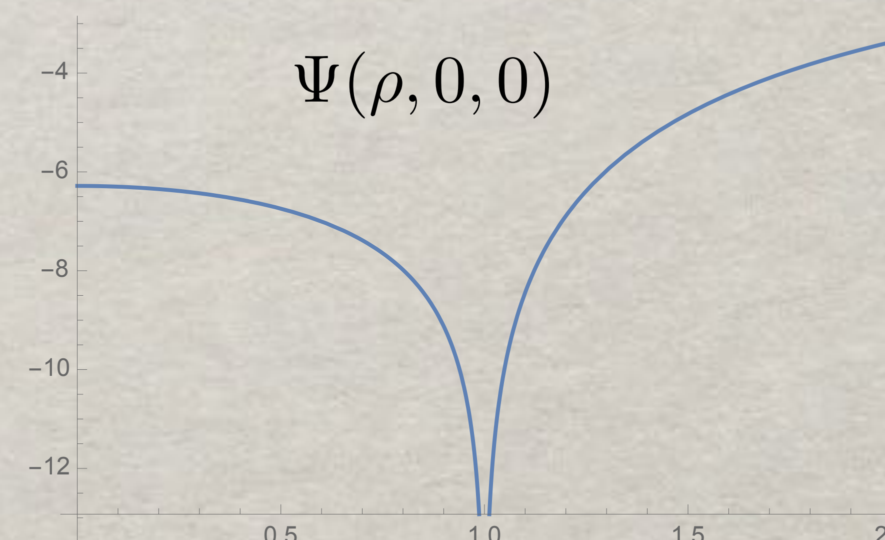


The BH and the boson star form an unstable equilibrium.

Intuition from a Newtonian toy model.

$$\Psi(\rho, z, \phi) = -\int_0^{2\pi} \frac{\chi \mathcal{R} d\varphi}{\sqrt{\rho^2 - 2\rho\mathcal{R}\cos(\phi - \varphi) + \mathcal{R}^2 + z^2}}$$





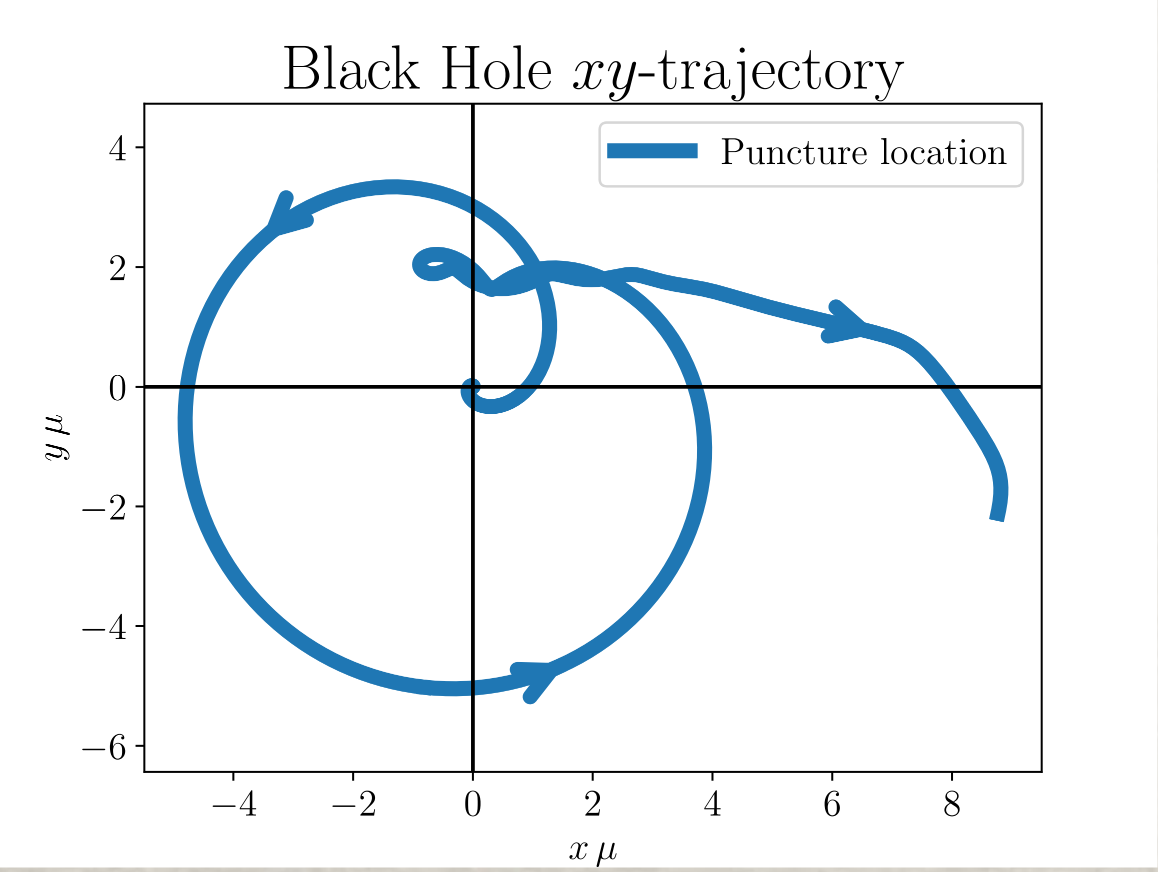
$$\Psi(\rho, 0, 0) \overset{\rho \sim 0}{\simeq} -2\pi - \frac{\pi}{2}\rho^2$$

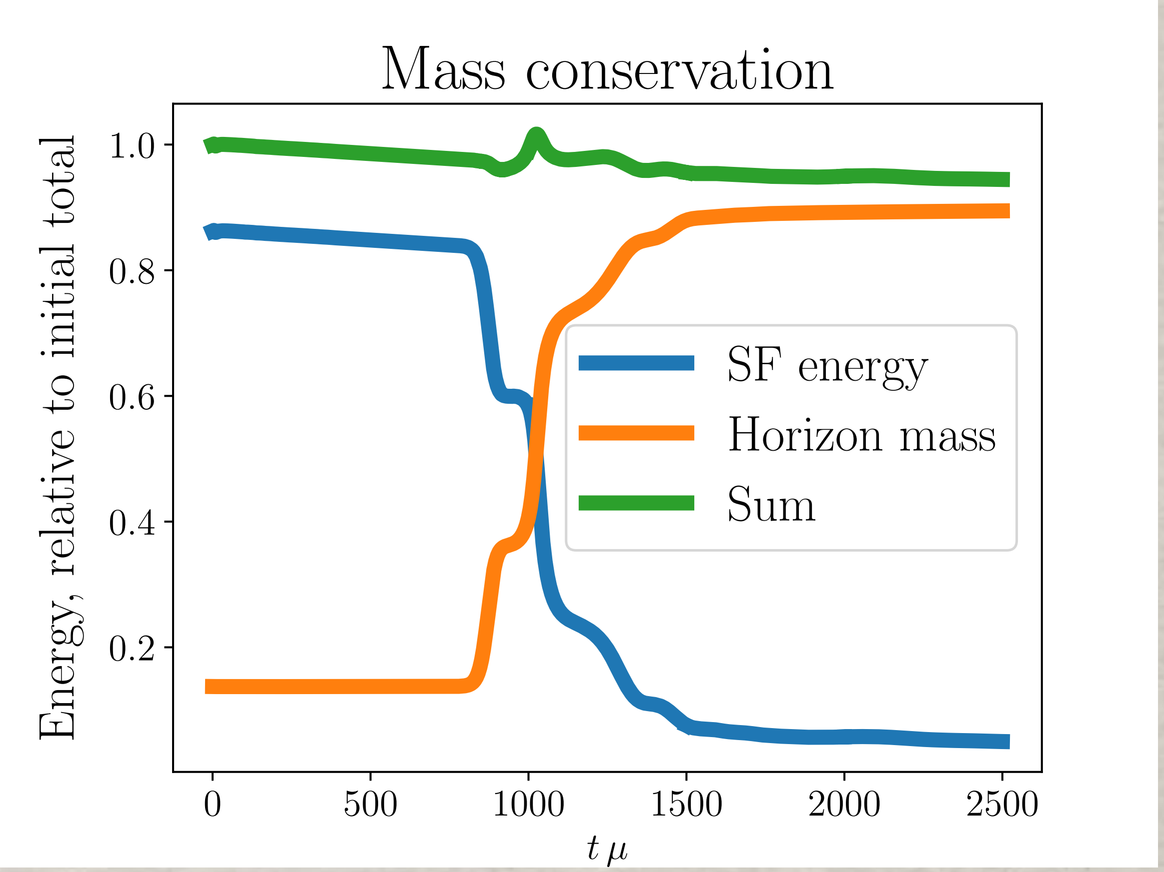
Dynamical evolutions of BHs with synchronised (scalar) hair

[Preliminary results] Simulations: J. Nicoules and M. Zilhão (to appear)

Solution	m	ω	r_H	M	J	M_H/M	J_H/J
"Close" to Boson Star	1	0.90	0.20	1.01049	0.91193	0.11754	0.01217

~88% of total energy in "hair"



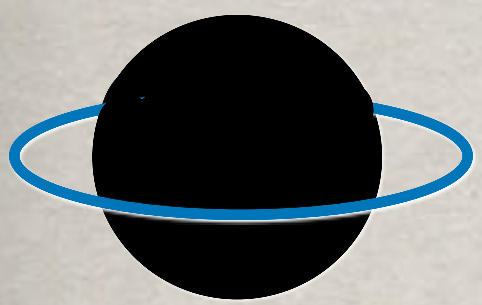


Very hairy scalar sync BHs are dynamically unstable - "migration"

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \nabla_\alpha\Phi^*\nabla^\alpha\Phi - \mu^2|\Phi|^2\right)$$

$$-V(|\Phi|)$$

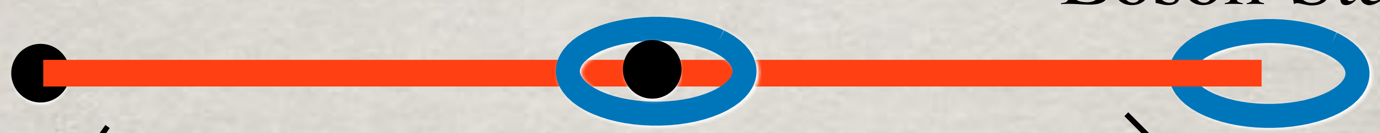
Summary on dynamics of BHs with sync scalar hair (simplest model):

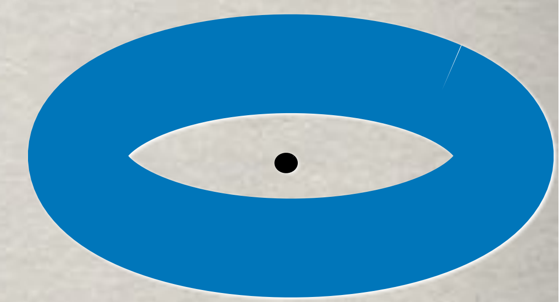


Kerr BH

Vanishing horizon

Rotating Boson Stars





Vanishing scalar field

- Kerr is unstable against superradiance forming hairy BHs as transient states;

- Hairy BHs formed from superradiance (up to ~10% energy in hair) could be sufficiently long lived to be astrophysical players (supermassive BHs);

- Superradiance efficiency selects a preferable scale (Universality of the Kerr hypothesis);

- ~~Spinning BSs are afflicted by a non-axisymmetric instability;~~

- Hairy BHs where the scalar environment is gravitationally dominant are in an unstable equilibrium;

- The development leads to a migration to an approximately Kerr BH.

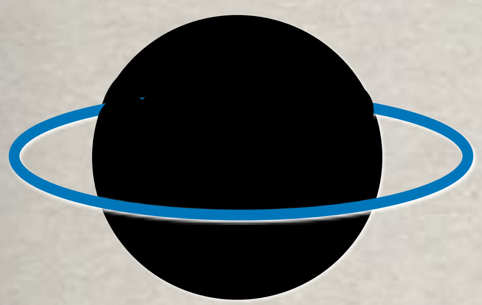
Impact of self-interactions?

Di Giovanni et al., PRD 102 (2020) 124009;
Siemonson and East, PRD 103 (2021) 044022

Geodesic approximation: origin never a stable equilibrium point Delgado et al., PRD 105 (2022) 6, 064026

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R - \frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha\right) \ .$$

Dynamics of BHs with sync Proca hair (simplest model):

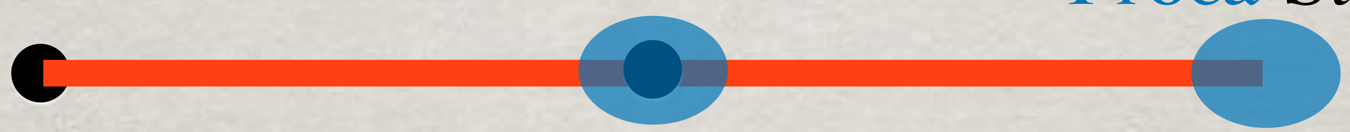


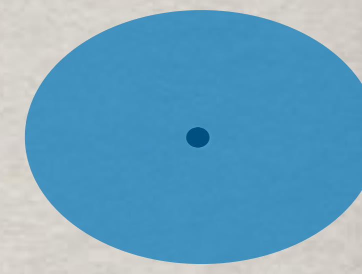
Kerr BH

Vanishing horizon

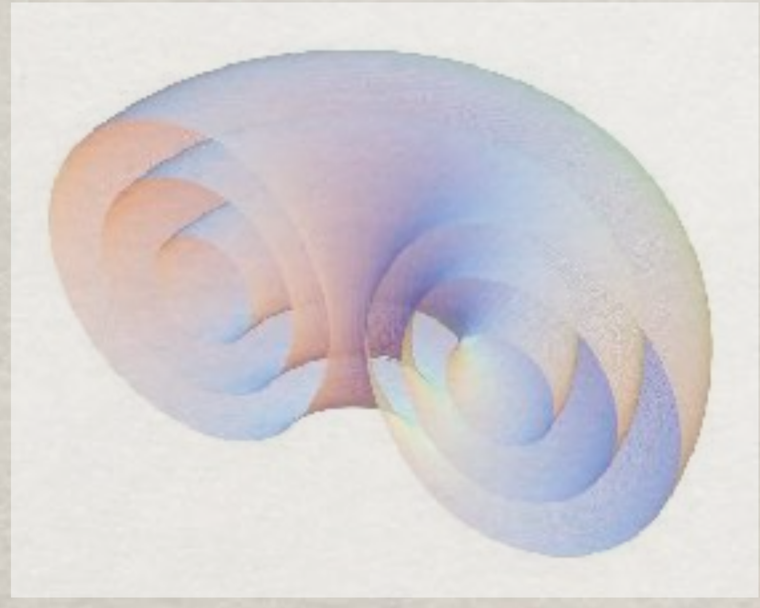
Rotating

Proca Stars





Vanishing Proca field



Rotating boson stars

Rotating Proca stars

Brito, Cardoso, CH and Radu, PLB 752 (2016) 291
CH, Radu and Rúnarsson, CQG 33 (2016) 154001
CH, Perapechka, Radu and Shnir, PLB 797 (2019) 134845

The non-axisymmetric instability of the scalar model has not been seen here

Sanchis-Gual et al., PRL 123 (2019) 221101

Geodesic approximation: origin can be a stable equilibrium point Delgado et al., PRD 105 (2022) 6, 064026

Sync is a generic mechanism to endow spinning BHs with hair:

scalar -> Proca hair
CH+ CQG 33 (2016) 154001;
JCAP 07 (2024) 081
Santos+ JHEP 07 (2020) 010; …

add scalar self-interactions
CH+ PRD 92 (2015) 8, 084059
Delgado+ PRD 103 (2021) 104029;…

other dimensions
Brihaye+ PLB 739 (2014) 1
CH+ PLB 748 (2015) 30
…

other dimensions and asymptotics
Dias+ JHEP 07 (2011) 115
Izuka+ JHEP 08 (2015) 112; …

add electric and magnetic charges,
Delgado+ PLB 761 (2016) 234
…

"Simplest model" considered before
D=4,
asymptotically flat (AF),
free-scalar

Non-canonical kinetic terms
CH+ JHEP 10 (2018) 119;
JHEP 02 (2019) 111; …

Non-minimal couplings (beyond GR)
Kleihaus+ PLB 744 (2015) 406
CH+ IJMPD 27 (2018) 1843009
….

Excited states
Wang+ PRD 99 (2019) 064036
Delgado+ PLB 792 (2019) 436
…

Multipolar (odd-parity) scalar hair
Kunz+ PRD 100 (2019) 064032;…

2-AF-BHs in equilibrium
CH+ PRL 131 (2023) 121401

Purely charged version (resonance condition)
CH+ EPJC 80 (2020) 390;
Hong+ PRL 125 (2020) 111104;
Hong+ PLB 803 (2020)135324;…

Many theoretical and phenomenological aspects;
E.g. both scalar and Proca BHs with sync hair can have multiple LRs
Cunha et al. PRL 115 (2015) 211102; Sengo et al. JCAP 01 (2023) 047

3) Final comments and outlook

Final comments and outlook

- One can make statements fully independent of the equations of motion about the geometric properties of BHs, e.g. the existence of LRs.

- BHs in GR (and beyond) can have hair of bosonic fields (with no Gauss law associated). Sync provides a generic mechanism to endow spinning BHs with bosonic hair;

- In the family of models detailed here, the hairy BHs co-exist with Kerr (or the appropriate generalisation); they do not replace them;

- In D=4, AF, the model may be motivated in relation to the dark matter problem, as ultralight scalars are a candidate for ("fuzzy") dark matter;

- The dynamics of the BHs with sync hair is complex. Ongoing effort. Likely, no solution in these models is **absolutely** (classically) stable against small perturbations, not even Kerr (only Schwarzschild). But time scales are important if one is considering possible astrophysical roles.

Final comments and outlook

- Theoretically/phenomenologically the space of solutions is diverse:

- Sharp non-Kerr features can be present:
a) multiple LRs and exotic fundamental photon orbits;
b) multiple ergo-regions CH, Radu PRD 89 (2014) 124018;
c) disconnected regions with stable timelike circular orbits;
….

- These can endow these BHs with very non-Kerr phenomenology:
a) unusual images;
b) backwards chirp Collodel, Doneva, Yazadjiev, PRD 105 (2022) 044036;
c) unusual X-ray spectra under some disk model Ni et al. JCAP 07 (2016) 049;
….

- Perhaps the most exotic features are in (quickly) unstable solutions (scalar vs. Proca?);

- These families of solutions provide a rich arena for theoretical BH research in fairly simple models where, e.g., the initial value problem is well-defined.

- Unfortunately, the numerical nature of the solutions restricts its wider use by the community. Ways around this would be welcome.

Epilogue

"Jim was traveling through Omaha, Nebraska, with Warren Buffett at the end of 2003.
The global economy was bad. One could see closed shops and bankrupted companies. Jim said to Warren:
- Everything is bad. How can the economy recover from such a thing?
- Jim, do you know what was the top-sellling candy bar in 1962?
- No.
- Snickers - said Warren - and do you know what is the top-sellling candy bar today?
- No.
- Snickers.
Silence followed - "

Same as Ever:
Looking for (In)variants in the Black Holes Landscape

Carlos Herdeiro
Gravitational Geometry and Dynamics Group, Aveiro University, Portugal

Thank you for your attention!
Muito obrigado pela vossa atenção!

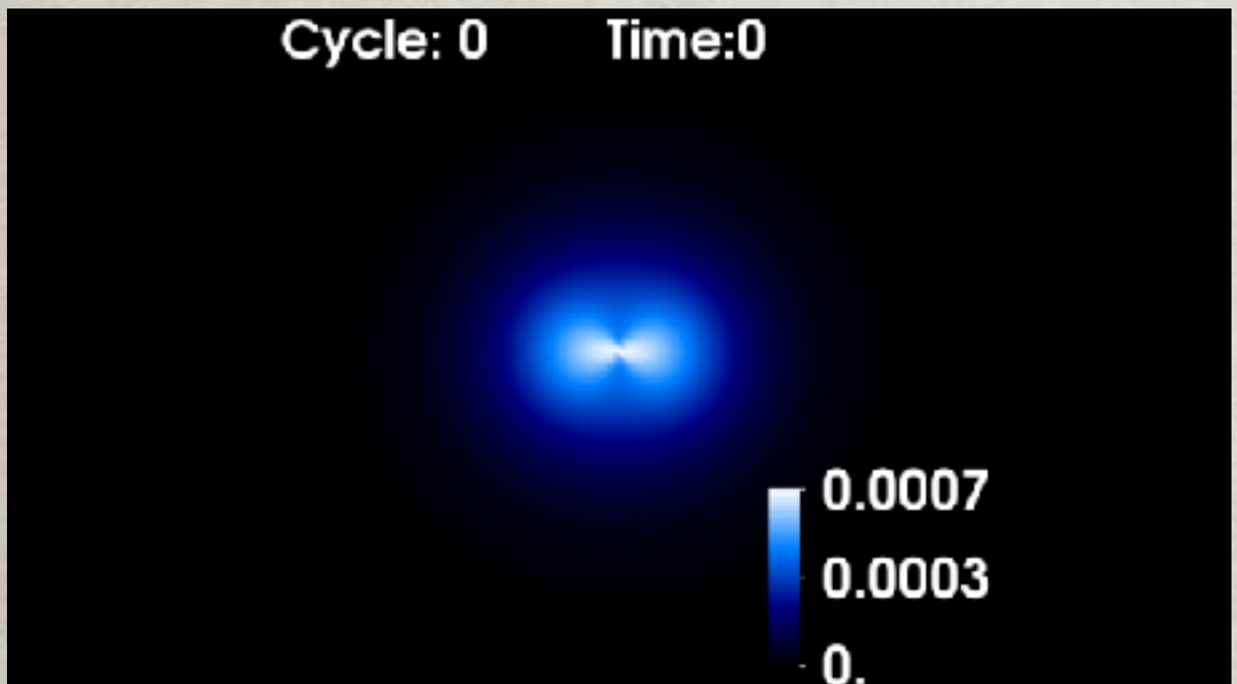
18 November 2024

Spinning Proca stars do not exhibit such instability.

Sanchis-Gual, Di Giovanni, Zilhão, CH, Cerda-Duran, Font and Radu, PRL 123 (2019) 221101

http://gravitation.web.ua.pt/node/1740

Evolution of a perturbed spinning Proca star



Evolution of an excited spinning Proca star

Thus, spinning Proca stars are dynamically more robust in these simplest models.

But in models with self-interactions, the spinning scalar stars instability can be mitigated.

Di Giovanni, Sanchis-Gual, Cerdan-Duran, Zilhão, CH, Font and Radu, PRD 102 (2020) 124009;
Siemonson and East, PRD 103 (2021) 044022