Ergoregion instability in a fluid with vorticity

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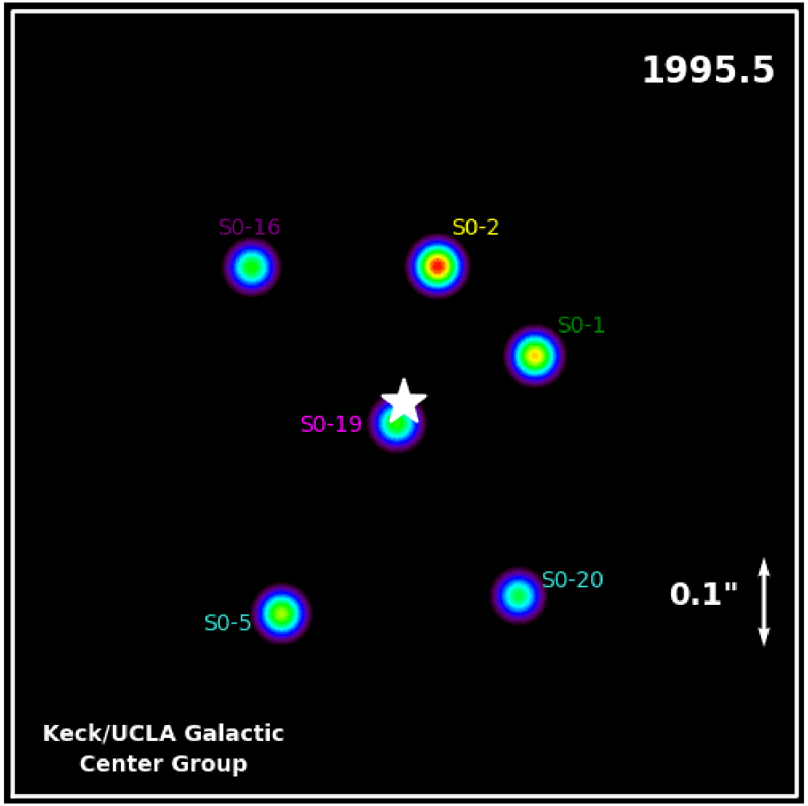
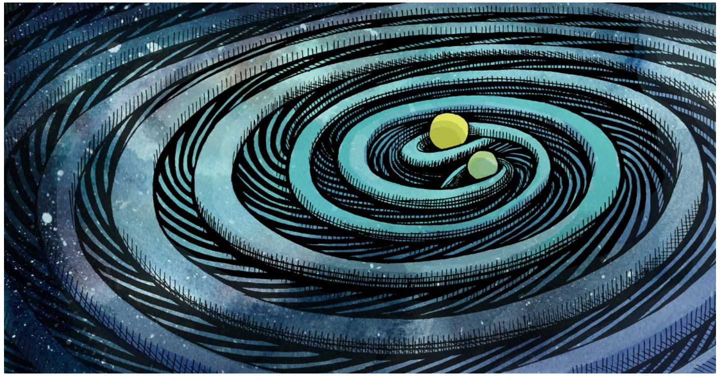
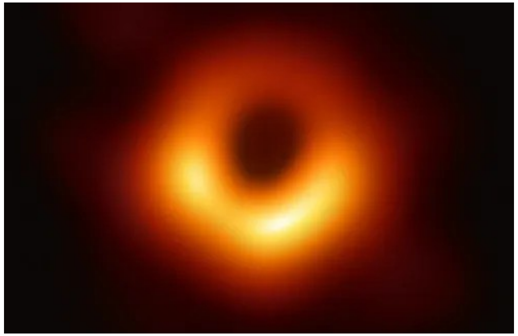
[arXiv:2410.24161]

Introduction





Credit: Sandbox Studio



Credit: EHT

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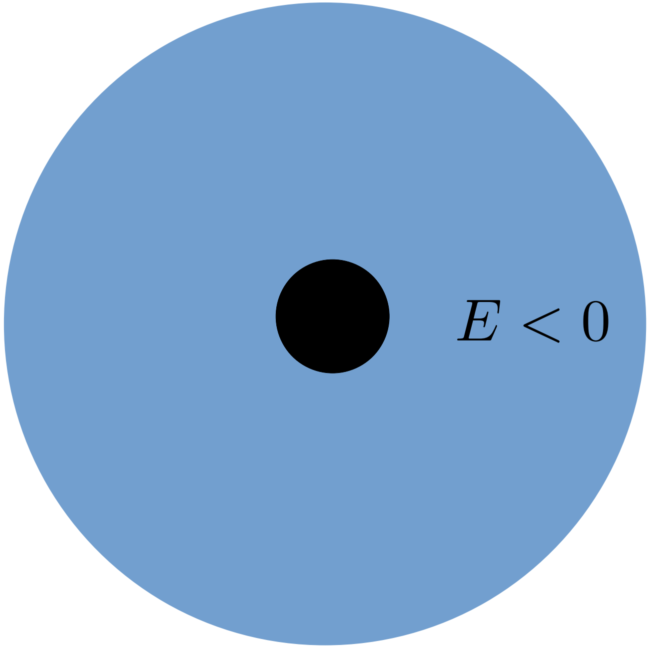

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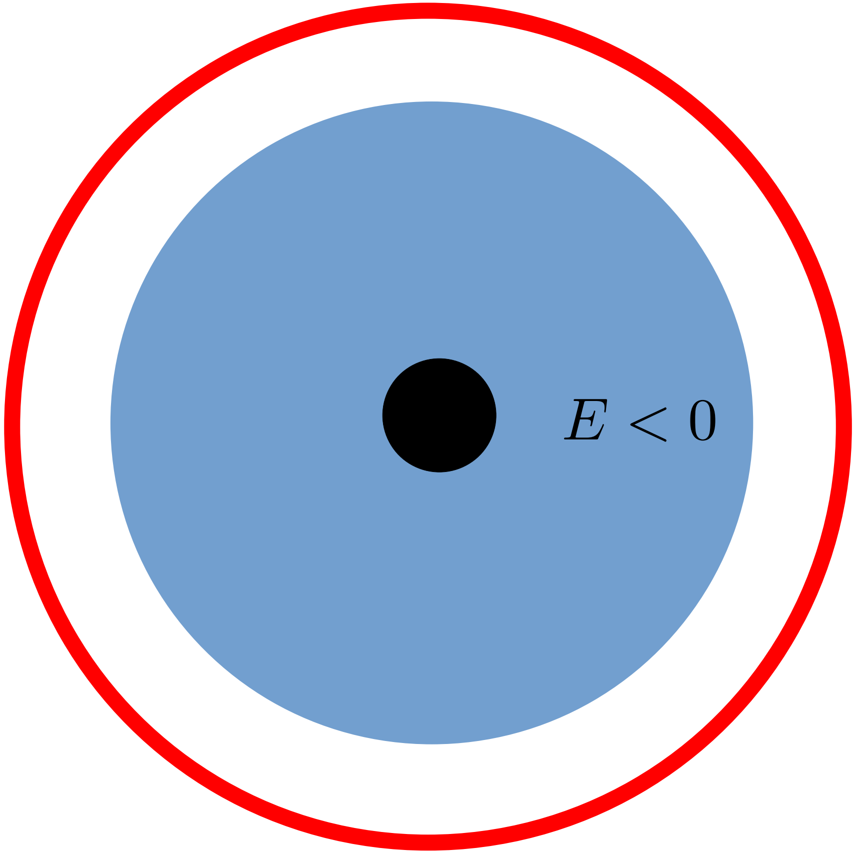
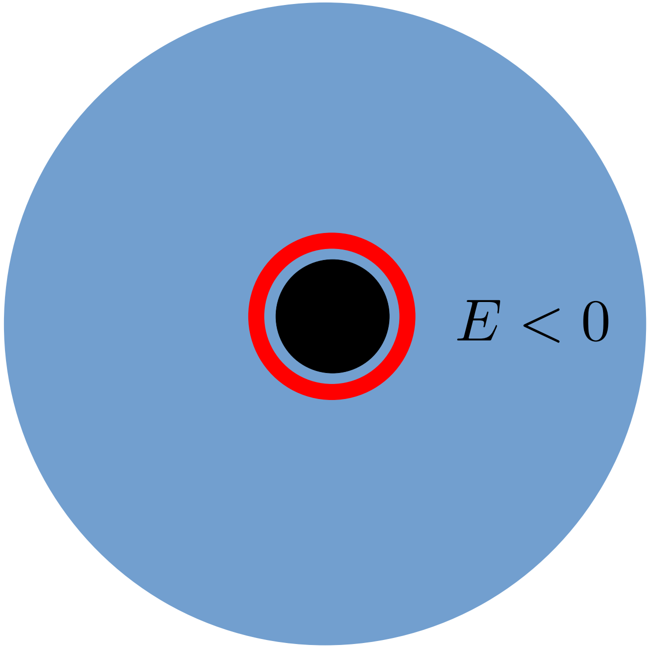


Press&Teukolsky
Nature 238, 211 (1972)

Introduction



Press&Teukolsky
Nature 238, 211 (1972)

Friedman
Commun. Math. Phys 63, 243 (1978)

Oliveira+
Phys. Rev. D 89, 12 (2014)
[arXiv:1405.4038]

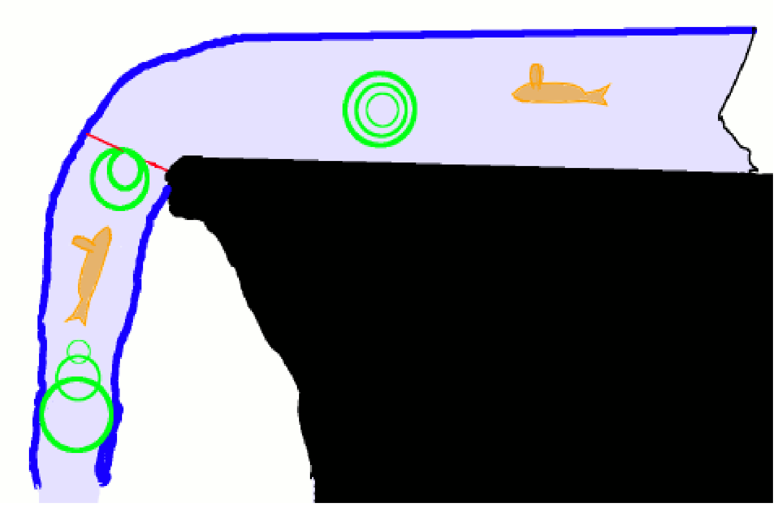
Introduction

Unruh
Phys.Rev.Lett. 46 (1981)

Visser (1998)
[arXiv:gr-qc/9901047]

Patrick
[arXiv:2009.02133]

Bergliaffa+
Physica D 191 (2004), 121
[arXiv:condmat/0106255]



Credit: Unruh

Perturbations in a fluid with vorticity

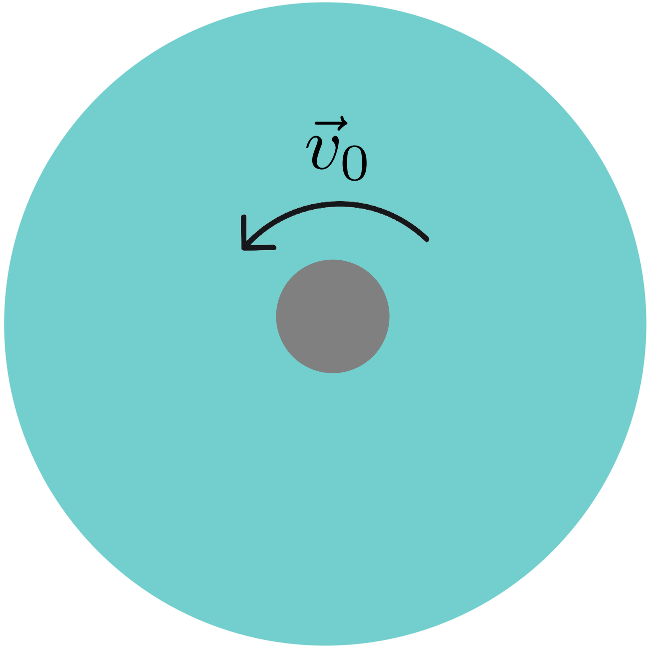
$$\vec{\omega}_0 = \nabla \times \vec{v}_0$$

$$\left(\Box + \frac{\omega_0^2}{c_s^2} \right) \Phi = 0$$

$$\Box \equiv \frac{1}{c_s^2} \frac{D^2}{Dt^2} - \nabla^2$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + (\vec{v}_0 \cdot \nabla)$$

Perturbations in a fluid with vorticity



$$\vec{v}_0 = v_\theta(r)\hat{\theta}$$

$$v_\theta(r) = \frac{Cr}{r_0^2 + r^2}$$

$$M = \left.\frac{|\vec{v}|}{c_\mathrm{s}}\right|_{r=r_\mathrm{e}} = 1$$

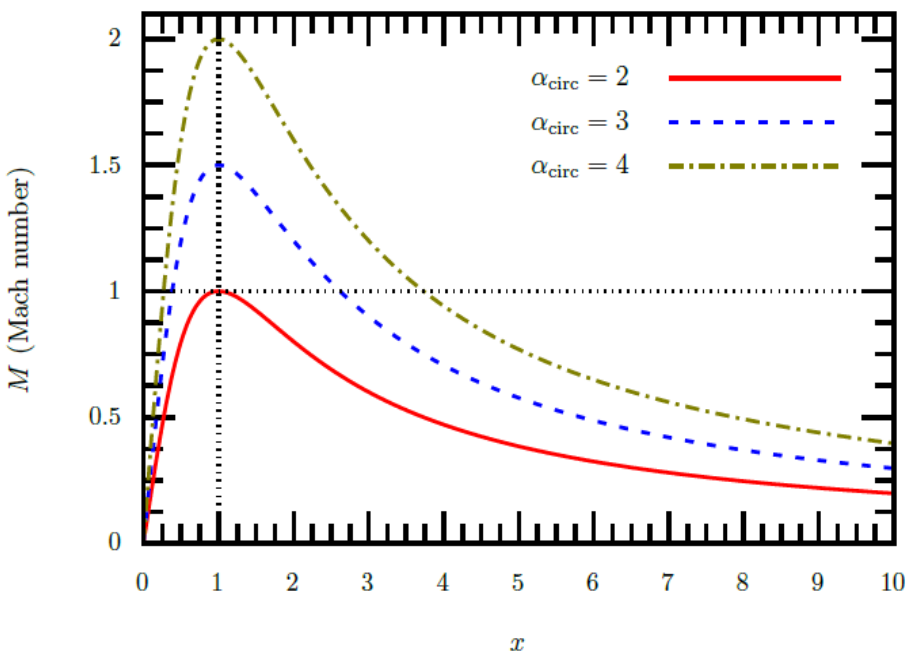
$$r_{\mathrm{e}\pm} = \frac{r_0}{2}\left(\alpha_\mathrm{circ} \pm \sqrt{\alpha_\mathrm{circ}^2 - 4}\right)$$

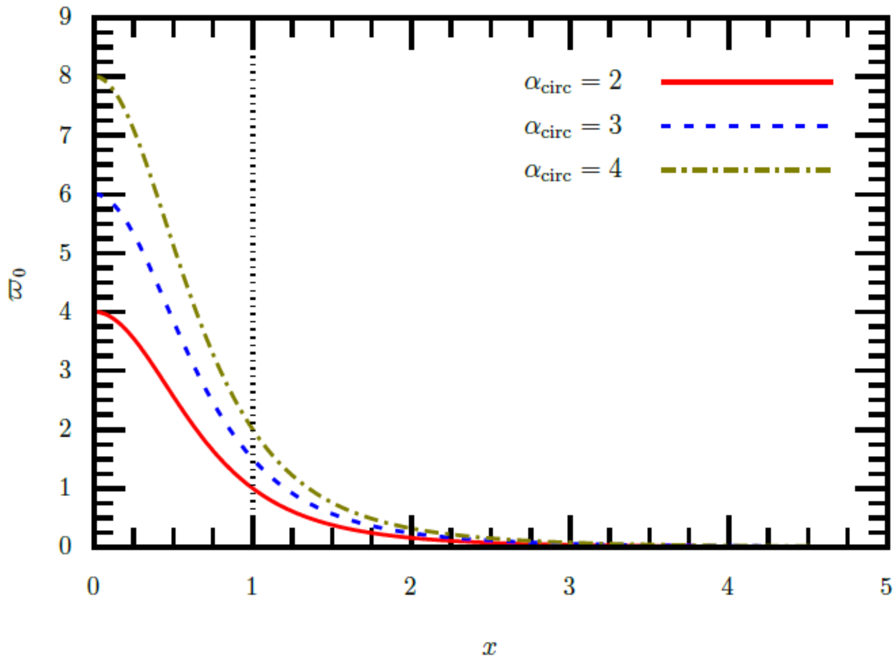
$$\alpha_\mathrm{circ} \equiv \pm\frac{|C|}{c_\mathrm{s} r_0}$$

Rosenhead
Proc. R. Soc. A 127, 590 (1930)

Perturbations in a fluid with vorticity

$$x = r/r_0, \quad \varpi_0 = \omega_0 r_0 / c_s$$





Perturbations in a fluid with vorticity

$$\Phi(t,r,\theta) = \frac{1}{\sqrt{r}} \sum_{m=-\infty}^{\infty} \phi_{\omega m}(r) \exp\left[i\left(m\theta - \omega t\right)\right]$$

$$\left[\frac{d^2}{dx^2} + \left(\varpi - \frac{\mathcal{M}m}{x}\right)^2 - \widetilde{V}_m(x)\right]\phi_{\varpi m}(x) = 0$$

$$\widetilde{V}_m(x) = \frac{m^2 - 1/4}{x^2} + \varpi_0^2$$

$$\mathcal{M} = \frac{\alpha_{\rm circ}\, x}{1 + x^2}$$

Boundary conditions

$$\phi_{\varpi m}\left(x \to \infty\right) \sim \exp\left(i\varpi x\right)$$

$$\left[\frac{d}{dx}\left(\frac{\phi_{\varpi m}(x)}{\sqrt{x}}\right)\right]_{x=x_{\rm in}} = 0$$

Methods: direct integration

$$\phi_{\varpi m}\left(x \to \infty\right) = \exp\left(i\varpi x\right) \sum_{j=0}^{j_{\max}} \frac{a_j}{x^j}$$

$$\infty > x \geq x_{\text{in}}$$

Methods: continued fraction

$$\phi_{\varpi m}(x) = \exp\left(i\varpi x\right)\sum_{n=0} a_n \left(1 - \frac{x_{\text{in}}}{x}\right)^n$$

$$\alpha_1 a_2 + \beta_1 a_1 + \gamma_1 a_0 = 0,$$
$$\alpha_2 a_3 + \beta_2 a_2 + \gamma_2 a_1 + \delta_2 a_0 = 0,$$
$$\alpha_3 a_4 + \beta_3 a_3 + \gamma_3 a_2 + \delta_3 a_1 + \varepsilon_3 a_0 = 0,$$
$$\alpha_4 a_5 + \beta_4 a_4 + \gamma_4 a_3 + \delta_4 a_2 + \varepsilon_4 a_1 + \zeta_4 a_0 = 0,$$
$$\alpha_5 a_6 + \beta_5 a_5 + \gamma_5 a_4 + \delta_5 a_3 + \varepsilon_5 a_2 + \zeta_5 a_1 + \eta_5 a_0 = 0,$$
$$\alpha_6 a_7 + \beta_6 a_6 + \gamma_6 a_5 + \delta_6 a_4 + \varepsilon_6 a_3 + \zeta_6 a_2 + \eta_6 a_1 + \lambda_6 a_0 = 0,$$
$$\alpha_7 a_8 + \beta_7 a_7 + \gamma_7 a_6 + \delta_7 a_5 + \varepsilon_7 a_4 + \zeta_7 a_3 + \eta_7 a_2 + \lambda_7 a_1 + \mu_7 a_0 = 0,$$
$$\alpha_8 a_9 + \beta_8 a_8 + \gamma_8 a_7 + \delta_8 a_6 + \varepsilon_8 a_5 + \zeta_8 a_4 + \eta_8 a_3 + \lambda_8 a_2 + \mu_8 a_1 + \nu_8 a_0 = 0,$$
$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} + \delta_n a_{n-2} + \epsilon_n a_{n-3} + \zeta_n a_{n-4} + \eta_n a_{n-5} + \lambda_n a_{n-6} + \mu_n a_{n-7} + \nu_n a_{n-8} + \xi_n a_{n-9} = 0, \quad \text{for } n \geq 9$$

Methods: continued fraction

$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} = 0, \qquad \text{for } n \geq 1$$

$$\frac{a_1}{a_0} = -\cfrac{\gamma_1}{\beta_1 - \cfrac{\alpha_1\gamma_2}{\beta_2 - \cfrac{\alpha_2\gamma_3}{\beta_3 - \ldots}}}$$

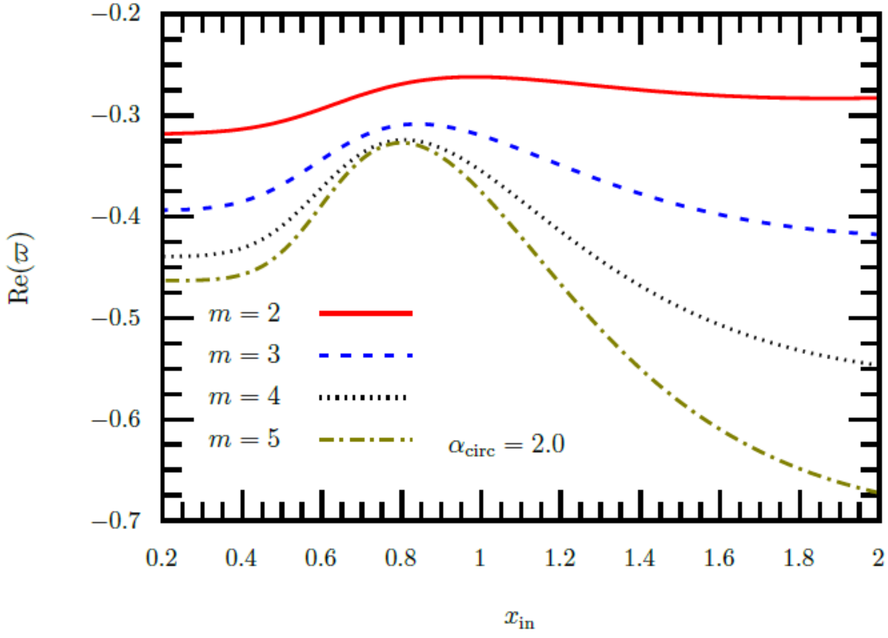
$$\frac{a_1}{a_0} = \frac{1}{2} - i\varpi x_{in}$$

$$1 - 2i\varpi x_{\mathrm{in}} + \cfrac{2\gamma_1}{\beta_1 - \cfrac{\alpha_1\gamma_2}{\beta_2 - \cfrac{\alpha_2\gamma_3}{\beta_3 - \ldots}}} = 0$$

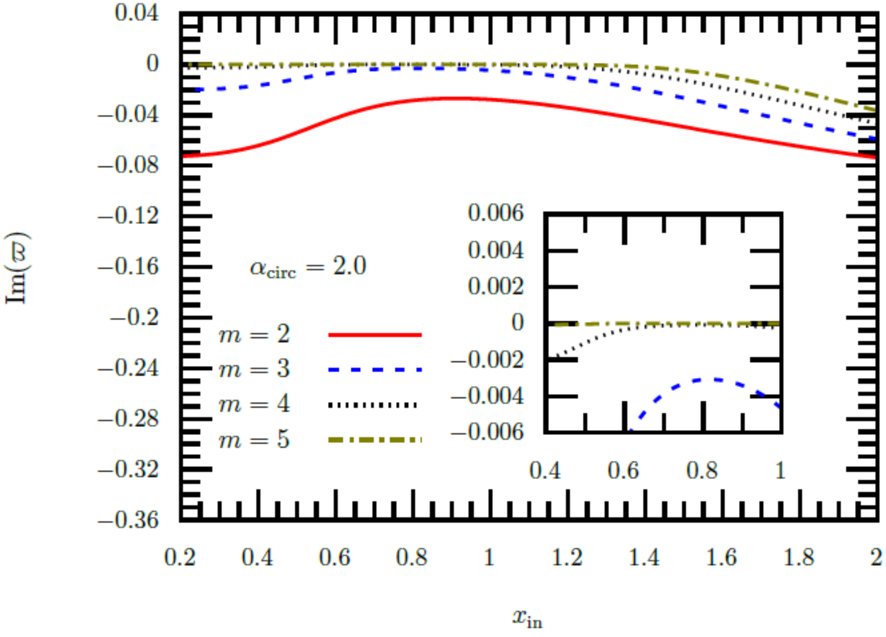
Results

$m = 2$				
	$\alpha_{\text{circ}} = 2.0$		$\alpha_{\text{circ}} = 4.0$	
Method	$\text{Re}(\varpi)$	$\text{Im}(\varpi)$	$\text{Re}(\varpi)$	$\text{Im}(\varpi)$
DI	$-0.2738630610(6)$	$-0.030470272(6)$	$+0.8519075782544$	$+0.005030287188(6)$
CF	$-0.2738630610(1)$	$-0.030470272(5)$	$+0.8519075782544$	$+0.005030287188(8)$
$m = 3$				
	$\alpha_{\text{circ}} = 2.0$		$\alpha_{\text{circ}} = 4.0$	
Method	$\text{Re}(\varpi)$	$\text{Im}(\varpi)$	$\text{Re}(\varpi)$	$\text{Im}(\varpi)$
DI	$-0.313570188531(6)$	$-0.003311230409(6)$	$+2.00568207201(1)$	$+0.002400344286(5)$
CF	$-0.313570188531(2)$	$-0.003311230409(3)$	$+2.00568207201(0)$	$+0.002400344286(4)$

Results

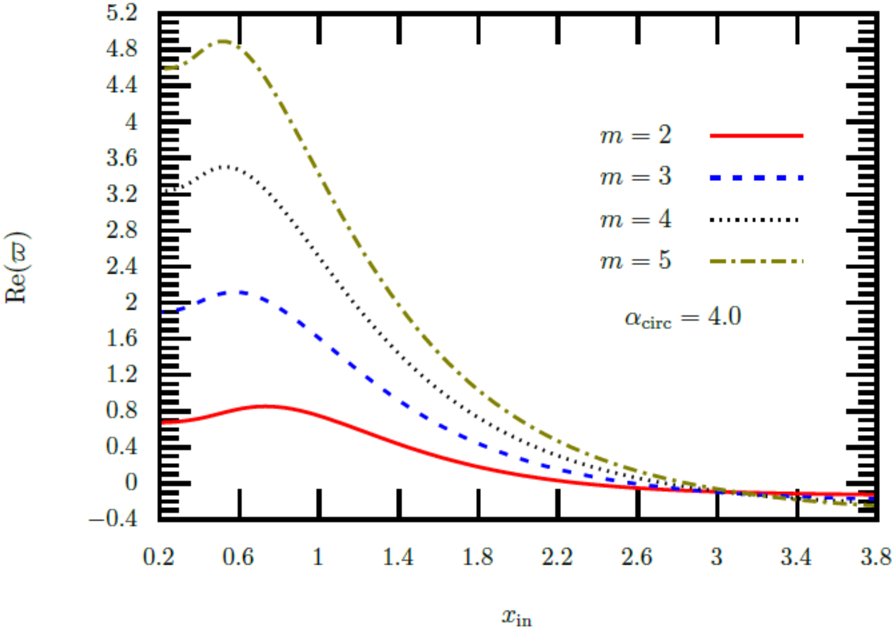


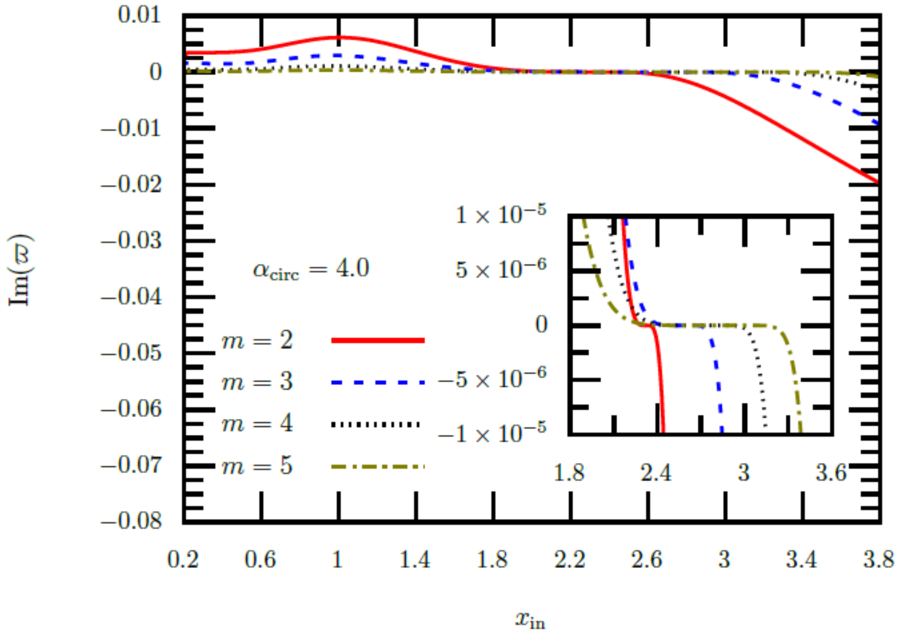
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
Re(ϖ)
$m = 2$
$m = 3$
$m = 4$
$m = 5$
$\alpha_{\rm circ} = 2.0$
0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
$x_{\rm in}$



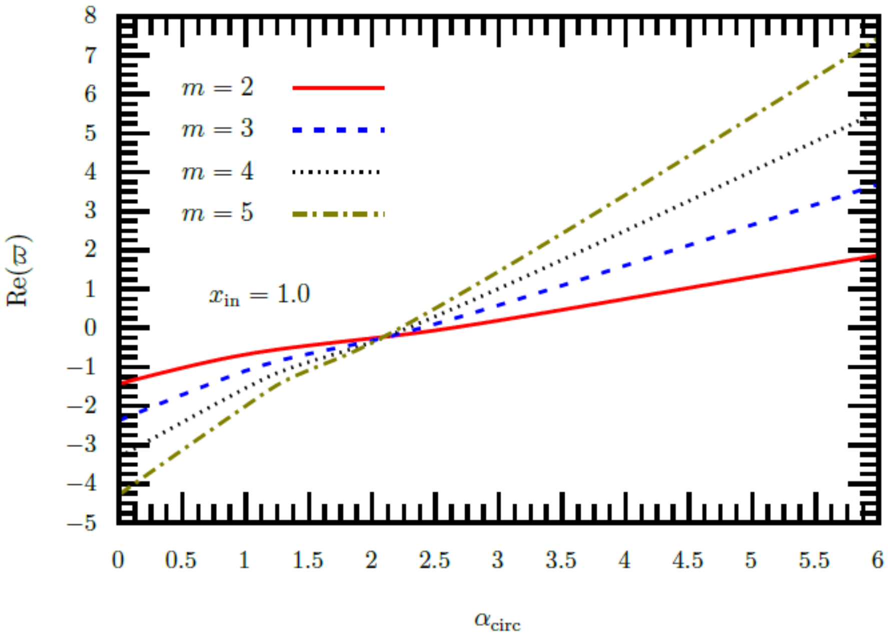
0.04
0
−0.04
−0.08
−0.12
−0.16
−0.2
−0.24
−0.28
−0.32
−0.36
Im(ϖ)
$\alpha_{\rm circ} = 2.0$
$m = 2$
$m = 3$
$m = 4$
$m = 5$
0.006
0.004
0.002
0
−0.002
−0.004
−0.006
0.4 0.6 0.8 1
0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
$x_{\rm in}$

Results

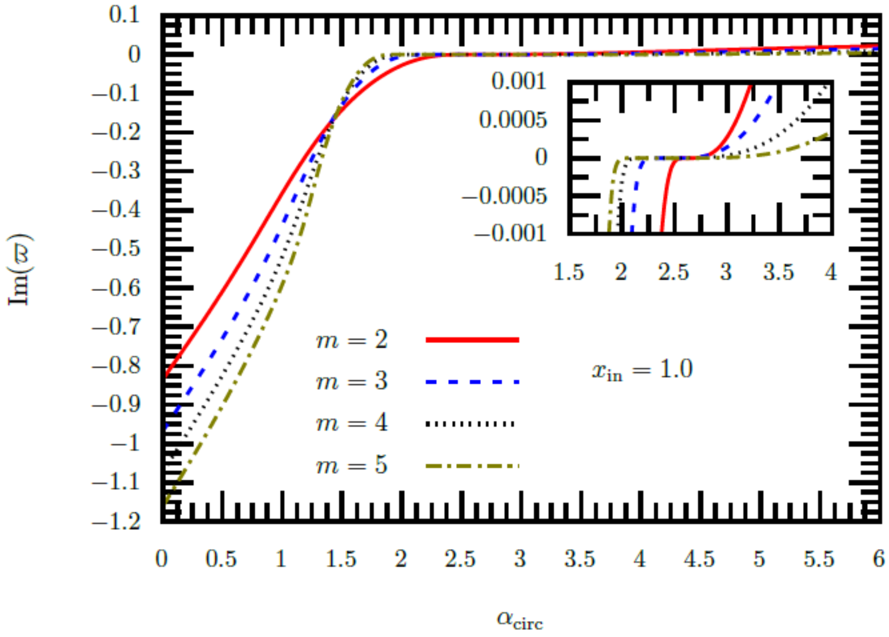

$m = 2$
$m = 3$
$m = 4$
$m = 5$
$\alpha_{\mathrm{circ}} = 4.0$
$\mathrm{Re}(\varpi)$
x_{in}


$\alpha_{\mathrm{circ}} = 4.0$
$m = 2$
$m = 3$
$m = 4$
$m = 5$
$\mathrm{Im}(\varpi)$
x_{in}
1×10^{-5}
5×10^{-6}
-5×10^{-6}
-1×10^{-5}

Results

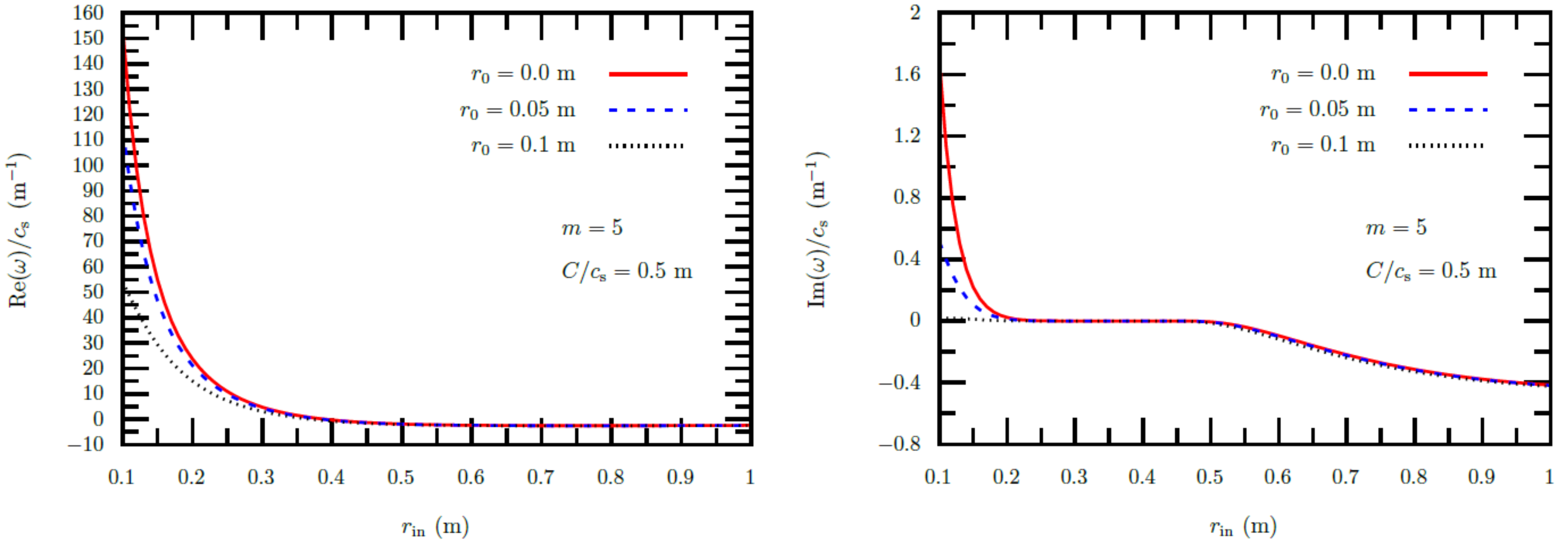


$m = 2$
$m = 3$
$m = 4$
$m = 5$
$x_{\rm in} = 1.0$
Re(ϖ)
$\alpha_{\rm circ}$



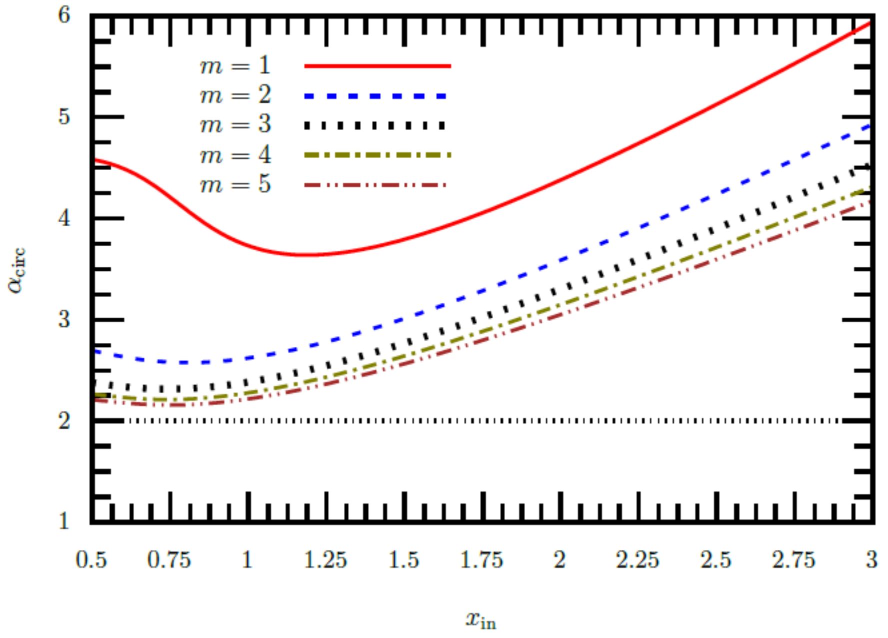
$m = 2$
$m = 3$
$m = 4$
$m = 5$
$x_{\rm in} = 1.0$
Im(ϖ)
$\alpha_{\rm circ}$

Results



160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
−10
$\mathrm{Re}(\omega)/c_s\ (\mathrm{m}^{-1})$
$r_0 = 0.0$ m
$r_0 = 0.05$ m
$r_0 = 0.1$ m
$m = 5$
$C/c_s = 0.5$ m
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r_{in} (m)
2
1.6
1.2
0.8
0.4
0
−0.4
−0.8
$\mathrm{Im}(\omega)/c_s\ (\mathrm{m}^{-1})$
$r_0 = 0.0$ m
$r_0 = 0.05$ m
$r_0 = 0.1$ m
$m = 5$
$C/c_s = 0.5$ m
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r_{in} (m)

Results



6
5
4
3
2
1
$\alpha_{\rm circ}$
$m = 1$
$m = 2$
$m = 3$
$m = 4$
$m = 5$
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
$x_{\rm in}$

Final remarks

- Systems with ergoregion but no event horizon present an instability
- The ergoregion instability is a large-m phenomenon
- The vorticity tends to diminish the instability of the system

Coming soon!

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Obrigada! Thank you!

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