Shadow of a collapsing star in a regular spacetime

Juan Carlos Degollado
Instituto de Ciencias Físicas, UNAM

work in collaboration with

Daniel Núñez Trigueros

VI Amazonian Simposium on Physics

Outline

1. Singularities and black holes.

2. Lensing by black holes, photon sphere.

3. Spherical collapse.

4. Description of the angular size of shadow.

Objective

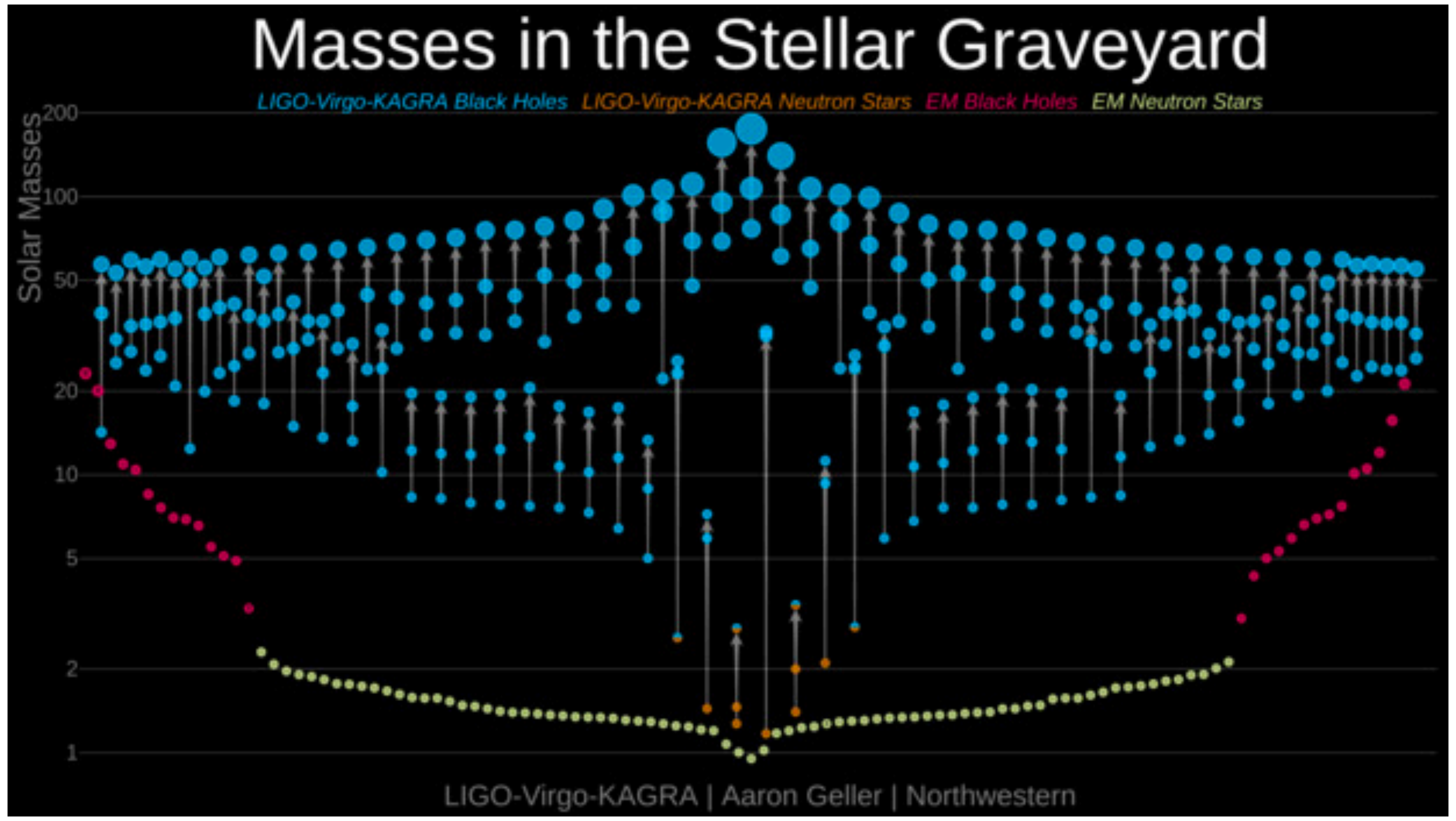
The shadow of a black hole refers to the projection of the region around the hole in which circular orbits of light exist on an observer's sky.

In this work we present a theoretical model of how this shadow forms over time when a black hole forms through a gravitational collapse in a spacetime without singularities.

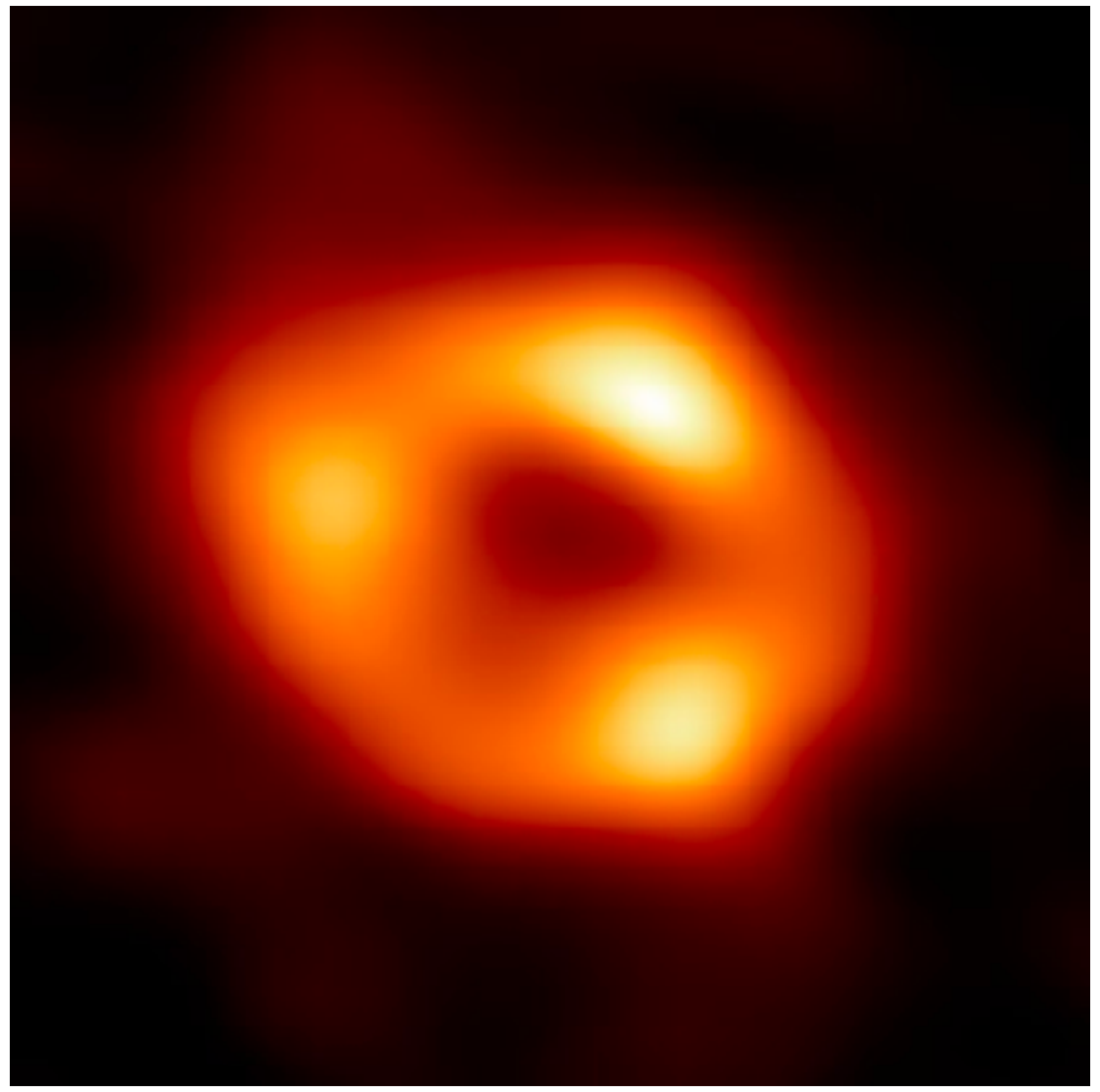
We examine a star that is spherically symmetric, dark, and opaque, assuming it begins to collapse in free fall, resembling a pressure less fluid.

General Relativity

Einstein's theory of general relativity gave rise to three main predictions: black holes, gravitational waves, and spacetime singularities. These have been the core of the most interesting developments in theoretical and experimental physics in recent decades. The most precise observations to date have detected the first two of these predictions during the last few years.



LIGO-Virgo / Aaron Geller / Northwestern University.



https://eventhorizontelescope.org

Singularities in GR

The Penrose and Hawking singularity theorems state that, under the assumption of the presence of matter satisfying physically reasonable energy conditions, the existence of singularities is inevitable in General Relativity.

Singularities must appear as the final state of collapse once a series of conditions are met:

i) The validity of General Relativity during the collapse,

ii) The validity of some energy condition.

iii) The global hyperbolicity of space-time and

iv) The formation of trapped surfaces at some point during the collapse.

Hawking, S.W.; Penrose, R. , Proc. R. Soc. Lond. A, 314, 529–548 (1970)

Rise of Singularities

Energy conditions refer to the behavior of matter at macroscopic scales and are not necessarily valid to apply to matter fields in the strong curvature regime, in particular, near the formation of the singularity where quantum effects may be relevant.

Therefore, models that allow violations of energy conditions towards the end of the collapse can be considered even within General Relativity.

To prevent the formation of a singularity while retaining the formation of trapped surfaces, we must demand that either General Relativity does not apply throughout the collapse and/or that the energy conditions are violated at some point.

Regular black holes

Bardeen proposed the first model of an asymptotically flat, static, and spherically symmetric black hole with a regular center.

This type of black hole is called a *regular black hole* or *non-singular black hole*. Initially, this model was not obtained as an exact solution of Einstein's equations, but later, Ayón-Beato and García showed that the Bardeen model can be considered as a solution of Einstein's equations coupled with a physical source of a magnetic monopole in a nonlinear electrodynamics.

J .M . Bardeen, Proceedings of International Conference GR5. Tbilisi, USSR, 1968.

E. Ayón-Beato and A. García, Phys. Lett. B 493, 149-152 (2000)

Static and spherically symmetric spacetime

Starting from the Schwarzschild solution, new non-empty spacetimes can be designed, where the non-zero energy-momentum tensor is interpreted as an effective correction. These solutions do not present a central singularity. As a first attempt, one can consider the outer line element in coordinates $\{T, R, \theta, \phi\}$, written as:

$$ds^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Omega^2 \, , \qquad f(R) = 1 - \frac{2M(R)}{R} \, ,$$

Hayward spacetime

Spherically symmetric spacetime used to describe a black hole-like structure with a regular core

$$ds^2 = r^2 dS^2 + dr^2/F(r) - F(r)dt^2$$

$$F(r) = 1 - \frac{2mr^2}{r^3 + 2l^2 m}$$

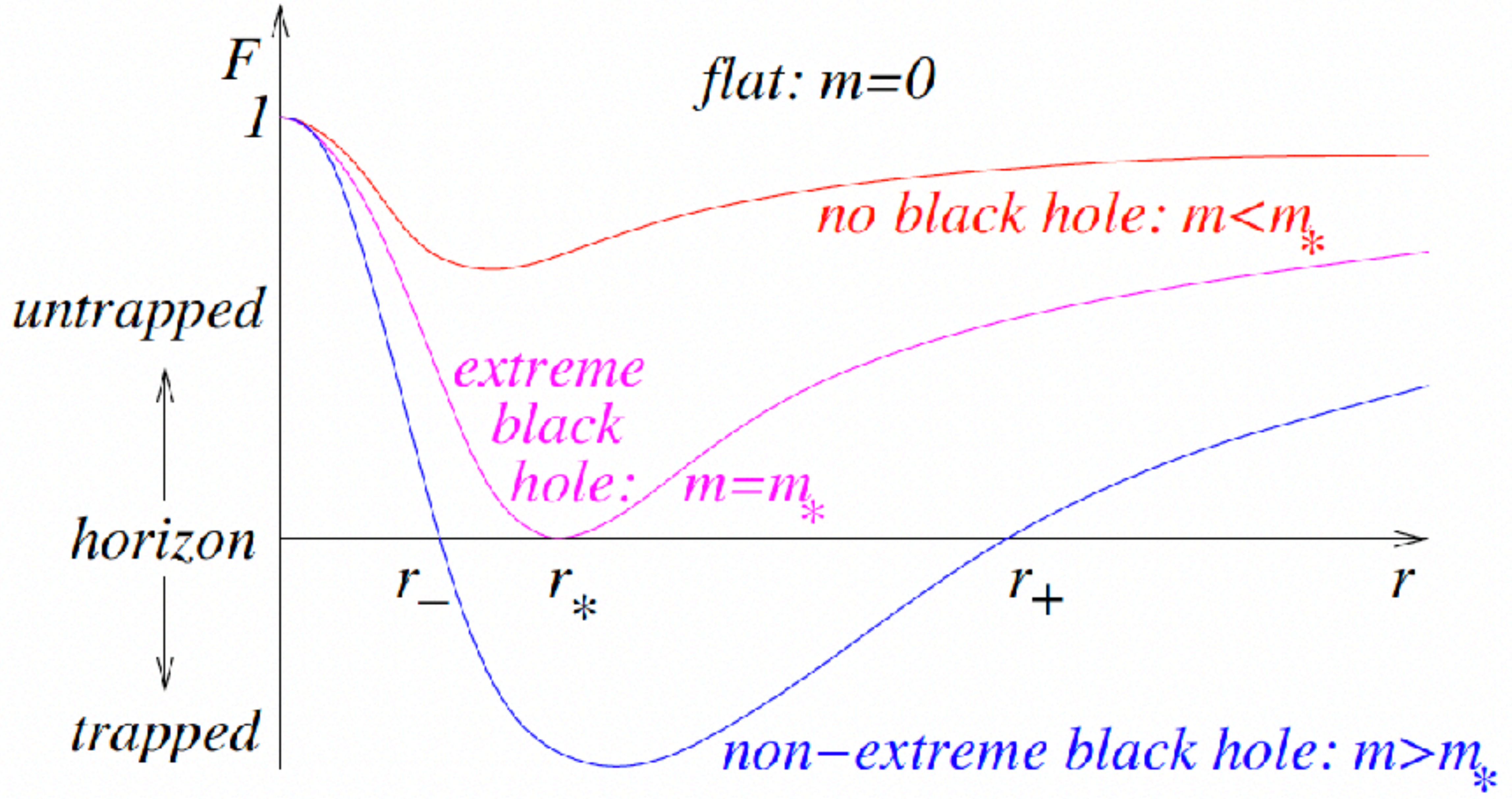
This spacetime is regular in the sense that the Kretschmann scalar is finite in the limit R -> 0.

$r \to \infty.$

$$F(r) \sim 1 - 2m/r$$

$r \to 0$

$$F(r) \sim 1 - r^2/l^2$$



$$m_* = (3\sqrt{3}/4)l$$

Sean A. Hayward. Phys.Rev.Lett. 96 (2006)

Hayward spacetime

The metric was not originally derived from any particular alternative theory of gravity, nevertheless it provides a framework to test the formation and evaporation of non-singular black holes both within general relativity and beyond.

Later, Fang and Wang, found that could be associated with a nonlinear electromagnetic field.

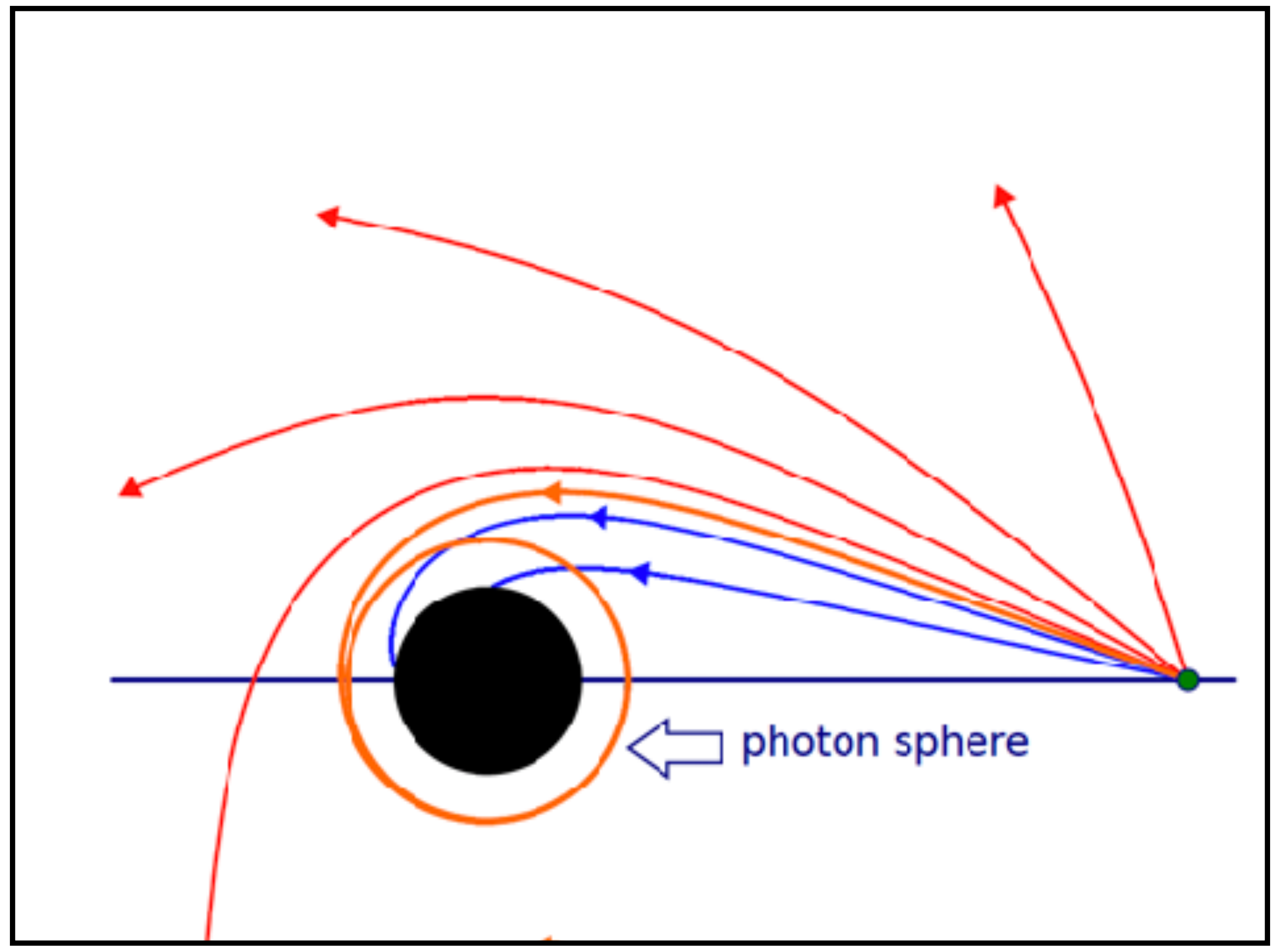
$$\mathcal{L} = \frac{4\mu}{\alpha} \frac{(\alpha \mathcal{F})^{\frac{\mu+3}{4}}}{\left(1 + (\alpha \mathcal{F})^{\frac{\mu}{4}}\right)^2} . \qquad A = Q_m \cos\theta \, d\phi$$

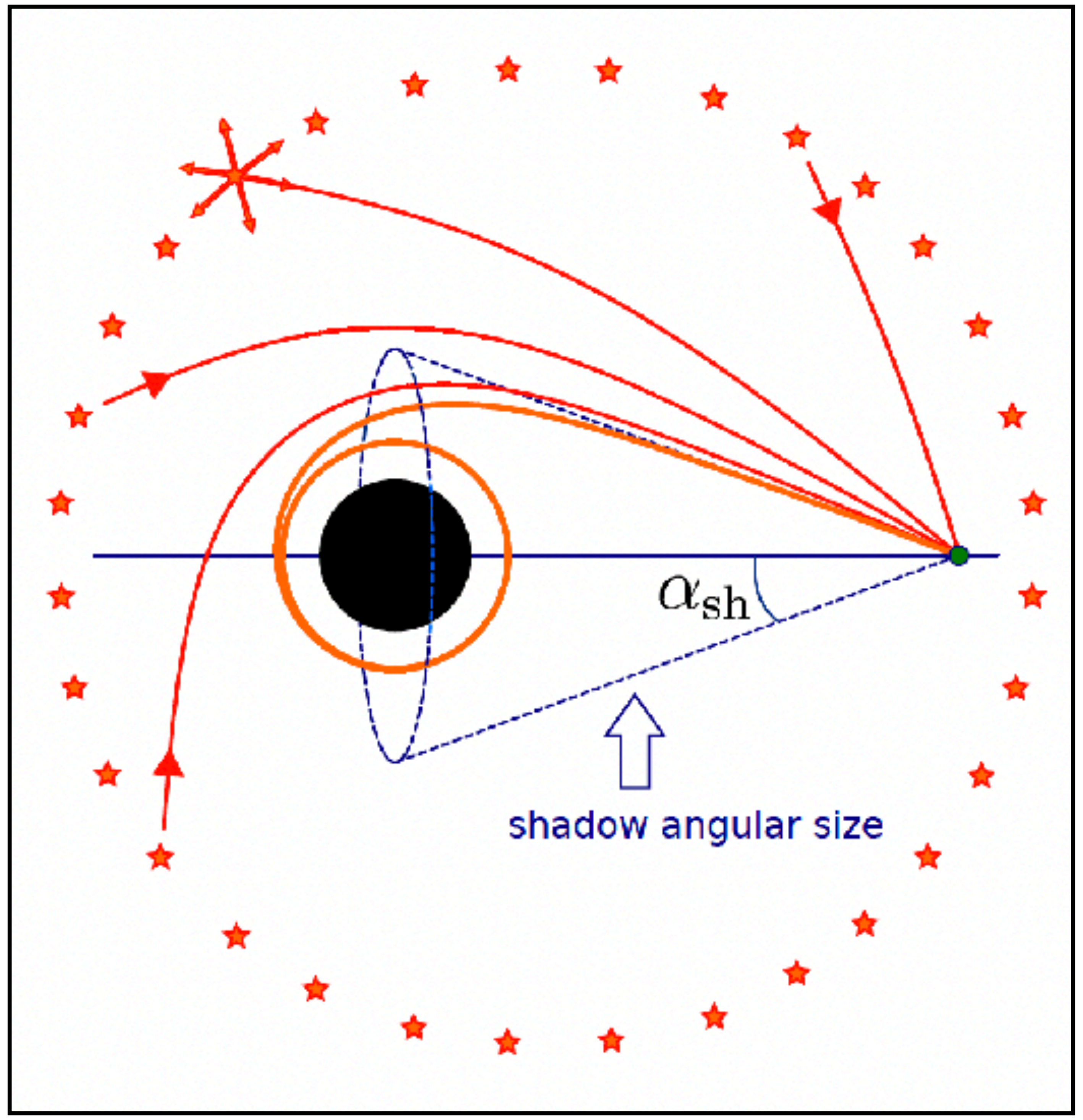
Fang, Wang. Phys. Rev. D 94, 124027 (2016)

Angular size

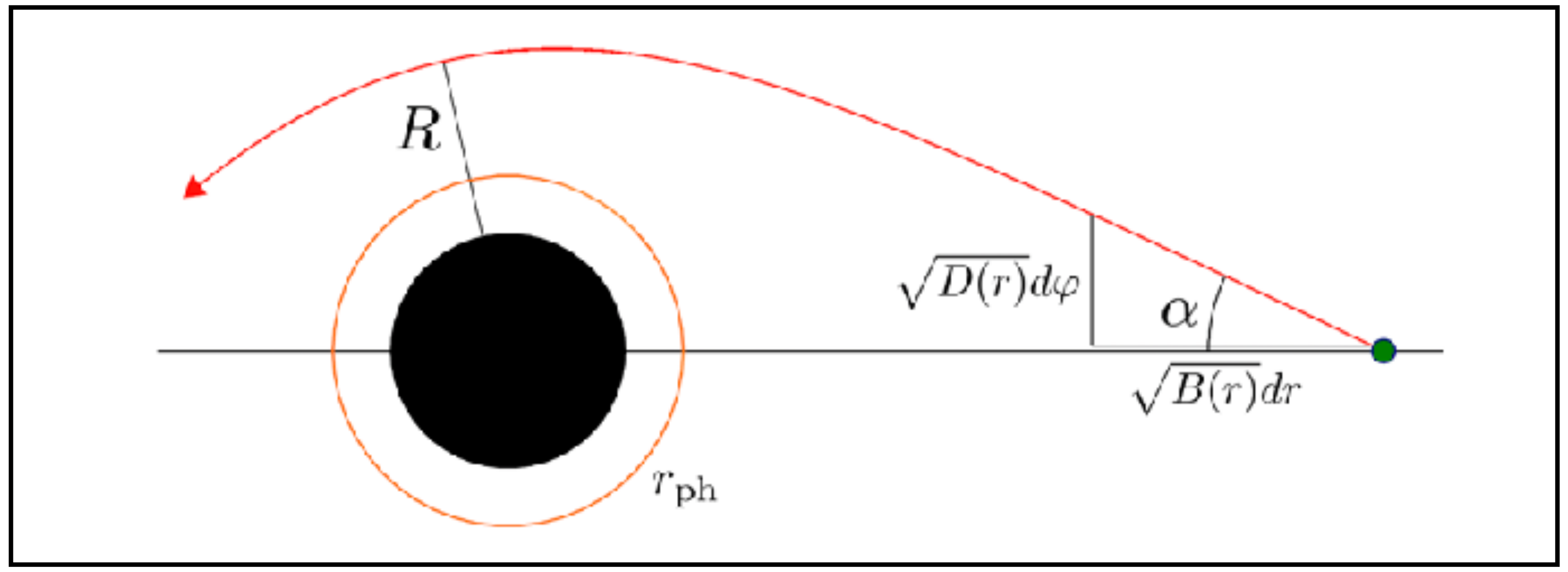
The ring diameter surrounding the shadow of a black hole is 5.2 (Schwarzschild) times the diameter of the event horizon and the size in the plane of the distant observer scales with the mass of the black hole and its distance from the observer.

Ray tracing





Perlik, V., Tsupk Physics Reports, 947, 1-39 (2022).



Schneider, S., Perlik, V. Gen. Rel. Grav., 50(6):58, 2018

$$\cot\,\alpha = \frac{\sqrt{g_{rr}}}{\sqrt{g_{\varphi\varphi}}}\,\frac{dr}{d\varphi}\bigg|_{r=r_O}$$

Gravitational dust collapse

The gravitational collapse of dust in spherical symmetry, the Oppenheimer-Snyder (OS) collapse, is obtained by solving the field equations for a sphere of homogeneous fluid without pressure (dust).

During the collapse of a star to a black hole the geometry, on the outside of the sphere, is the Schwarzschild geometry, whereas the geometry inside is the Robertson-Walker geometry.

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left(\frac{\mathrm{d}r^2}{1 - kr^2} + r^2 \mathrm{d}\theta^2 + r^2 \sin^2\theta \, \mathrm{d}\phi^2 \right)$$

Gravitational dust collapse

In an attempt to consider other effects that contribute to regularizing space-time, the following action is considered:

$$\mathcal{A} = \frac{1}{16\pi} \int d^4x \sqrt{|g|} \left(\mathrm{R} - \mathcal{L}_{\mathrm{Dust}} - \mathcal{L}_{\mathrm{corr}} \right) ,$$

The effective stress energy tensor has two contributions : $\mathrm{T}^{\mathrm{eff}}_{\kappa\lambda} = \mathrm{T}^{\mathrm{Dust}}_{\kappa\lambda} + \mathrm{T}^{\mathrm{corr}}_{\kappa\lambda} ,$

One for the dust, with four velocity u^κ $\mathrm{T}^{\kappa\lambda}_{\mathrm{Dust}} = \epsilon u^\kappa u^\lambda ,$

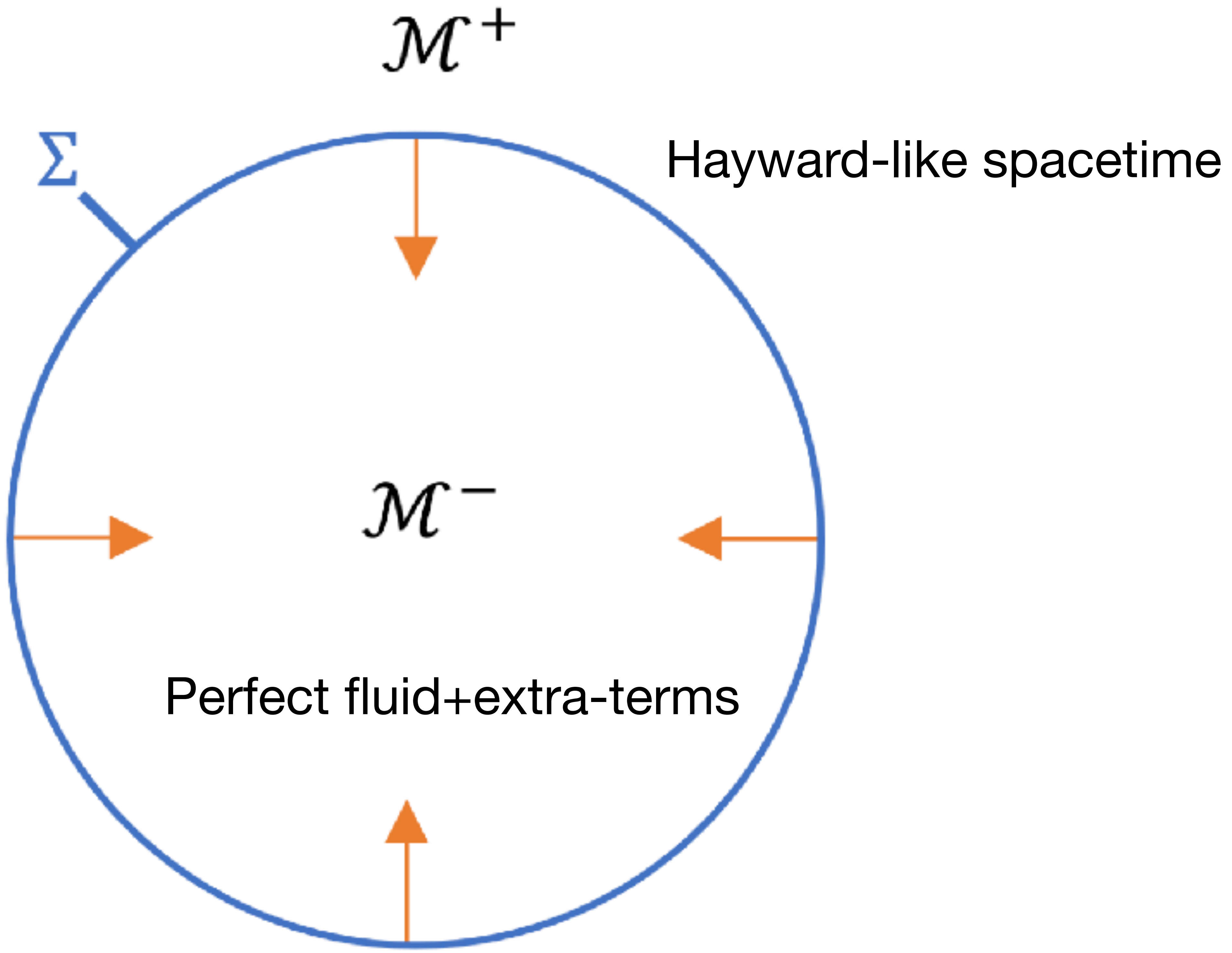
An effective energy density is expanded as

$$\epsilon_{\mathrm{eff}} = \epsilon + \alpha_1 \epsilon^2 + \alpha_2 \epsilon^3 + \dots .$$

Daniele Malafarina, Bobir Toshmatov. Phys. Rev. D 105, L121502 (2022)

Matching conditions

The matching conditions are applied in order connect the exterior with the region inside the collapsing shell



$$[\gamma_{ab}] = \gamma^+_{ab} - \gamma^-_{ab} = 0$$

$$[K_{ab}] = K^+_{ab} - K^-_{ab} = 0.$$

$$K_{ab} = g_{\mu\nu} n^\mu \nabla_a e^\nu_b$$

Extrinsic curvature

Gravitacional collapse OS

The line element to describe the collapse dynamics is

$$ds^2 = -dt^2 + B'^2 dr^2 + B^2 d\Omega^2 \,,$$

That in terms of the expansion factor *a*

$$B(r,t) = ra(t)\,,$$

The solution for OS collapse has the form:

$$\dot{a} = -\sqrt{\frac{m_0}{a}}\,, \qquad \epsilon = \frac{3m_0}{a^3}\,,$$

m_0 is the total mass of the distribution

Gravitacional collapse, Regular ET

We consider the exterior region is described by a metric of the form

$$ds^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Omega^2 \,, \qquad f(R) = 1 - \frac{2\mathcal{M}(R)}{R} \,,$$

A family of regular black holes is described by

$$\mathcal{M}(R) = \frac{M R^{\mu}}{(R^{\nu} + q_*^{\nu})^{\mu/\nu}} \,.$$

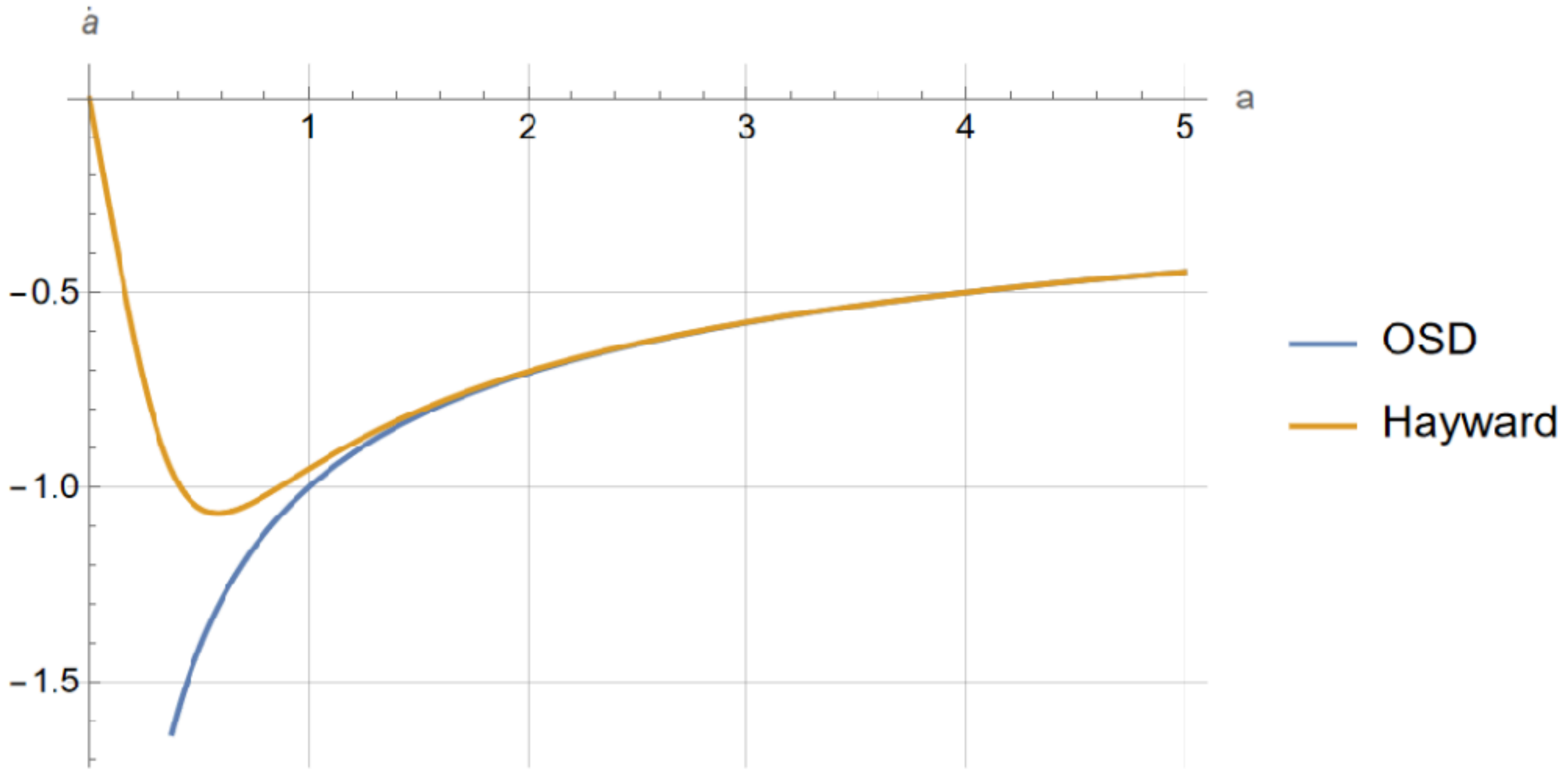
It reduces to the Shwarzschild spacetime when *q* -> 0*

Hayward spacetime formation

With a general parameterization, one can write the energy density and scaling factor in terms of the values for OS collapse

$$\epsilon_{\rm ef} = \frac{3m_{\rm ef}}{a^3} \qquad \dot{a} = -\sqrt{\frac{m_0}{a}\left[1 \pm \frac{a_{\rm cr}^3}{a^3}\right]^\gamma}$$

$$m_{\rm ef} = a\dot{a}^2.$$



and the Hayward spacetime

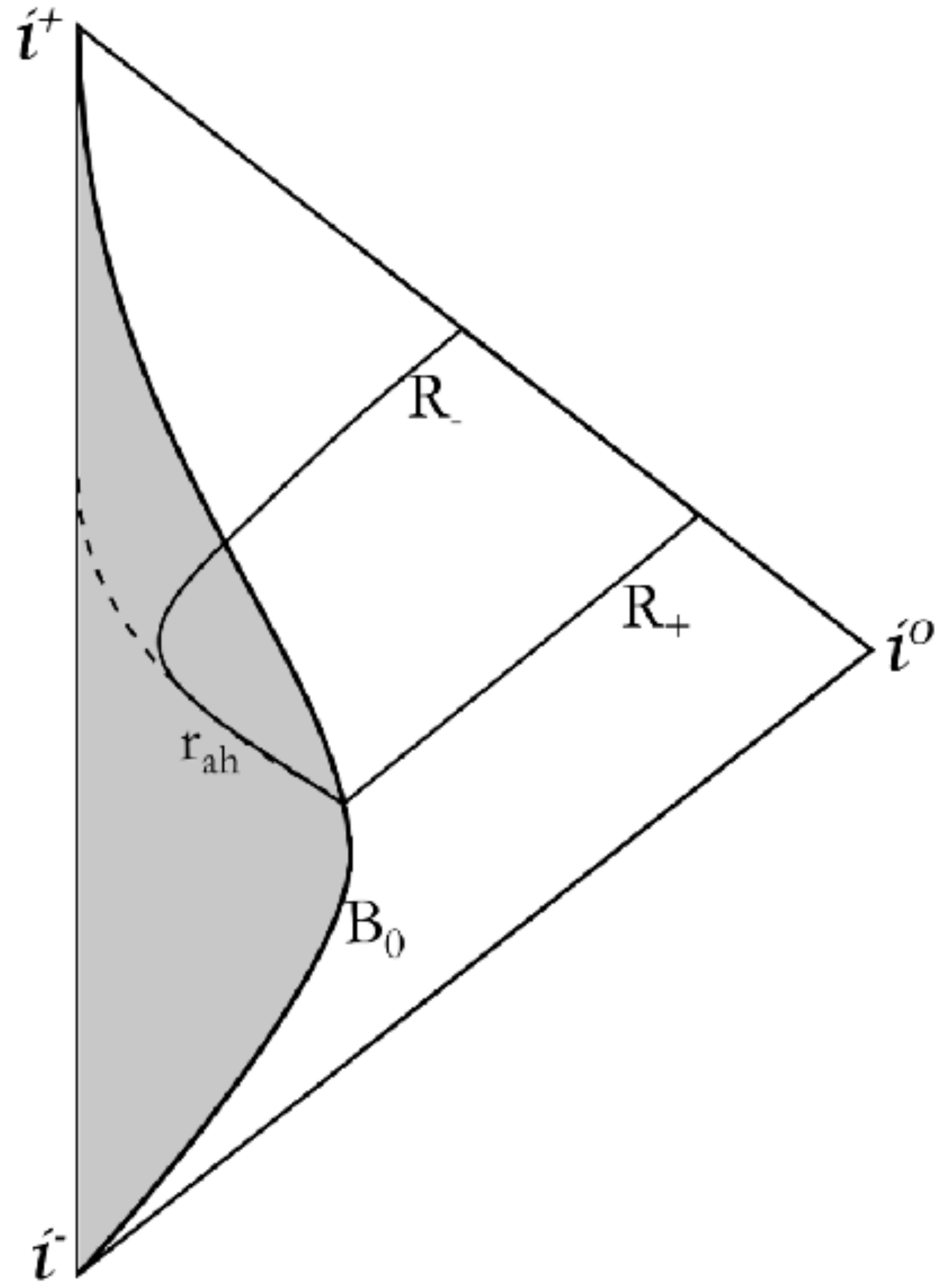
$$\epsilon_{\rm eff} = \epsilon\left(1 - \frac{\epsilon}{\epsilon_{\rm cr} + \epsilon}\right). \qquad \dot{a} = -\sqrt{m_0 \frac{a^2}{a^3 + q^3}}.$$

$$\epsilon_{\rm corr} = -\frac{\epsilon^2}{\epsilon_{\rm cr} + \epsilon}$$

Hayward spacetime formation

The effective matter content can be associated with a effective stress energy tensor of a perfect fluid

$$T^{\mu\nu}_{\text{ef}} = (\epsilon_{\text{ef}} + p_{\text{ef}}) u^\mu u^\nu + p_{\text{ef}} g^{\mu\nu} \qquad p_{\text{ef}} = -\frac{\epsilon^2 \epsilon_{\text{cr}}}{(\epsilon_{\text{cr}} + \epsilon)^2}$$



Penrose diagram of gravitational collapse

that leads to the formation of a Hayward BH

Angular size of the shadow

We determine the time dependent angular radius of the star's shadow, from the perspective of a stationary observer positioned at a fixed distance and from the viewpoint of an observer falling toward the center along with the collapsing star.

In Painlevé-Gullstrand coordinates:

$$ds^2 = -f(r)dT^2 + 2dTdr\sqrt{\frac{2M(r)}{r}} + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,, \qquad f(r) = 1 - \frac{2M(r)}{r}\,, \qquad M(r) = \frac{M_0 r^3}{r^3 + q^3}\,,$$

Static observer

We define a tetrad for a static observer:

$$e_0 = \frac{1}{\sqrt{1-\frac{2M(r)}{r}}}\frac{\partial}{\partial T}\,, \qquad e_1 = \sqrt{1-\frac{2M(r)}{r}}\frac{\partial}{\partial r} + \frac{\sqrt{2M(r)/r}}{\sqrt{1-\frac{2M(r)}{r}}}\frac{\partial}{\partial T}\,, \qquad e_2 = \frac{1}{r}\frac{\partial}{\partial \theta}\,, \qquad e_3 = \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\,.$$

From the equations for null geodesics:

$$\dot{T}\frac{\partial}{\partial T} + \dot{r}\frac{\partial}{\partial r} + \dot{\varphi}\frac{\partial}{\partial \varphi} = \chi(e_0 + \cos\alpha e_1 - \sin\alpha e_3)\,. \qquad \dot{r} = \chi\sqrt{1-\frac{2M(r)}{r}}\cos\alpha\,. \qquad \dot{\varphi} = \frac{-\chi\sin\alpha}{r}\,.$$

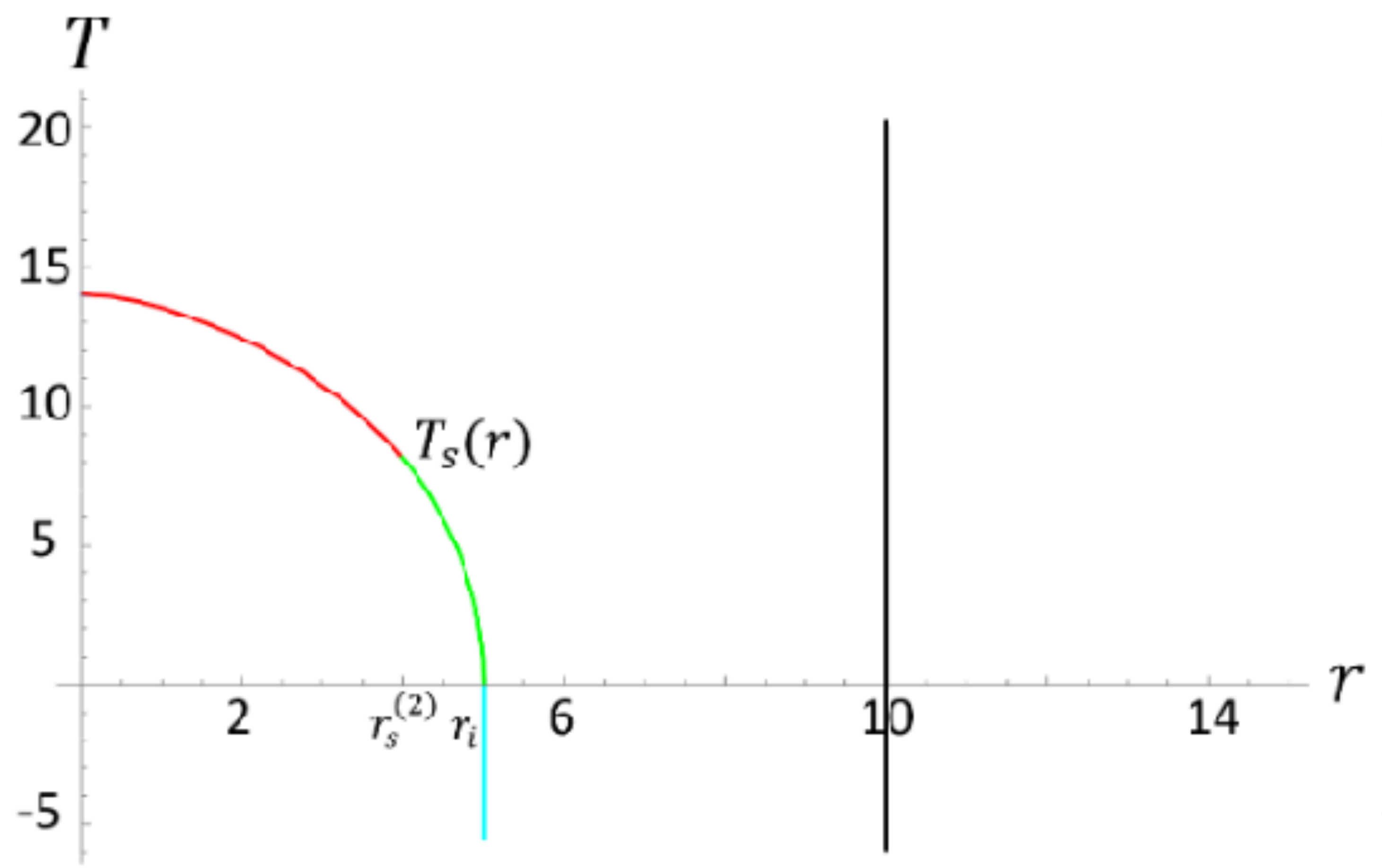
The angle made by the photons that the observer receives:

$$\sin\alpha = \sqrt{\mathcal{R}(r, r_m)} \qquad \mathcal{R}(r, r_m) = \frac{r_m^3(r-2M(r))}{r^3(r_m-2M(r_m))}\,.$$

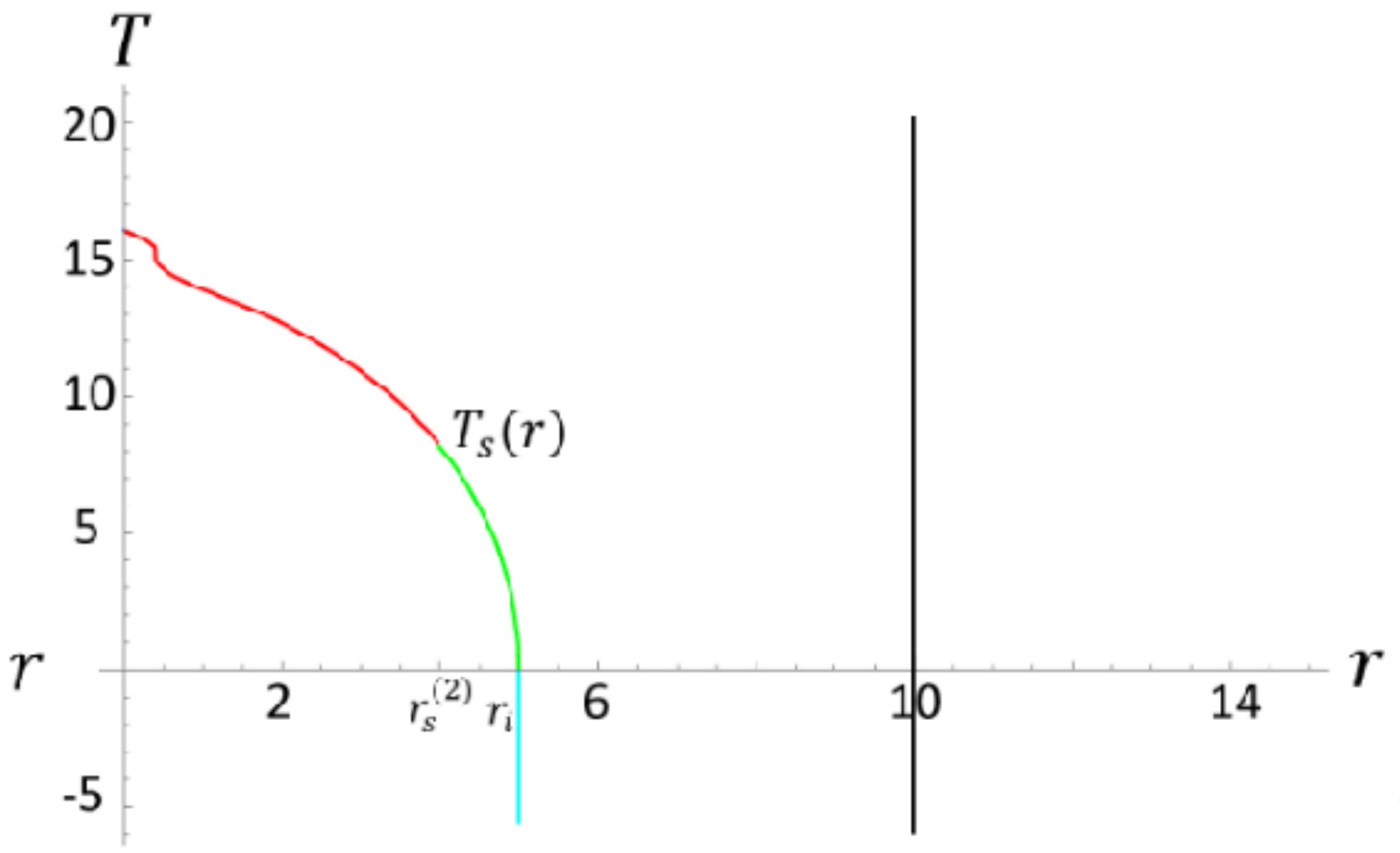
Angular size of the dark region

Trajectory of the surface of the dust cloud

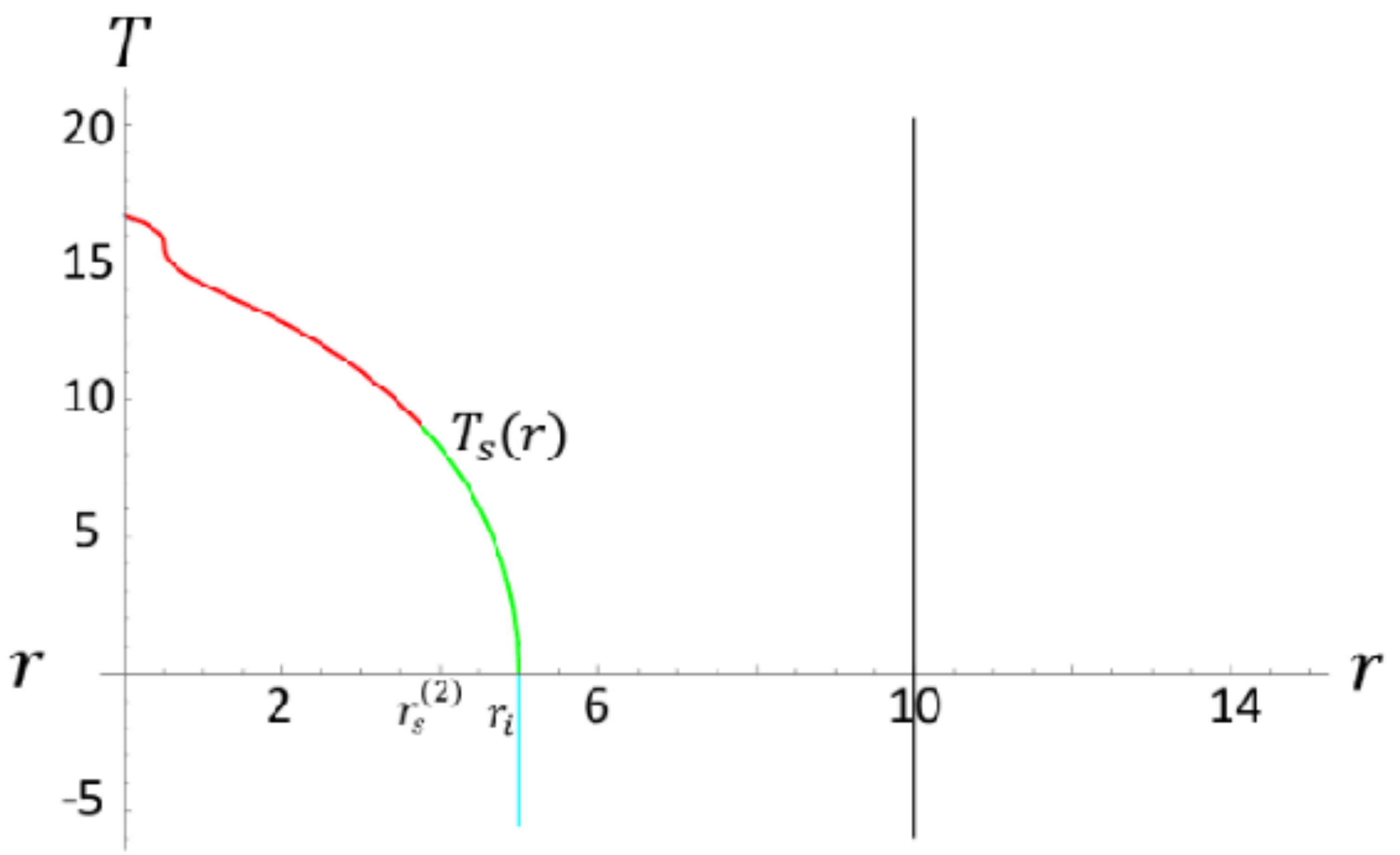
Static observer



(a) $q_* = 0.0$

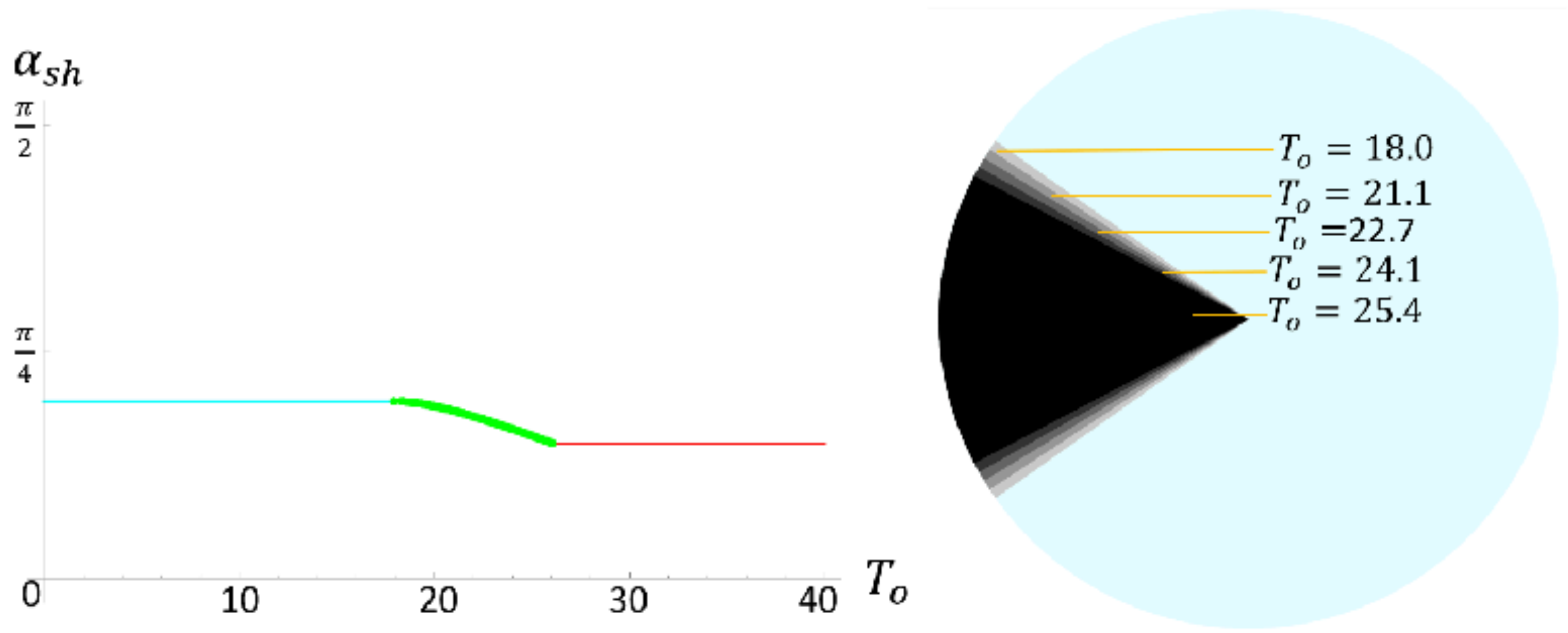


(b) $q_* = 0.8967$

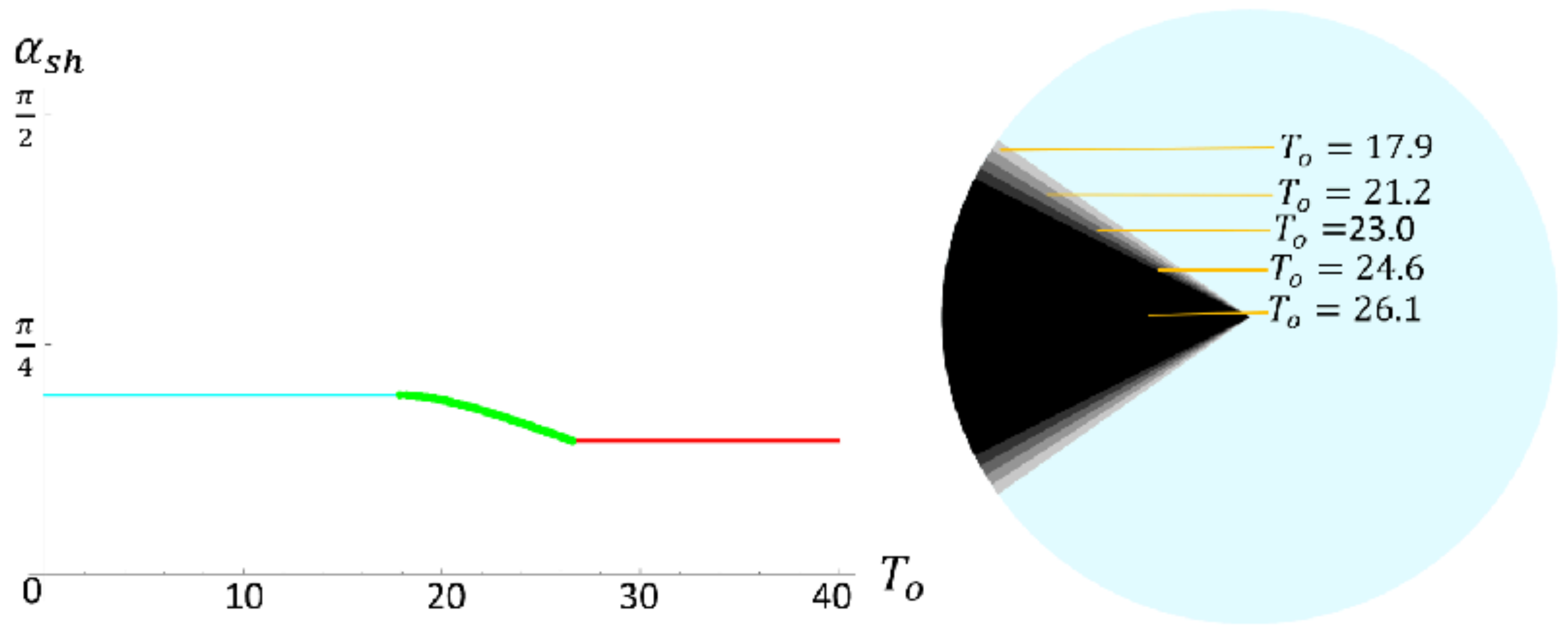


(c) $q_* = 1.0583$

Projection in the observer's sky



(a) $q_* = 0.0, r_o = 10$



(b) $q_* = 0.8967, r_o = 10$

Free falling observer

For a free falling observer the tetrad becomes

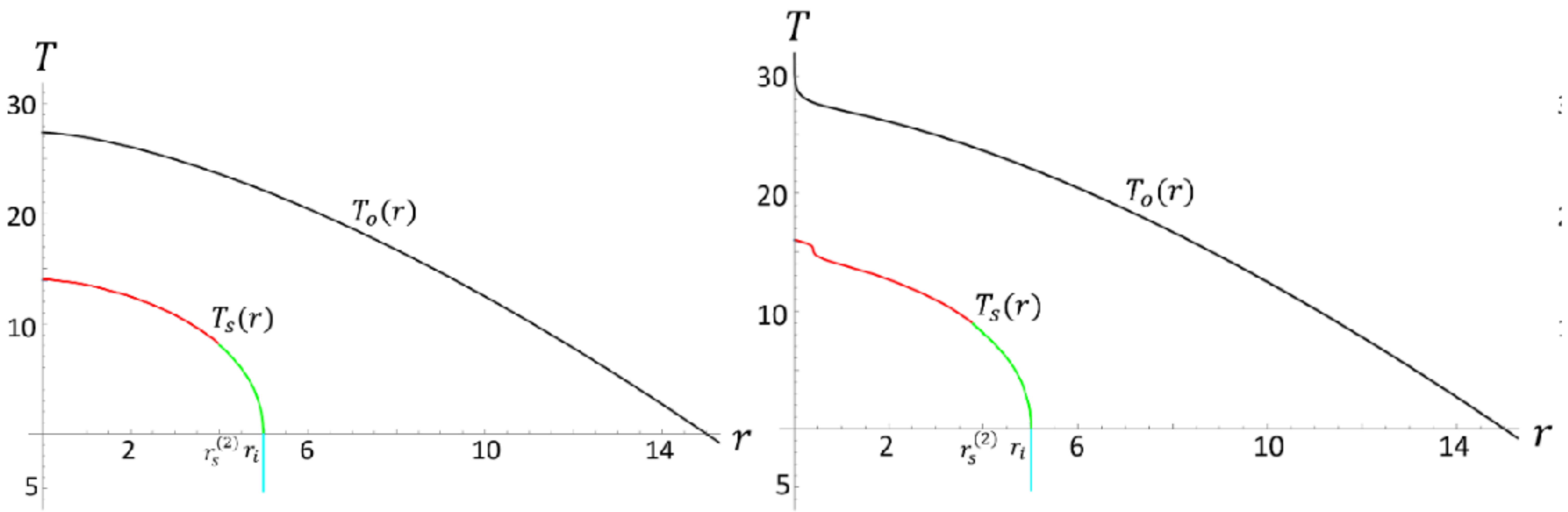
$$\tilde{e}_0 = \frac{\varepsilon - \sqrt{2M(r)/r}\sqrt{\varepsilon^2 - 1 + 2M(r)/r}}{1 - 2M(r)/r}\frac{\partial}{\partial T} - \sqrt{\varepsilon^2 - 1 + 2M(r)/r}\frac{\partial}{\partial r} \qquad \tilde{e}_1 = \varepsilon\frac{\partial}{\partial r} + \frac{\varepsilon\sqrt{2M(r)/r} - \sqrt{\varepsilon^2 - 1 + 2M(r)/r}}{1 - 2M(r)/r}\frac{\partial}{\partial T}$$

For null trajectories

$$\dot{T}\frac{\partial}{\partial T} + \dot{r}\frac{\partial}{\partial r} + \dot{\varphi}\frac{\partial}{\partial \varphi} = \tilde{\chi}(\tilde{e}_0 + \cos\tilde{\alpha}\tilde{e}_1 - \sin\tilde{\alpha}\tilde{e}_3)\,. \qquad \dot{r} = \tilde{\chi}\left(\varepsilon\cos\tilde{\alpha} - \sqrt{\varepsilon^2 - 1 + 2M(r)/r}\right), \qquad \dot{\varphi} = -\frac{\tilde{\chi}\sin\tilde{\alpha}}{r}$$

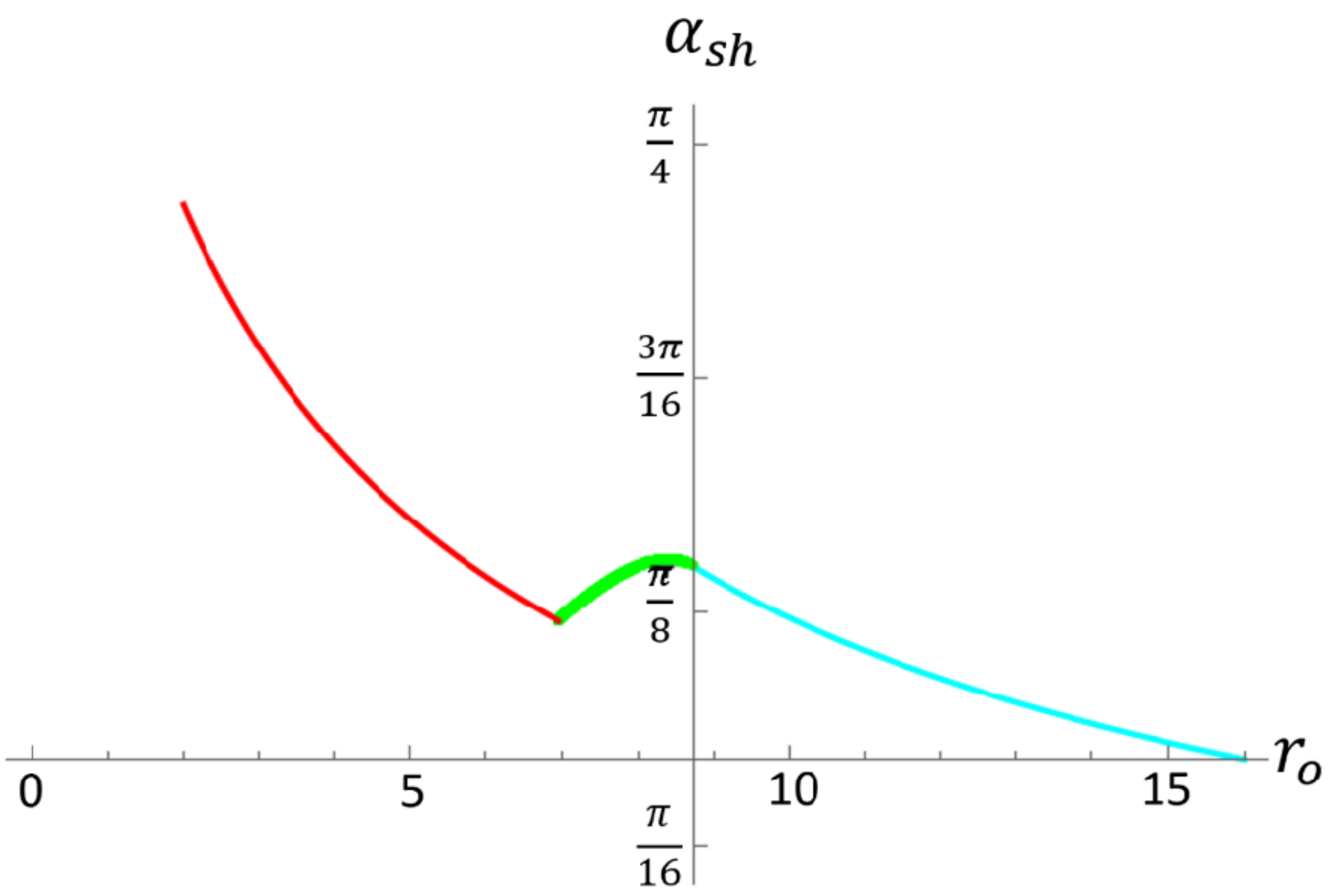
And the angular size

$$\sin\tilde{\alpha} = \frac{\sqrt{1 - 2M(r)/r}\sqrt{\mathcal{R}(r, r_m)}}{\varepsilon \pm \sqrt{\varepsilon^2 - 1 + 2M(r)/r}\sqrt{1 - \mathcal{R}(r, r_m)}}\,.$$

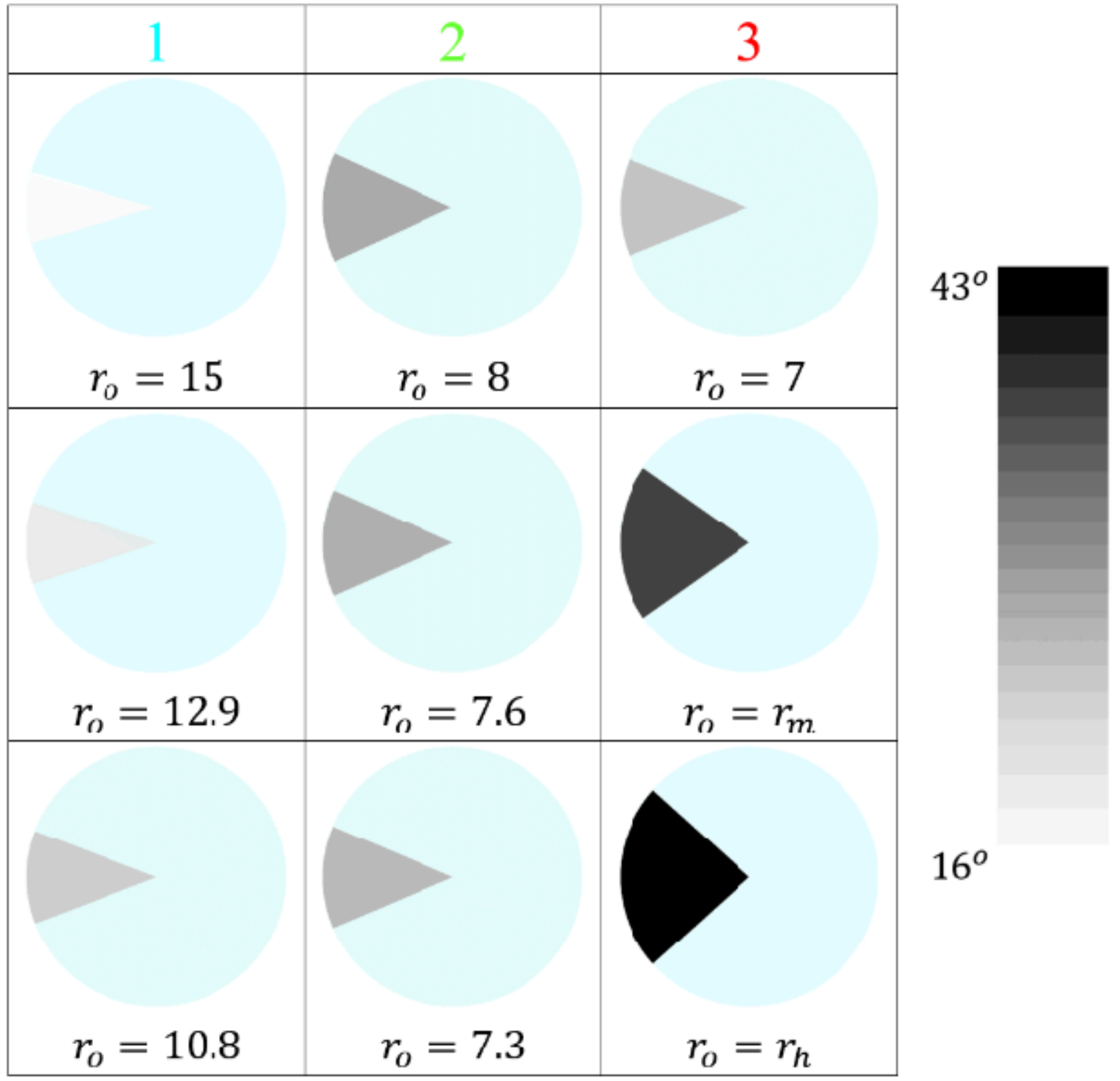


(a) $q_* = 0$ (b) $q_* = 0.8967$

Angular size of the shadow



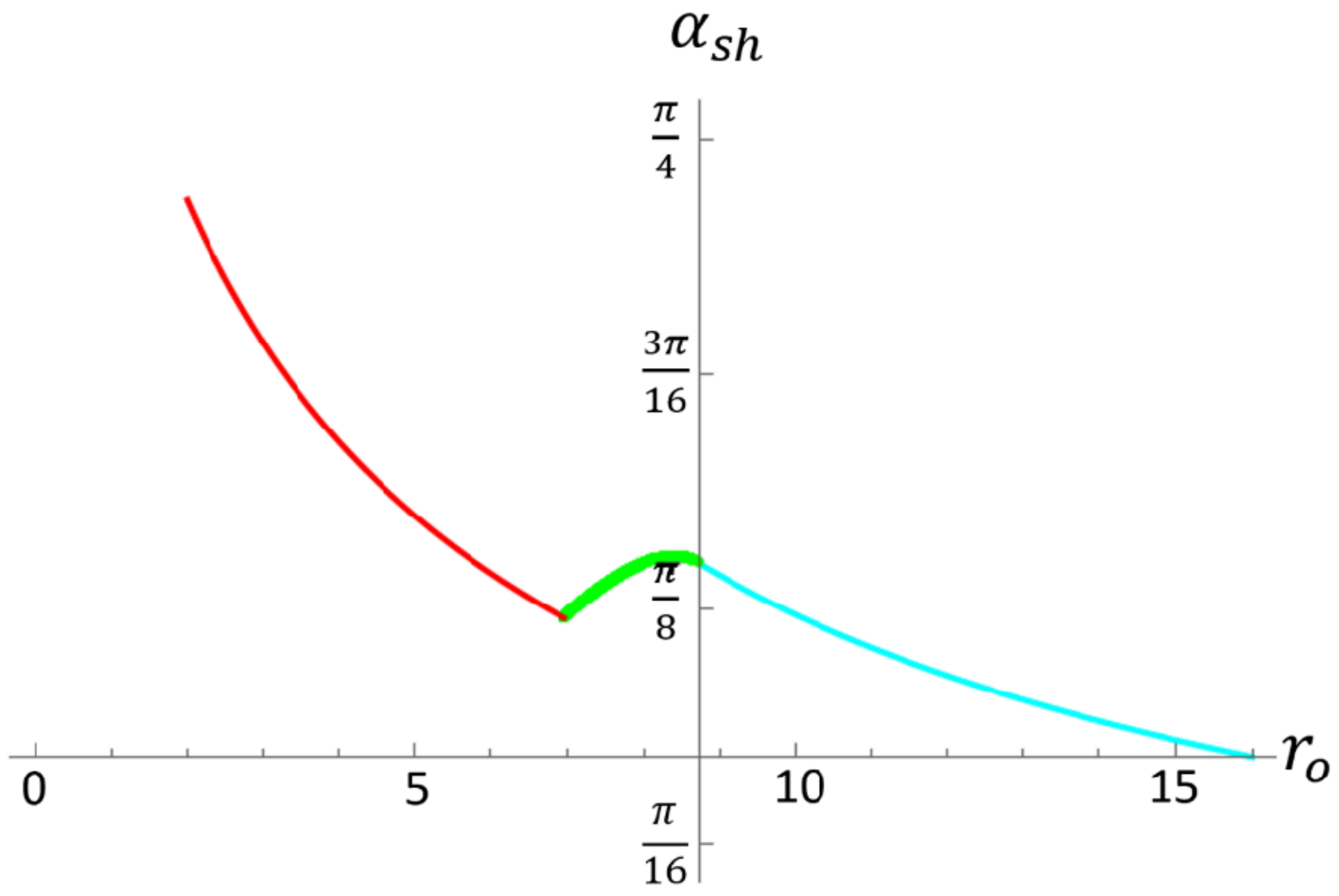
(a) $q_* = 0.0$



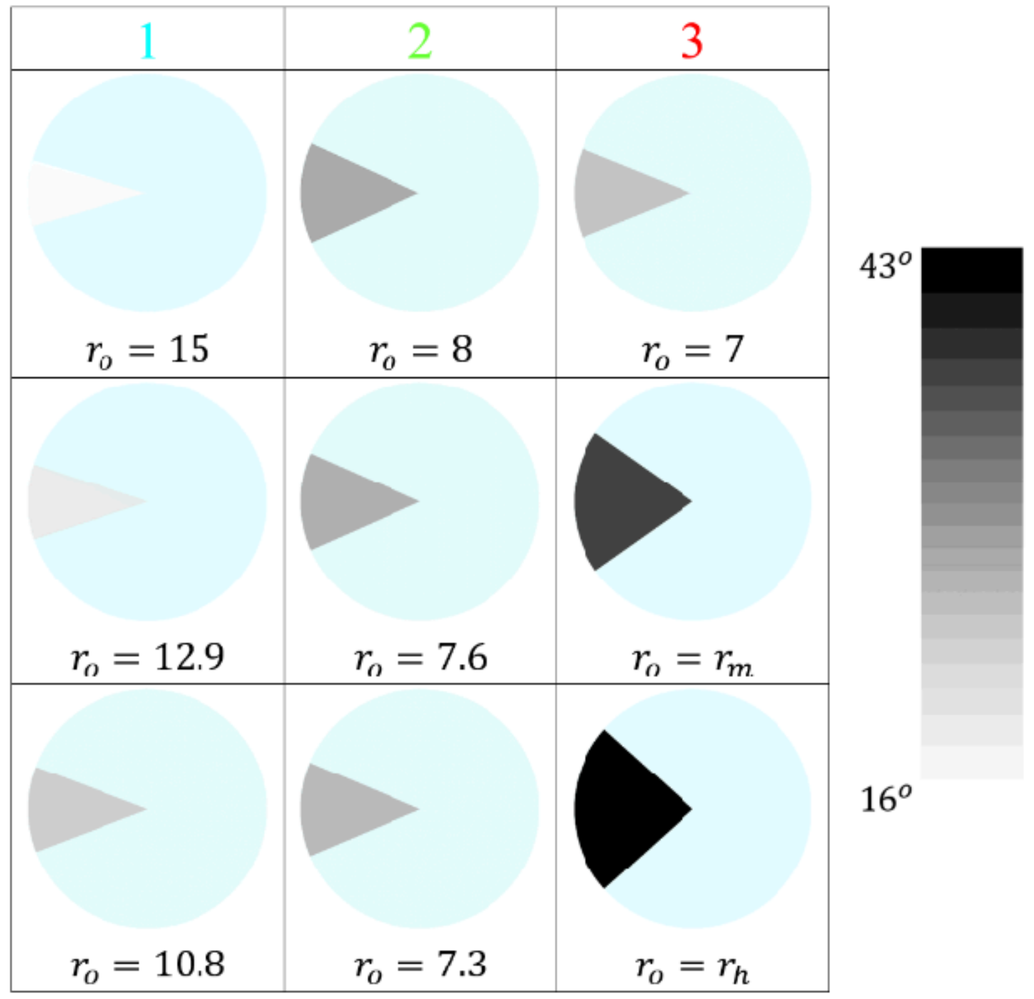
(b) $q_* = 0.0$

Schwarzschild spacetime

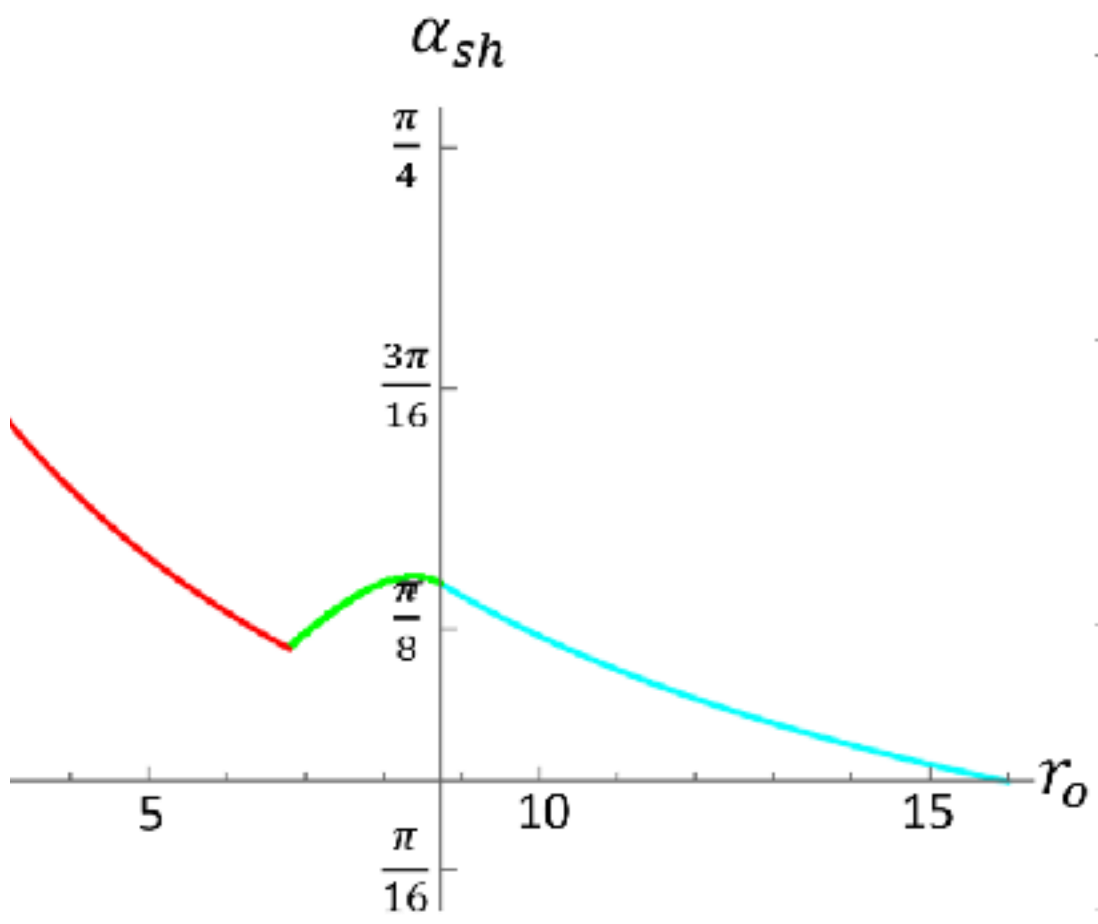
Angular size of the shadow



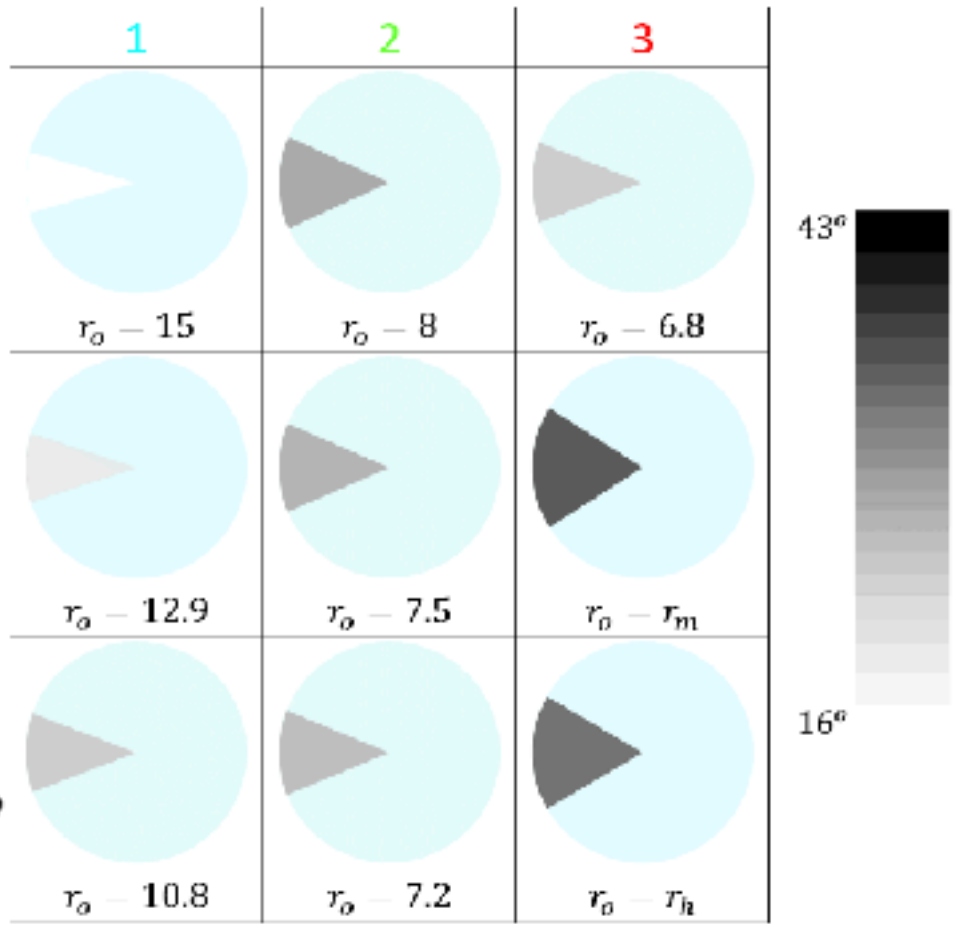
(a) $q_* = 0.0$



(b) $q_* = 0.0$

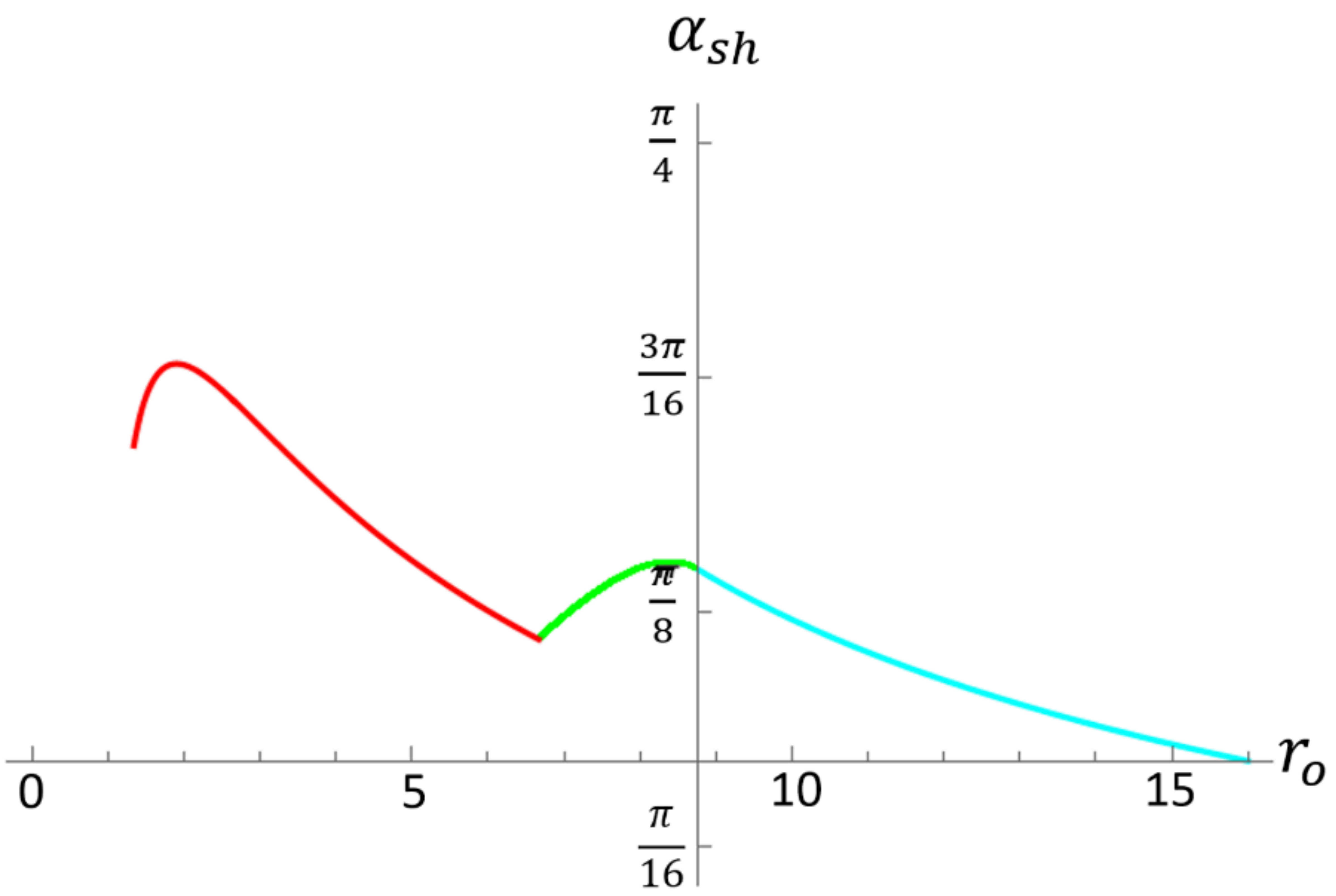


(c) $q_* = 0.8967$

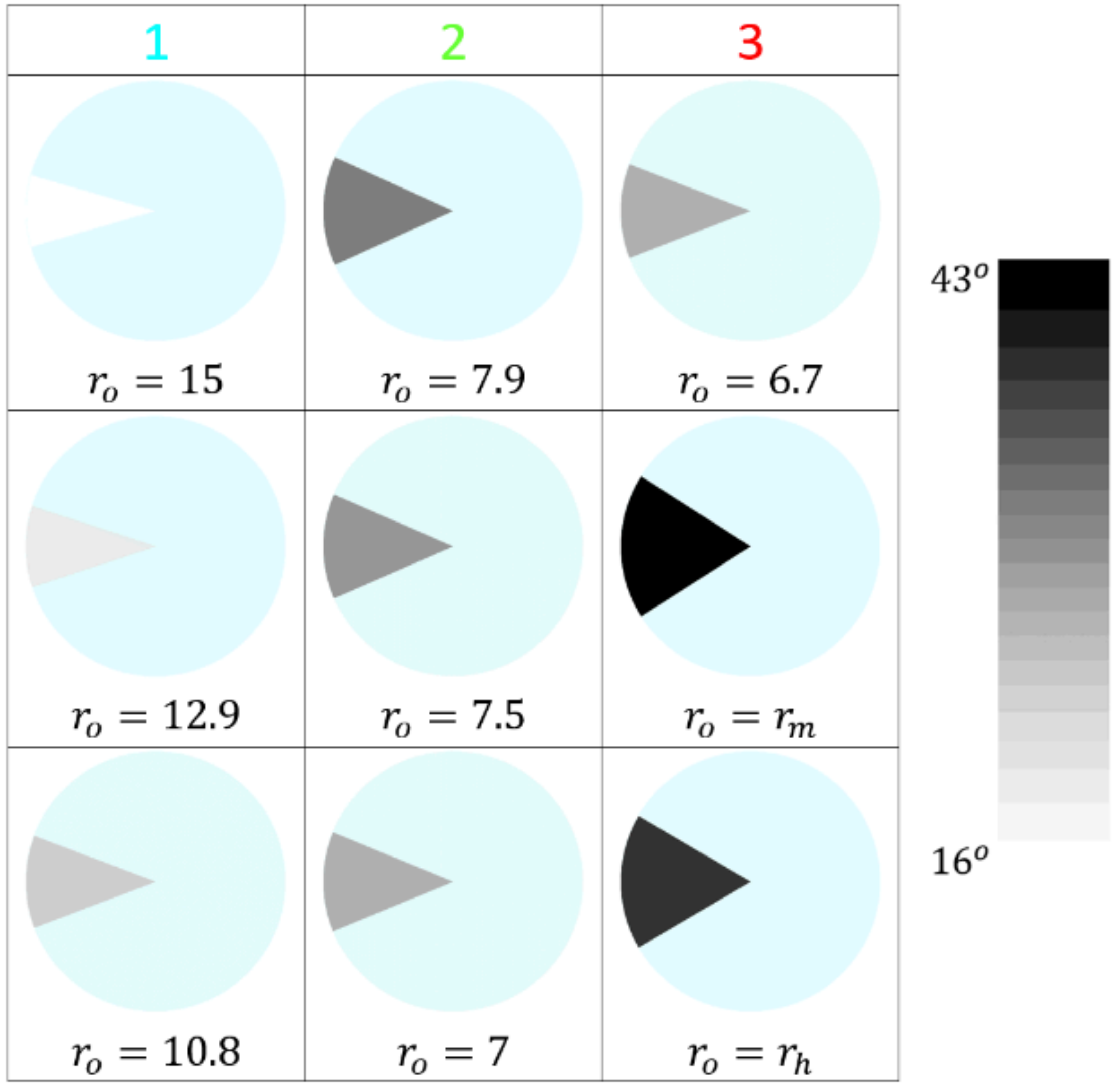


(d) $q_* = 0.8967$

Angular size of the shadow



(e) $q_* = 1.0583$



(f) $q_* = 1.0583$

Extreme Hayward spacetime

Final remarks

The evolution of the angle of the shadow produced during the collapse of a star in regular Hayward spacetime has a dependence on the deviation parameter.

A distant observer determines a dark circular region with an angular radius determined by the static black hole in a finite time, that is, the shadow forms in a finite time and is not an asymptotic process.

When the parameter is near the extreme, the shadow angle decreases near the end of the last stage of collapse. This decrease is attributed to the repulsion of the de Sitter nucleus that prevents the formation of the singularity during the collapse.