Falling charges into a Schwarzschild black hole: a quantum approach to radiation emission

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- **Outline**

- Introduction
- Electromagnetic field in Schwarzschild
- Radiation Emission
  - Zero-Frequency Limit
  - Numerical Results
- Conclusions
- References

- What are the emitted spectra and how do they change by varying v_0, r_0...?
- Connections with BH QNM?
- Can one use the analytical ZFL to estimate the emitted radiation?
- Issues in the point particle model.

- **Electromagnetic field:**

- Lagrangian:

$$S = \int \mathcal{L}_{\mathrm{FG}} d^4x$$

$$\mathcal{L}_{\mathrm{FG}} = \sqrt{-g}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\mathfrak{G}^2\right]$$

$$\mathfrak{G} \equiv \nabla^\sigma A_\sigma + K^\sigma A_\sigma$$

$$K^\sigma = [0, f'(r), 0, 0]$$

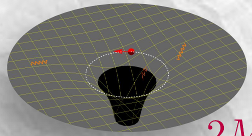

Figure 1: Embedding diagram of Schwarzschild.

$$f(r) = 1 - \frac{2M}{r}$$

• Electromagnetic field

- Lagrangian:

$$S = \int \mathcal{L}_{\rm FG} d^4x$$

$$\mathcal{L}_{\rm FG} = \sqrt{-g}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\mathfrak{G}^2\right]$$

$$\mathfrak{G} \equiv \nabla^\sigma A_\sigma + K^\sigma A_\sigma$$

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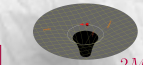

$$f(r) = 1 - \frac{2M}{r}$$

- Euler-Lagrange Equations:

$$\nabla_\mu F^{\mu\nu} + g^{\mu\nu}\nabla_\mu \mathfrak{G} - K^\nu \mathfrak{G} = 0$$

- Mode solutions:

Polarization

Angular q. n.

$$A_\mu^{\xi n;\omega lm} = \zeta_\mu^{\xi n\omega lm}(r,\theta,\phi)e^{-i\omega t}, \qquad (\omega > 0).$$

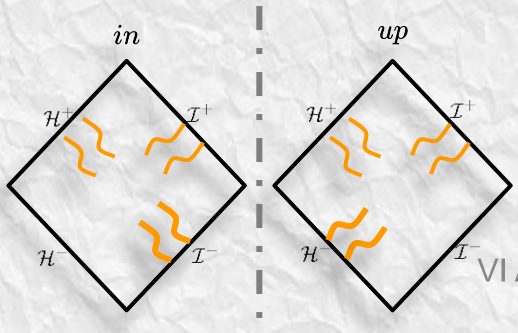

Figure 2: Penrose diagrams of Schwarzschild spacetime with *in* and *up* modes represented.

• Electromagnetic field

 Mode solutions:

$$A_\mu^{\xi n;\omega lm} = \zeta_\mu^{\xi n\omega lm}(r,\theta,\phi)e^{-i\omega t}, \qquad (\omega > 0).$$

Polarization:

$$\xi \equiv \begin{cases} \mathfrak{G} & \rightarrow \text{pure-gauge}, \\ I & \\ & \rightarrow \text{physical}, \\ II & \\ NP & \rightarrow \text{nonphysical}. \end{cases}$$

 Physical modes:

$$\mathfrak{G} = 0, \qquad l \geq 1$$

$$A_\mu^{In;\omega lm} = \left(0, \frac{\overline{\varphi_{\omega l}^{In}}}{r^2}Y_{lm}, \frac{f(r)}{l(l+1)}\frac{d\overline{\varphi_{\omega l}^{In}}}{dr}\partial_\theta Y_{lm}, \frac{f(r)}{l(l+1)}\frac{d\overline{\varphi_{\omega l}^{In}}}{dr}\partial_\phi Y_{lm}\right)e^{-i\omega t}$$

$$A_\mu^{IIn;\omega lm} = \left(0, 0, \overline{\varphi_{\omega l}^{IIn}}Y_\theta^{lm}, \overline{\varphi_{\omega l}^{IIn}}Y_\phi^{lm}\right)e^{-i\omega t}$$

Crispino *et al.* (1998)

PHYSICAL REVIEW D, VOLUME 58, 084027

Interaction of Hawking radiation and a static electric charge

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Atsushi Higuchi

Castiñeiras *et al.* (2005)

PHYSICAL REVIEW D **71**, 104013 (2005)

Semiclassical approach to black hole absorption of electromagnetic radiation emitted by a rotating charge

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(Received 8 March 2005; published 12 May 2005)

• Radiation Emission

Figure 3: Infalling charge representation.

Infalling charge:

$$j^\mu(x) = \frac{q}{\sqrt{-g}}\frac{1}{v^t}\delta(r-r_s)\delta(\theta-\theta_s)\delta(\phi-\phi_s)v^\mu$$

$$v^\mu \equiv \frac{dx^\mu}{d\tau} = \left(\frac{E}{f(r)}, -\sqrt{E^2 - f(r)}, 0, 0\right)$$

Charge proper time.

$$E = \sqrt{\frac{f(r_0)}{1-\frac{v_0^2}{f(r_0)^2}}}.$$

Radial velocity:

$$v_r = -\frac{dr_s}{dt} = \frac{f(r_s)\sqrt{E^2 - f(r_s)}}{E}.$$

It has **no** maximum if:

$$v_0 > \sqrt{(16M^3 - 12M^2 r_0 + r_0^3)/3r_0^3}$$

- Accelerates and deccelerates;
- Only decelerates.

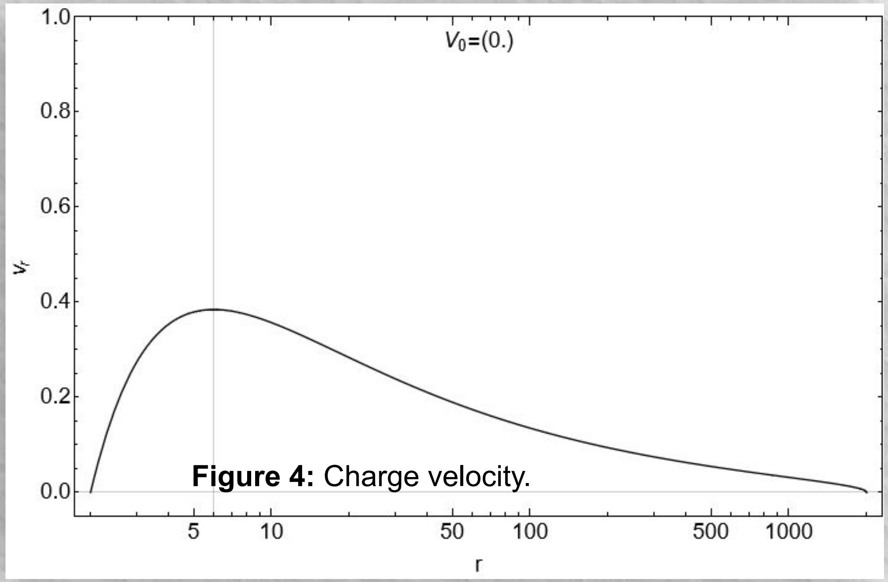
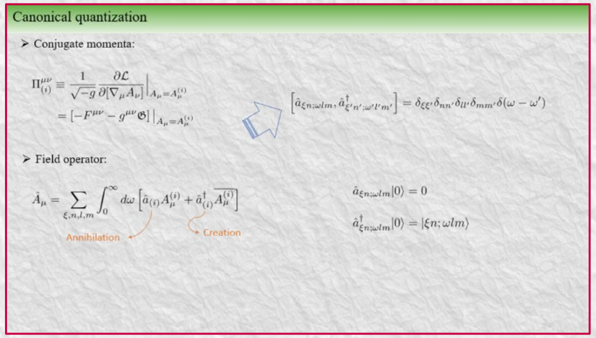


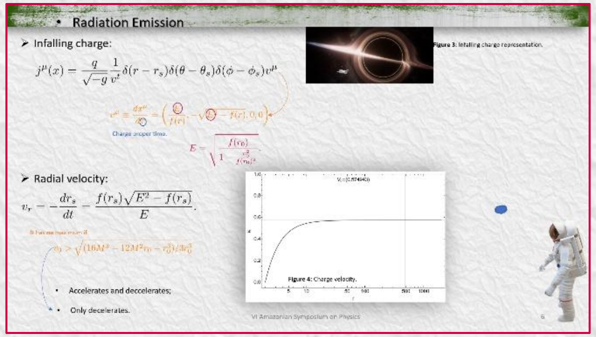
Figure 4: Charge velocity.

• Radiation Emission

- One-particle-emission amplitude:

$$\hat{S}_{\text{int}} = \int \sqrt{-g} j^{\mu} \hat{A}_{\mu} d^4 x$$





- **Radiation Emission**

- One-particle emission amplitude:

$$\hat{S}_{\text{int}} = \int \sqrt{-g} j^\mu \hat{A}_\mu d^4x$$

$$\mathcal{A}_{\text{em}} = \langle 1 | i\hat{S}_I | 0 \rangle.$$

$$\mathcal{A}_{\text{abs}} = \langle 0 | i\hat{S}_I | 1 \rangle.$$

$$\hat{a}|0\rangle \equiv 0$$

$$\hat{a}^\dagger |0\rangle = |1\rangle$$

$$S_{\text{tot}} = S + S_I$$

ld operator:

$$\hat{A}_\mu = \sum_{\xi,n,l,m} \int_0^\infty d\omega \left[\hat{a}_{(i)} A_\mu^{(i)} + \hat{a}^\dagger_{(i)} \overline{A_\mu^{(i)}} \right]$$

Annihilation

Creati

$$\mathcal{A}^{\xi n;\omega l m} = \langle \xi n; \omega l m | i \hat{S}_{\text{int}} | 0 \rangle,$$
$$= i \int \sqrt{-g} j^\mu \overline{A_\mu^{(i)}} d^4x.$$

alling source:

$$j^\mu(x) = \frac{q}{\sqrt{-g}} \frac{1}{v^t} \delta(r - r_s)\delta(\theta - \theta_s)\delta(\phi - \phi_s) v^\mu$$

$$v^\mu \equiv \frac{dx^\mu}{d\tau} = \left(\frac{E}{f(r)}, -\sqrt{E^2 - f(r \ldots} \right.$$

Particle's proper time

- **Radiation Emission**

 Partial energy spectrum:

$$\mathcal{E}^{n;\omega l} = \sum_{m=-l}^{l} \omega |\mathcal{A}^{n;\omega lm}|^2$$

$$\mathcal{E}^{n;\omega\ell} = \frac{(2\ell+1)q^2\omega}{4\pi} \left| \int_{2M}^{r_0} \frac{\varphi^n_{\omega\ell}(r_s)}{r_s^2} e^{i\omega t(r_s)} dr_s \right|^2.$$

 Total energy spectrum:

$$\mathcal{E}^{n;\omega} = \sum_{l\geqslant 1} \mathcal{E}^{n;\omega l}$$

 Partial and total emitted energies:

$$\mathcal{E}^{n;l} = \int d\omega \mathcal{E}^{n;\omega l} \qquad \mathcal{E}^{n} = \sum_{l\geqslant 1} \int d\omega \mathcal{E}^{n;\omega l}$$

• Zero-Frequency Limit

$$\mathcal{E}^{n;\omega\ell} = \frac{(2\ell+1)q^2\omega}{4\pi}\left|\int_{2M}^{r_0} \frac{\varphi^n_{\omega\ell}(r_s)}{r_s^2} e^{i\omega t(r_s)} dr_s\right|^2.$$

▯ ω → 0

- Spectrum observed asymptotically, for $r_0 \to \infty$

Integ. const.

$$\varphi^{in}_{\omega l} = C_\omega r\sqrt{\frac{2\omega}{\pi}} j_l(r\omega)$$

spherical Bessel functions

$$C_\omega = B^{I\,in}_{\omega l}\sqrt{2\pi\omega}$$

$$\mathcal{E}^{\text{in};0\ell} = q^2\frac{(2\ell+1)\ell(\ell+1)\Gamma(\ell)^2}{16\pi\cdot 4^\ell\Gamma(\ell+\frac{3}{2})^2} v_0^{2\ell}\left|{}_2F_1\left(\frac{\ell}{2},\frac{\ell+1}{2};\ell+\frac{3}{2};v_0^2\right)\right|^2$$

- **Zero-Frequency Limit**

$$\mathcal{E}^{\text{in};0\ell} = q^2 \frac{(2\ell+1)\ell(\ell+1)\Gamma(\ell)^2}{16\pi \cdot 4^\ell \Gamma(\ell+\frac{3}{2})^2} v_0^{2\ell} \left| {}_2F_1\left(\frac{\ell}{2}, \frac{\ell+1}{2}; \ell+\frac{3}{2}; v_0^2\right)\right|^2$$

ℓ	v_0	$q^{-2}\mathcal{E}^{\text{in};0\ell}$	Numerical
1	0.25	0.0010827	0.0010828
	0.75	0.0126318	0.0126317
	0.99	0.0347517	0.0347628
2	0.25	0.0000139	0.0000139
	0.75	0.0019812	0.0019813
	0.99	0.0170247	0.0170315
3	0.25	0.0000002	0.0000002
	0.75	0.0003610	0.0003611
	0.99	0.0102605	0.0102606

Table 1: Comparison between the analytical results for $\mathcal{E}^{\text{in};0\ell}$ and the numerically obtained partial energy spectrum with $\omega \to 0$.

• Numerical results

- Radiation emitted to infinity:

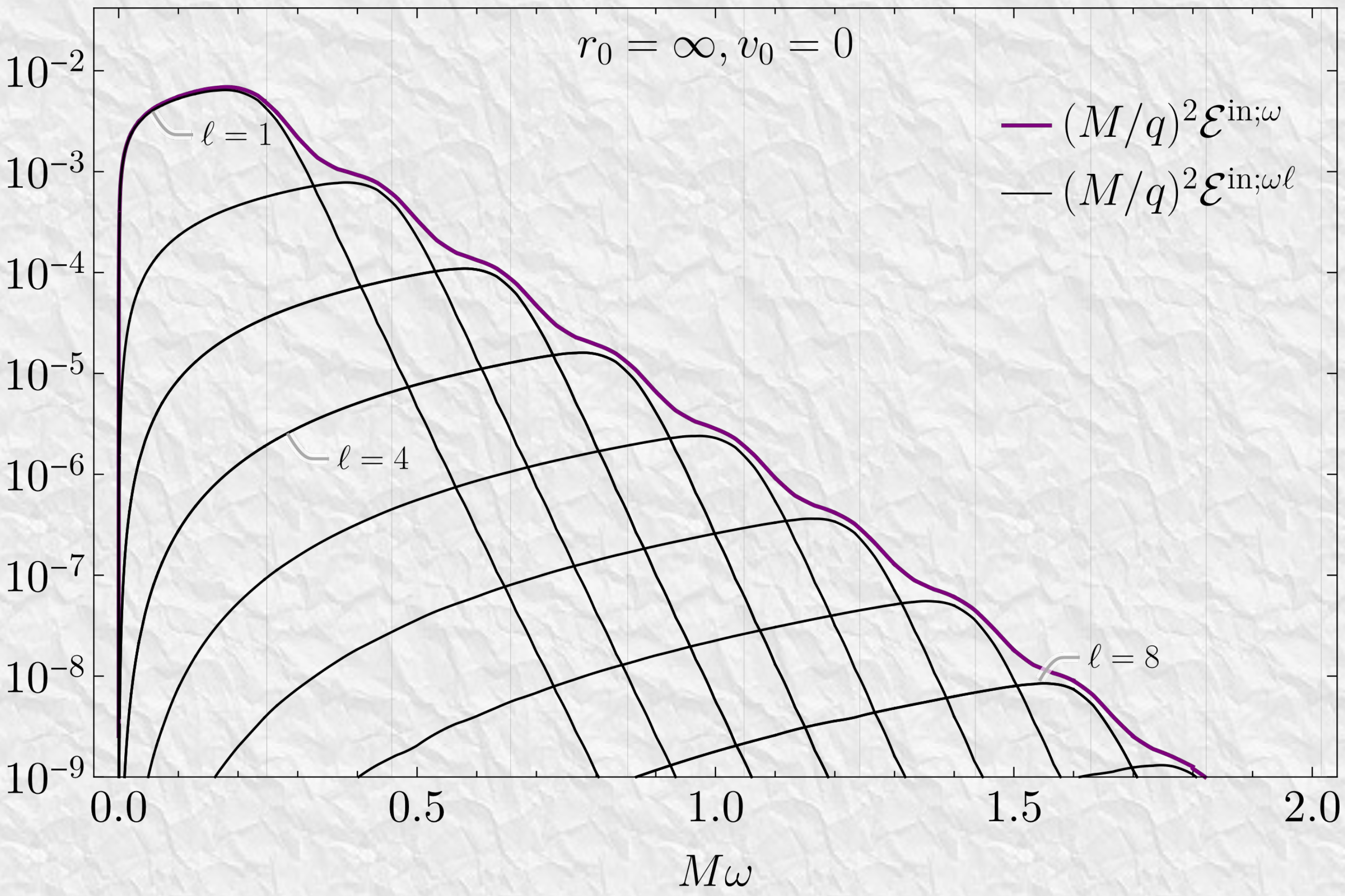


Figure 6: Partial and total energy spectra as a function of ω. We consider the first 9 multipoles.

Figure 5: Infalling charge from infinity.

- **Numerical results**

 Radiation emitted to infinity:

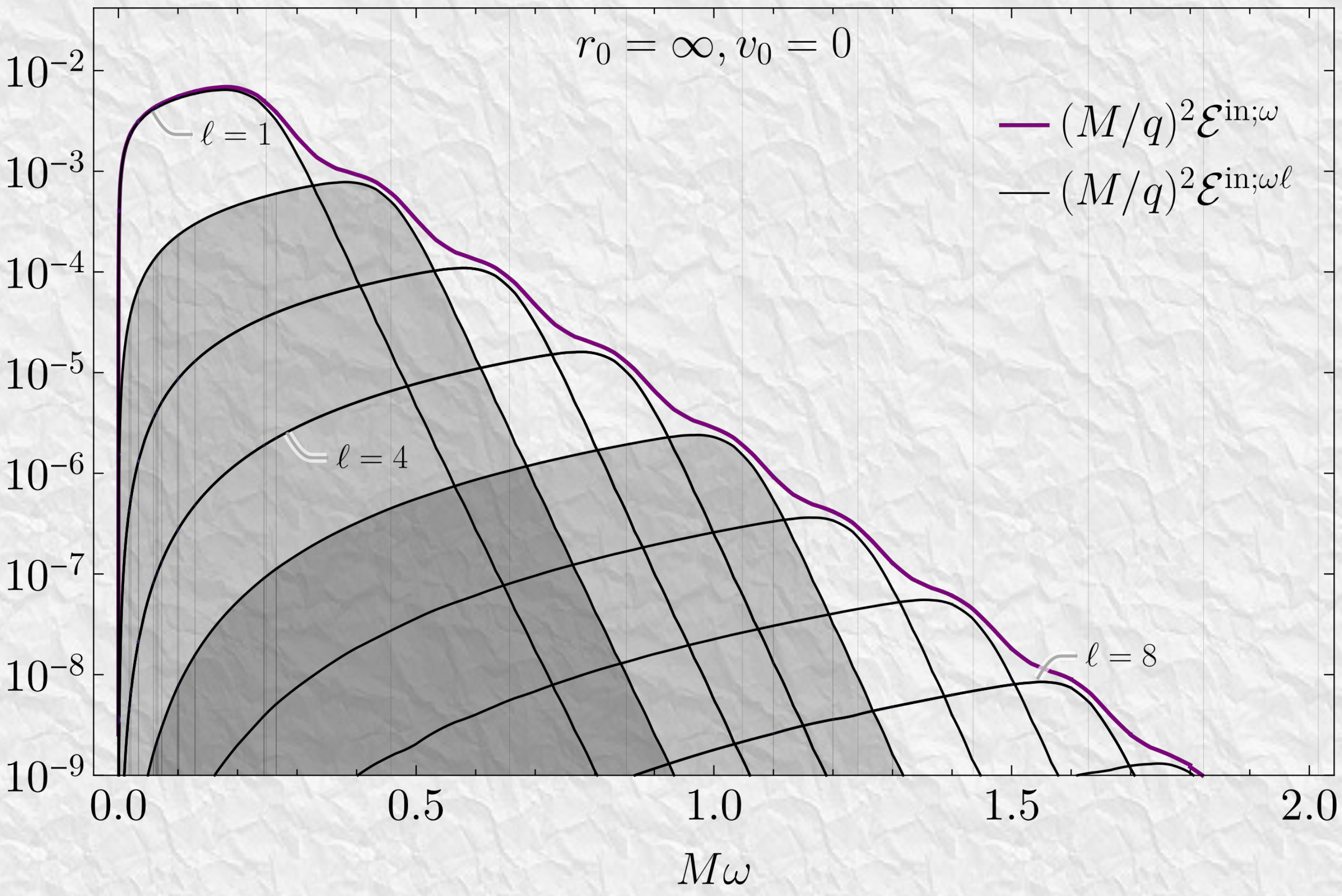


Figure 7: Partial and total energy spectra as a function of ω. We consider the first 9 multipoles. Notice that the plot is on a log scale.

 The contribution of each multipole:

$$\mathcal{E}^{in;2} \to 14.08\%$$

$$\mathcal{E}^{in;5} \to 0.06\%$$

$$(M/q^2)\mathcal{E}^{in} \to 0.170\% \text{ of } E$$

- **Numerical results**

 Radiation emitted to infinity:

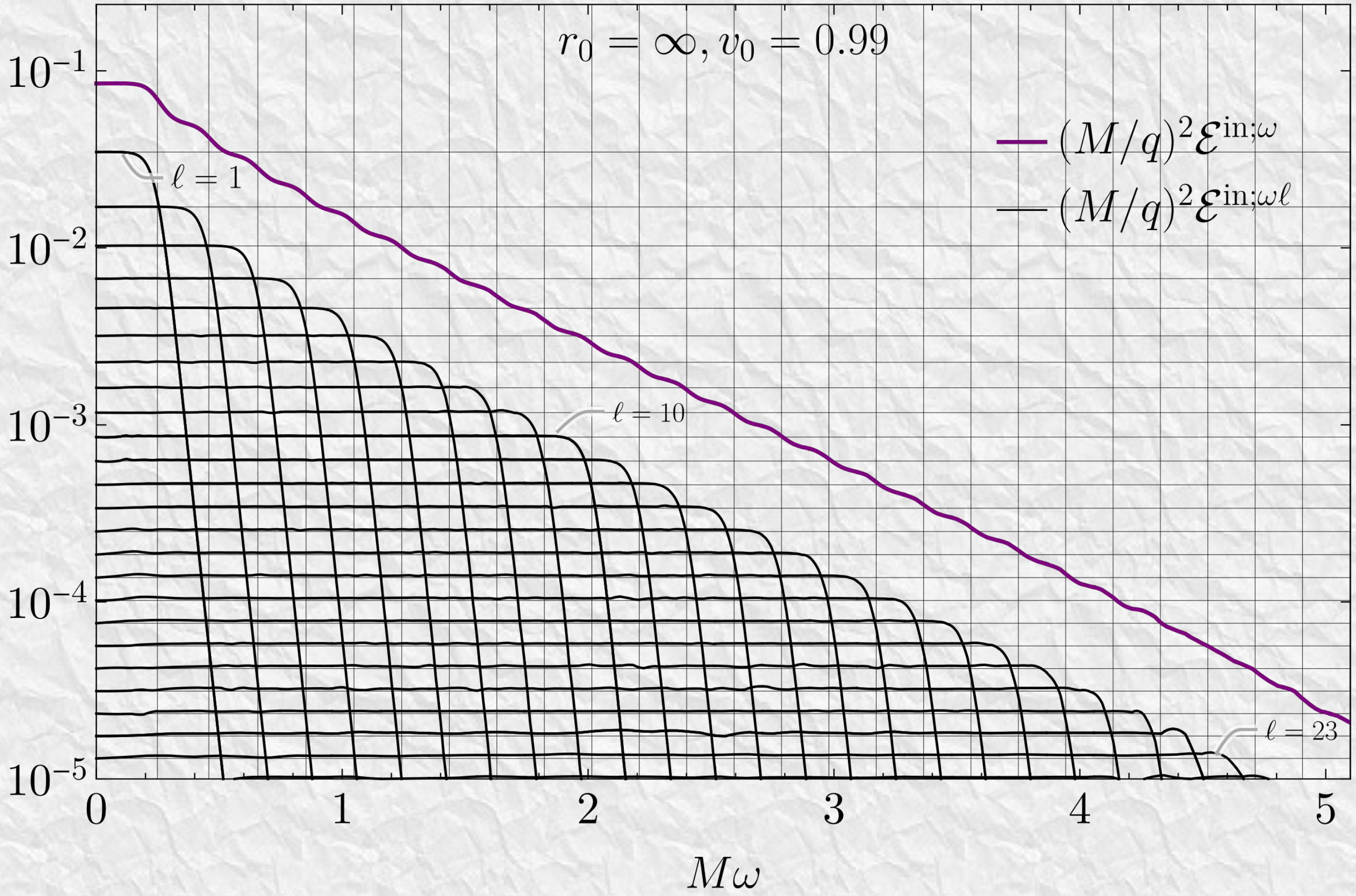
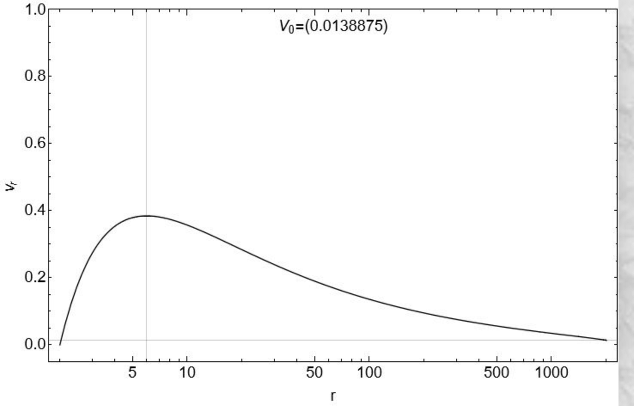


Figure 8: Partial and total energy spectra as a function of ω for an ultrarelativistic charge.



 The contribution of each multipole:

$$\mathcal{E}^{in;\ell=1} \rightarrow 16.25\%$$

$$\mathcal{E}^{in;\ell=5} \rightarrow 3.2\%$$

$$(M/q^2)\mathcal{E}^{in} \approx 0.8\% \text{ of } E$$

• Numerical results

 Radiation emitted to infinity:

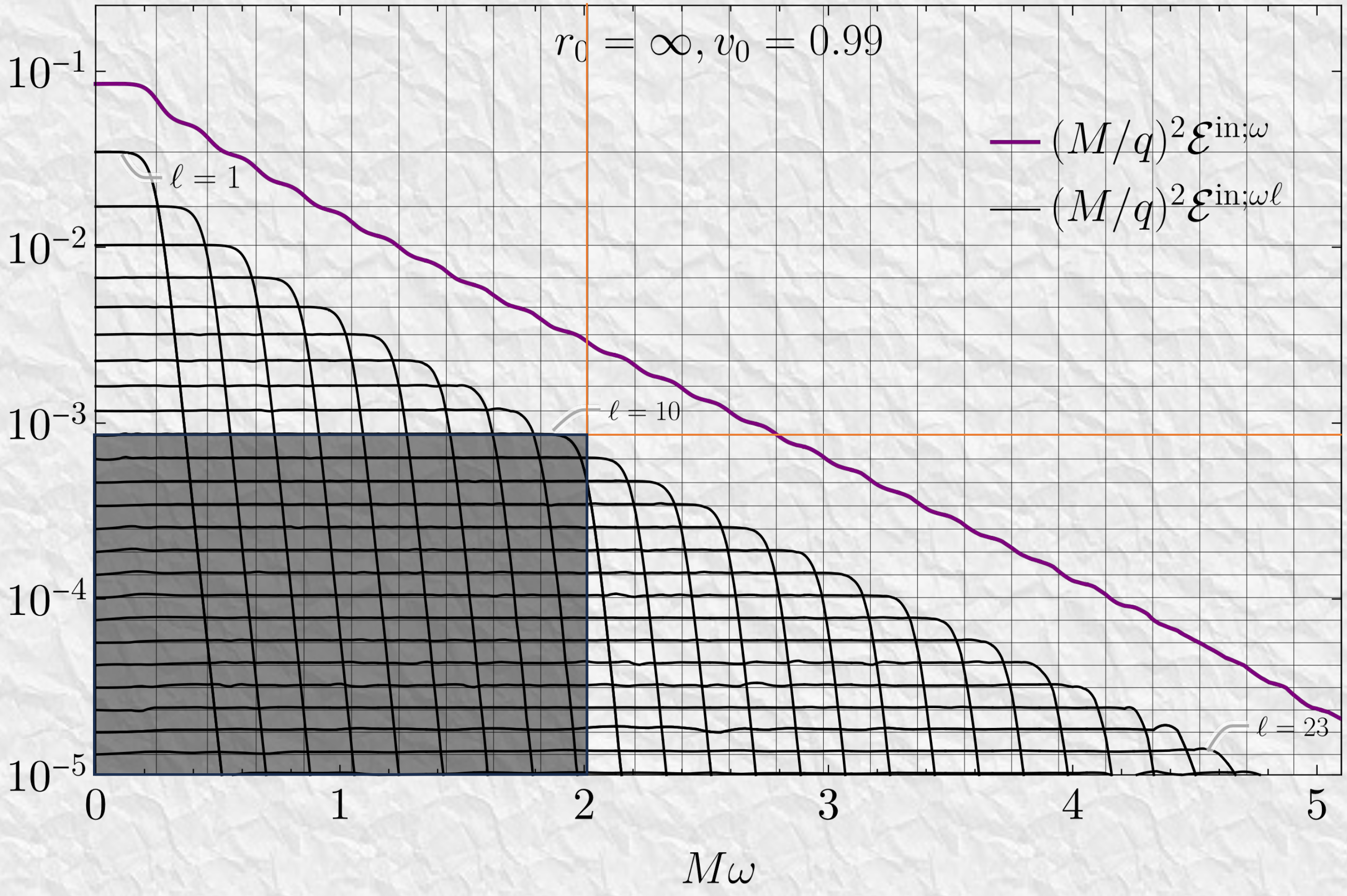


Figure 9: Partial and total energy spectra as a function of ω for an ultrarelativistic charge. Notice that the plot is on a log scale.

 Estimation with ZFL and ω^{qnf}:

$$\mathcal{E}^{\mathrm{in};0\ell} = q^2 \frac{(2\ell+1)\ell(\ell+1)\Gamma(\ell)^2}{16\pi \cdot 4^\ell \Gamma(\ell+\frac{3}{2})^2} v_0^{2\ell} \left| {}_2F_1\left(\frac{\ell}{2}, \frac{\ell+1}{2}; \ell+\frac{3}{2}; v_0^2\right)\right|^2$$

$$\mathcal{E}^{\mathrm{in};\ell} \approx \mathcal{E}^{\mathrm{in};0\ell}\omega_\ell^{\mathrm{qnf}} \text{ for } v_0 \approx 1$$

Agree very well (<3%)

- Important for:

$$v_0 \to 1,$$
$$\ell \to \infty.$$

• Numerical results

- Radiation emitted to infinity:

$$\mathcal{E}^{\text{in};\omega\ell} \approx \frac{\mathcal{E}^{\text{in};0\ell}}{1+e^{-\Lambda(\omega-\omega^{\text{qnf}})}}, \text{ for } v_0 \approx 1$$

Where Λ is a negative constant.

- Estimation with ZFL and ω^{qnf}:

$$\mathcal{E}^{\text{in};0\ell} = q^2 \frac{(2\ell+1)\ell(\ell+1)\Gamma(\ell)^2}{16\pi \cdot 4^\ell \Gamma(\ell+\frac{3}{2})^2} v_0^{2\ell} \left| {}_2F_1\left(\frac{\ell}{2}, \frac{\ell+1}{2}; \ell+\frac{3}{2}; v_0^2\right)\right|^2$$

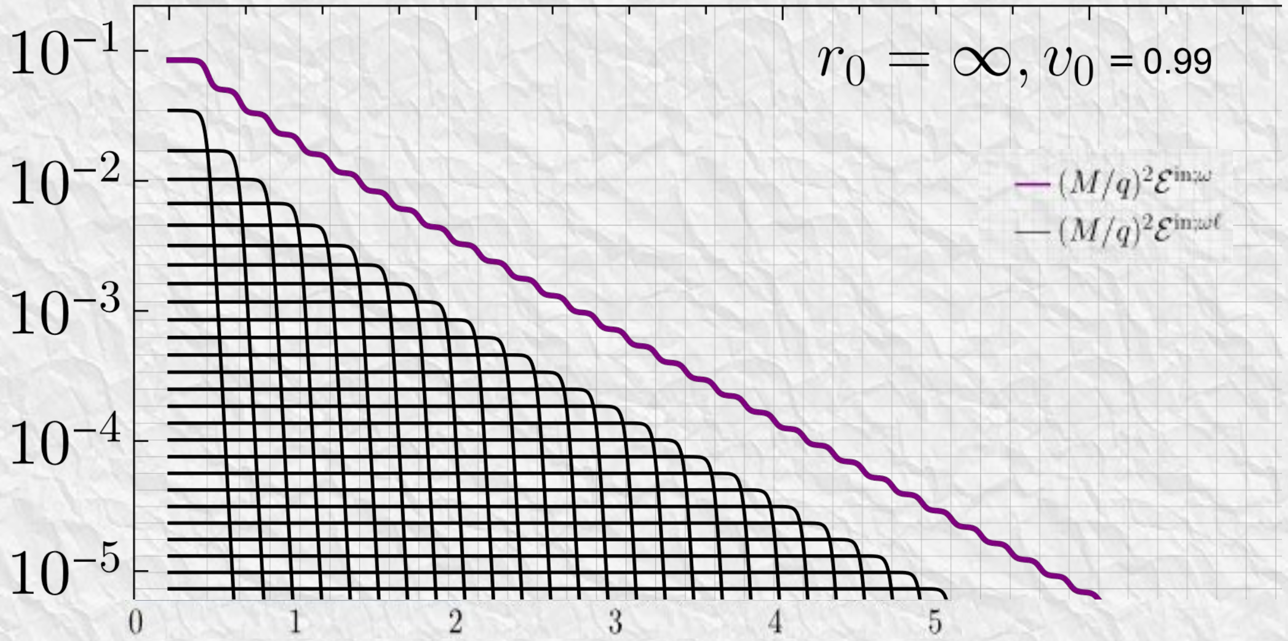


Figure 10: Partial and total energy spectra as a function of ω for an ultrarelativistic charge.

• Numerical results

 Radiation emitted to infinity:

$$\mathcal{E}^{\text{in};\omega\ell} \approx \frac{\mathcal{E}^{\text{in};0\ell}}{1+e^{-\Lambda(\omega-\omega^{\text{qnf}})}}, \text{ for } v_0 \approx 1$$

Where Λ is a negative constant.

 Estimation with ZFL and ω^{qnf}:

$$\mathcal{E}^{\text{in};0\ell} = q^2 \frac{(2\ell+1)\ell(\ell+1)\Gamma(\ell)^2}{16\pi\cdot 4^\ell \Gamma(\ell+\frac{3}{2})^2} v_0^{2\ell} \left| {}_2F_1\left(\frac{\ell}{2}, \frac{\ell+1}{2}; \ell+\frac{3}{2}; v_0^2\right)\right|^2$$

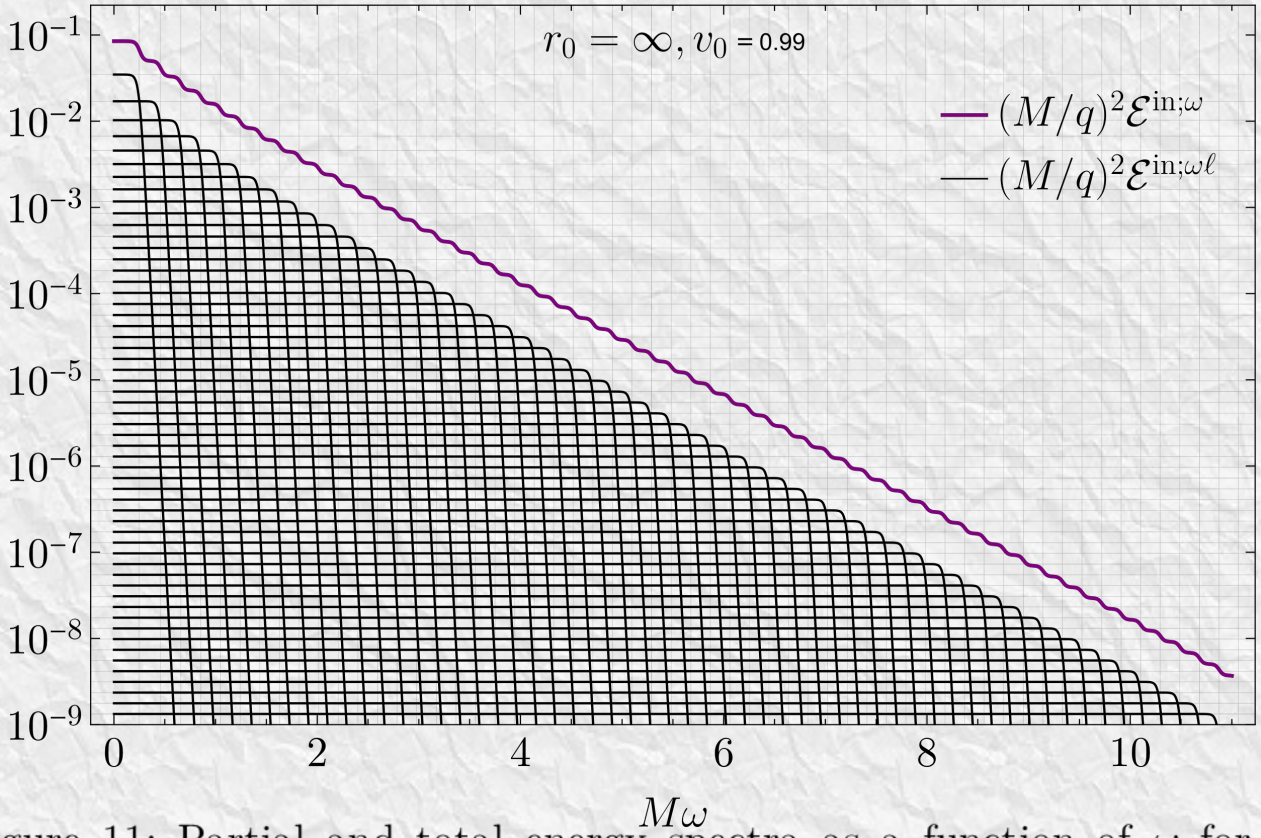


Figure 11: Partial and total energy spectra as a function of ω for an ultrarelativistic charge.

• Numerical results

 Radiation emitted to infinity:

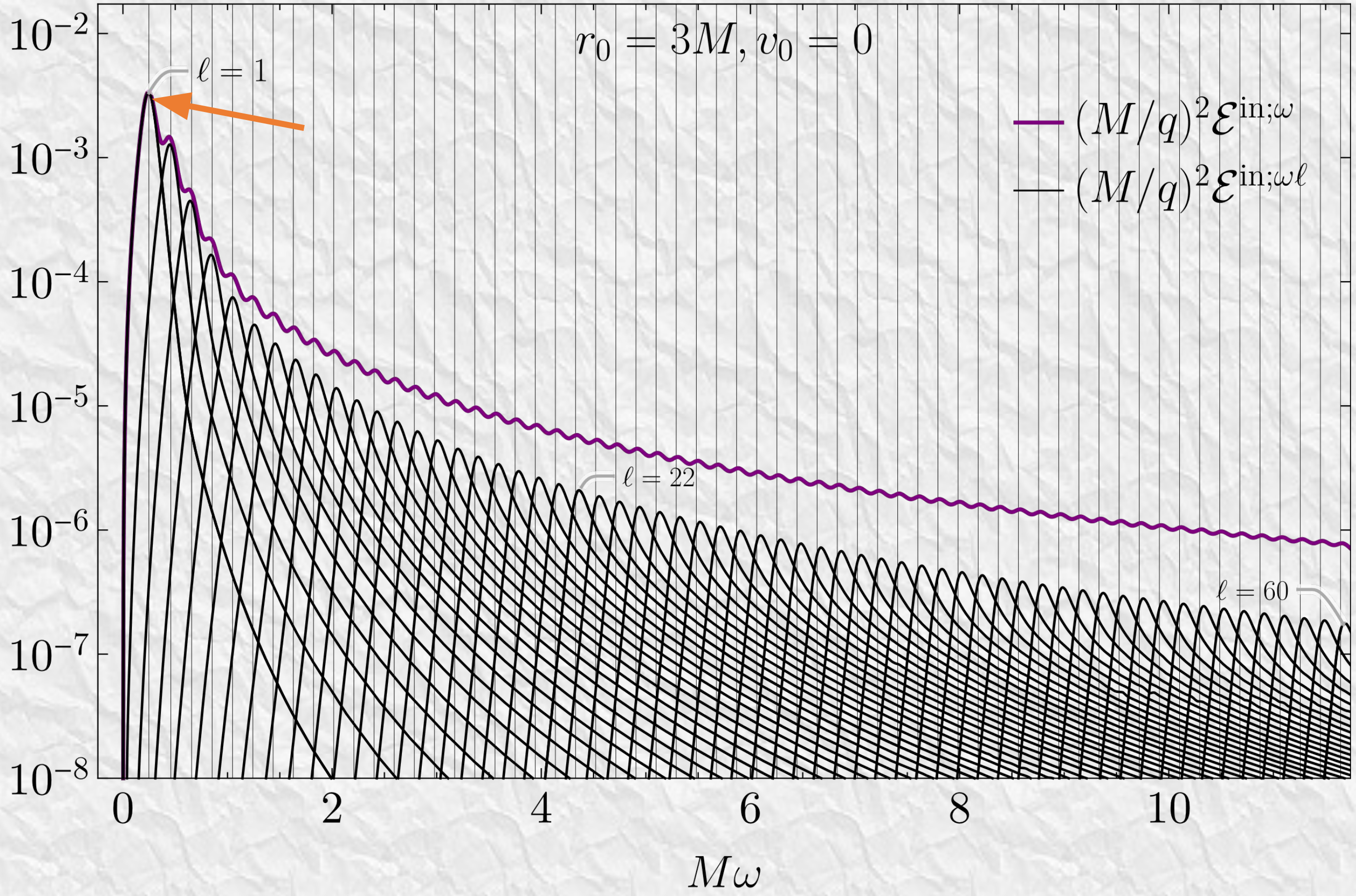


Figure 13: Partial and total energy spectra as a function of ω. We consider the first 60 multipoles.

Figure 12: Infalling charge from a finite distance.

• Maximum shifts around the QNF:

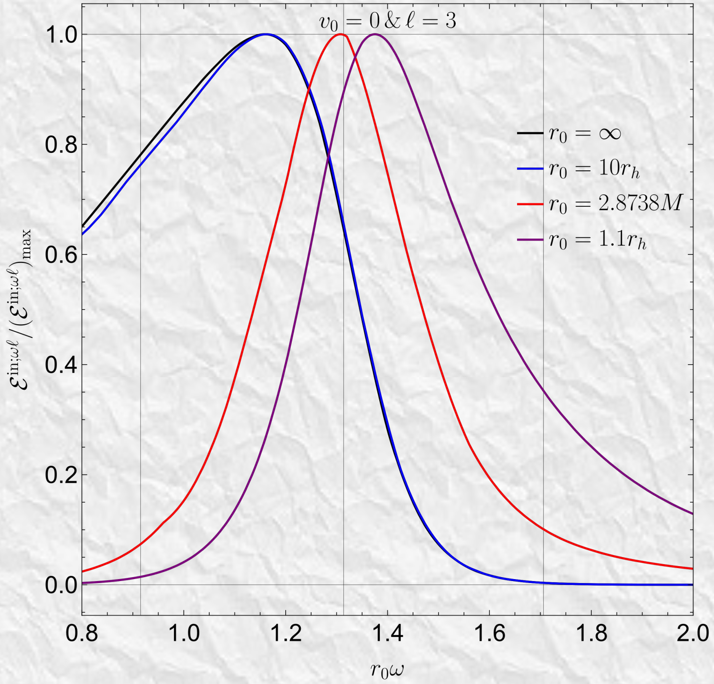


Figure 14: Maximum of the spectrum.

• Numerical results

- Radiation emitted to infinity:

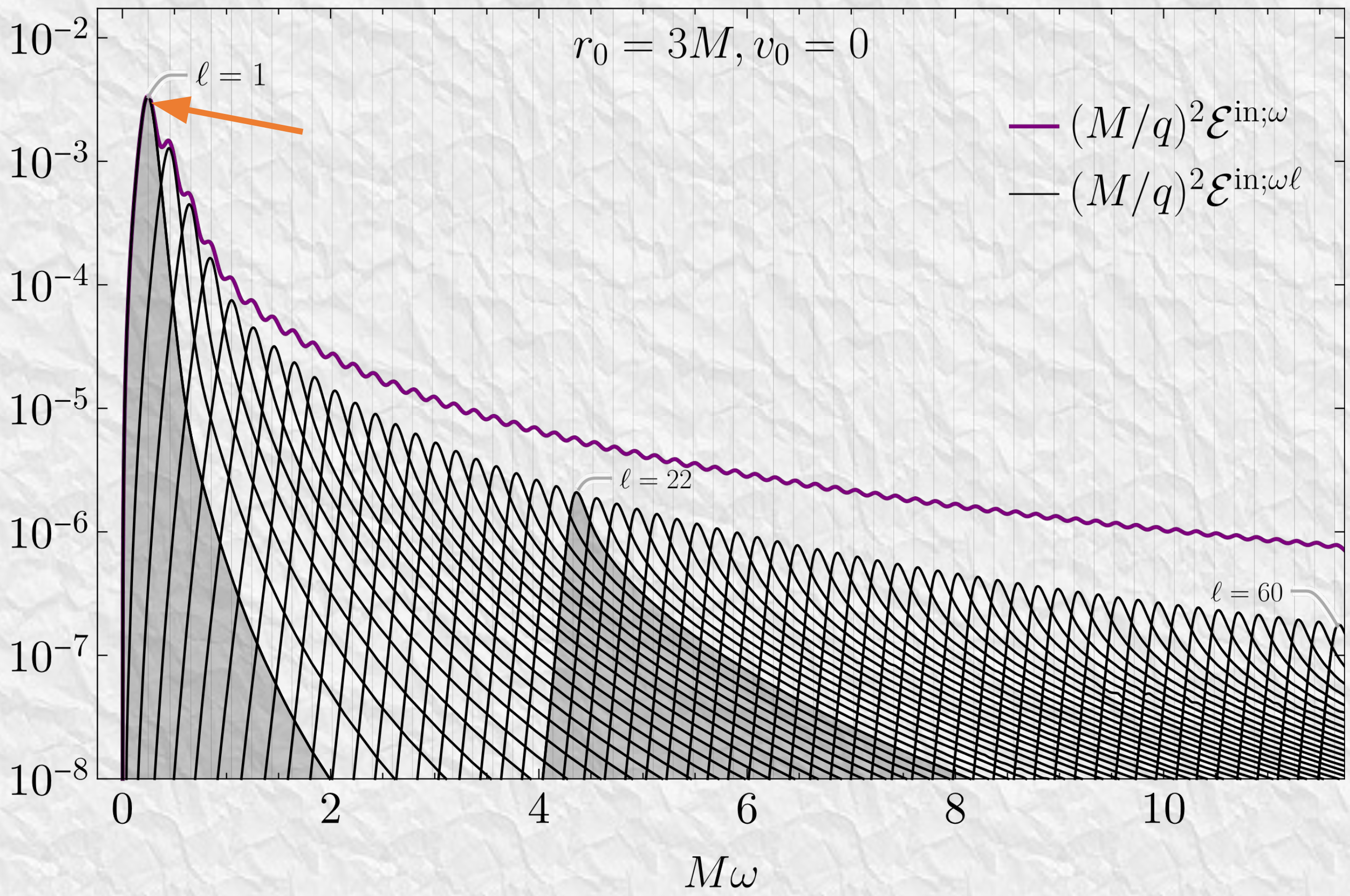


Figure 15: Partial and total energy spectra as a function of ω. We consider the first 60 multipoles. Notice that the plot is on a log scale.

$$\mathcal{E}^{in;\ell=1} \rightarrow 55.25\%$$

$$\mathcal{E}^{in;\ell=22} \rightarrow 0.1\%$$

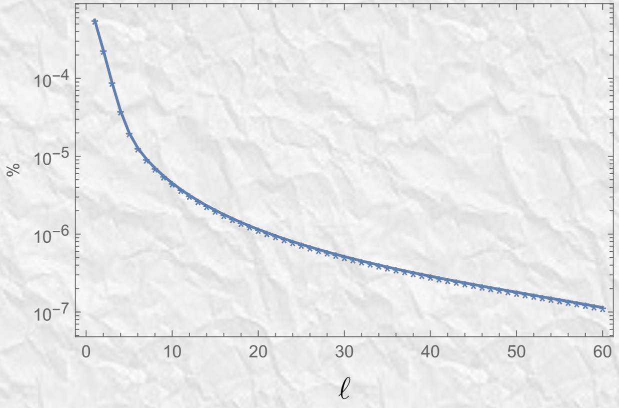


Figure 16: Multipolar distribution.

• Radiation due to an extended charge

- The energy spectrum of the "string:"

$$j^\mu(x) = \sum_{j=0}^{N-1} j_j^\mu(x). \longrightarrow \mathcal{A}_N^{n;\omega\ell m} = \left(\sum_{j=0}^{N-1} \frac{e^{i\omega \frac{j\Delta t}{N-1}}}{N}\right) \mathcal{A}^{n;\omega\ell m} \longrightarrow \mathcal{E}_N^{n;\omega\ell} = \zeta_N(\omega)\mathcal{E}^{n;\omega\ell}$$

For N particles, each with charge q/N.

Interference!

$$\zeta_\infty = \left(\frac{2\sin\frac{\Delta t\omega}{2}}{\Delta t\omega}\right)^2 \propto 1/\omega^2.$$

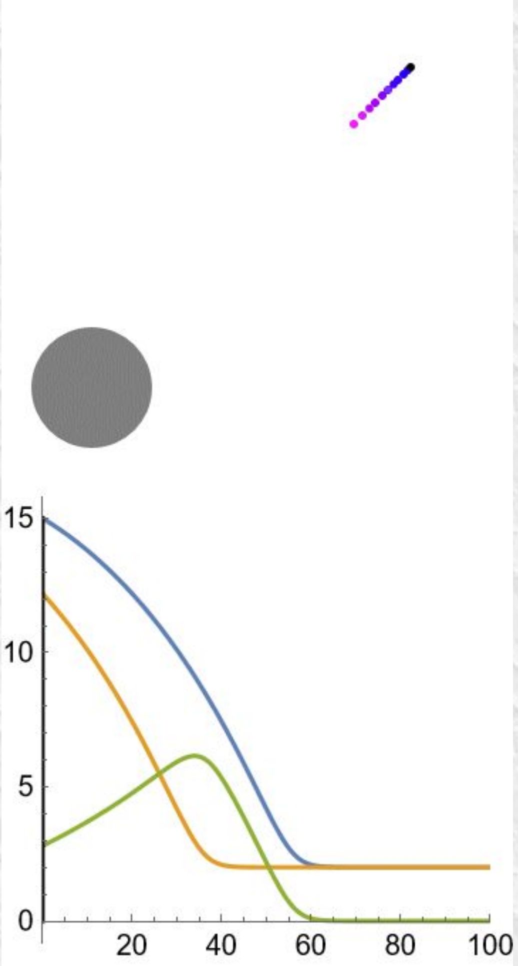
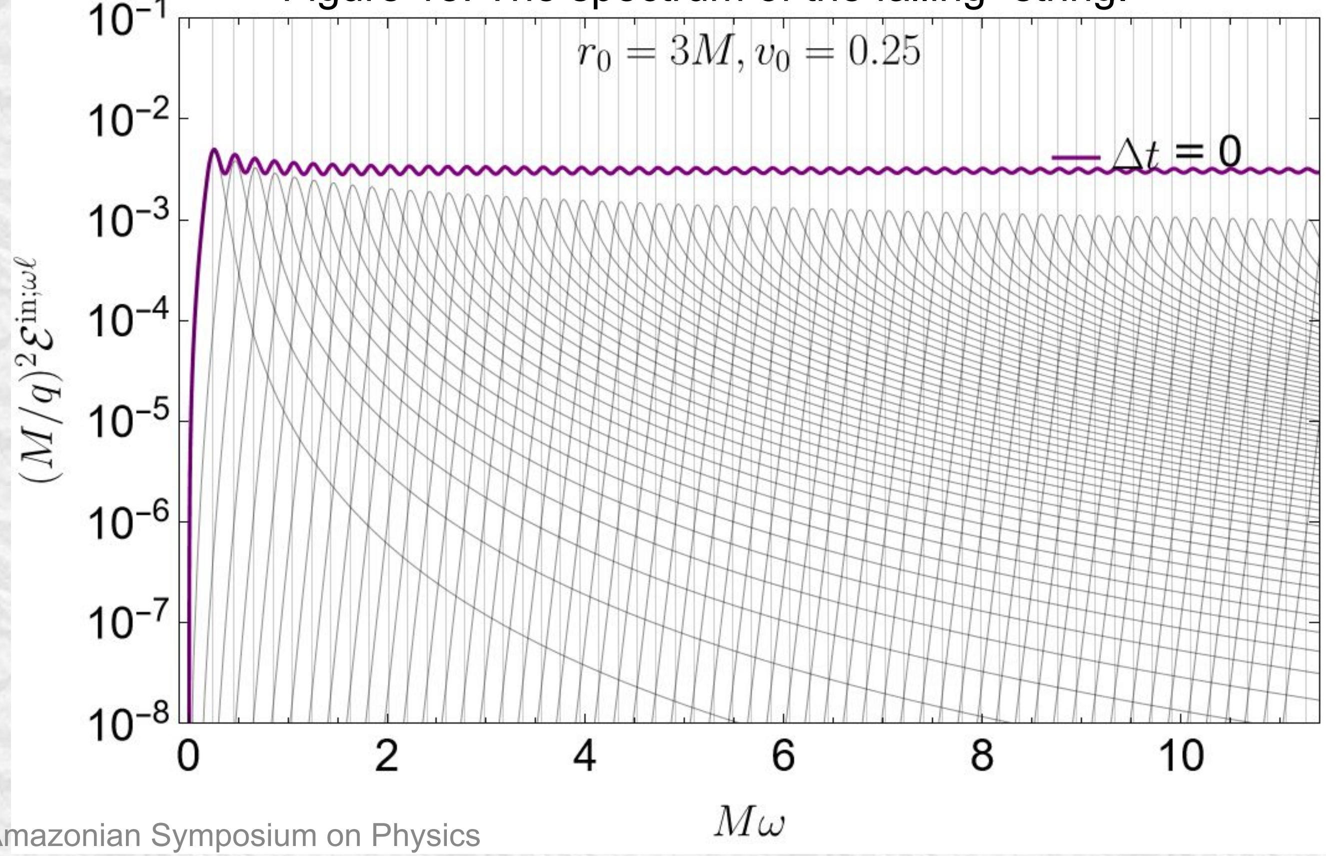


Figure 17: Representation of a falling "string."

Figure 18: The spectrum of the falling "string."



- **Conclusions**

❖ We can analyze radiation settings around BHs using quantum field theory in curved spaces.

❖ The radiation spectrum presents some signatures from the BH quasinormal modes.

❖ The emitted radiation to infinity is divergent when the initial velocity is the speed of light.

- **Conclusions**

❖ The radiation going into the BH is divergent, which is connected to the Coulombic field carried by the charge falling into the BH.

❖ Extended objects: a charged “string” does not eliminate but seems to tame the divergence behavior present in the absorbed radiation.

❖ Ride the inspiral?
  ❖ Analyze the case of inspiralling matter into the BH.

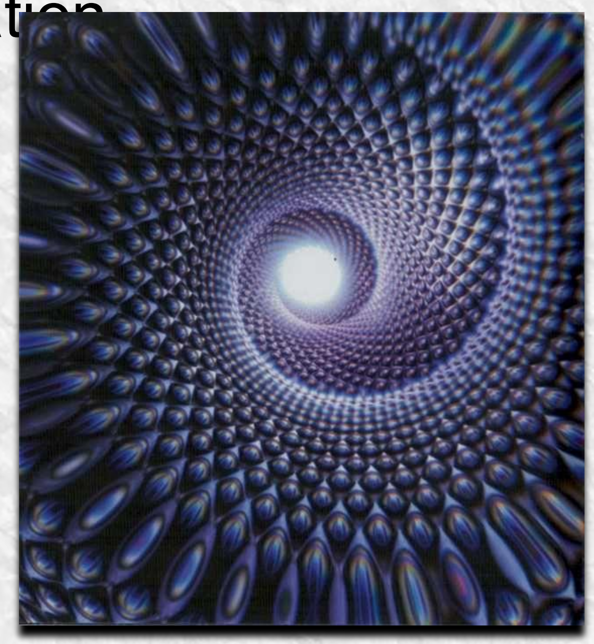


Figure 19: Alex Grey’s artwork for the new *Tool* album.

- **Acknowledgments:**

PPGF

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