THE MAXWELL FISHEYE LENS AND COLLAPSING SPHERES OF UNIFORM DENSITY

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Maxwell's fisheye is a gradient-index lens that perfectly focusses the rays from a point source on its rim at the opposite point on its rim. In a simple geometric reinterpretation, lensed rays map to null geodesics on the curved geometry of a hypersphere. Hyperspherical geometries emerge in General Relativity, up to an overall conformal factor, in the study of static or dynamical spheres of uniform density: well-known examples include the Friedmann spacetime, the Schwarzschild interior solution, and Oppenheimer-Snyder collapse. In this talk I will describe a general class of spheres of uniform density and isotropic pressure, embedded in the Schwarzschild spacetime, that are characterised by the radial motion of their surface (a free function). I show that these spacetimes are conformal to the hypersphere, and, by matching spacetime geometries at the surface, that the extent χ_0 of the three-sphere geometry encompassed by the interior is an elementary function of the energy and proper acceleration of the surface. If the proper acceleration remains constant, then the extent χ_0 also remains constant, and thus the internal geometry is conformal to part of a static Maxwell fisheye lens. We consider the special case of the Schwarzschild interior solution that collapses to form a black hole under constant proper acceleration.