QUANTUM SYSTEM ANALOGUE MODEL FOR A PARTICLE IN A BLACK HOLE BACKGROUND

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The thin-layer method proposed by da Costa [1] allows us to study the behavior of a wavefunction on a surface or curve embedded in an Euclidean space, with or without the presence of an electromagnetic field, and there are a number of papers on the exact solution of Schrödinger's and Pauli's equation on surfaces such as cylinders, spheres, tori, among others [2,3]. The surface of choice will determine the number, types and ranks of singularities of the differential equations, which in turn may be of easy solution once we examine the properties of the said singularities.

In particular, we propose [4] a system consisting of a particle on the surface of a spheroid. Upon solving Schrödinger's and Pauli's equation on a prolate spheroid with and without an external non-central potential, we repeatedly encounter differential equations of Heun type, ranging from three to five singularities, the last case pertaining to a generalized Heun or Lamé [5] type of equation. Because of this, we exactly solve and present the solutions to these problems using Heun's special functions. Additionally, we introduce Schäfke η-function [6] to be used as Pauli's equation solution on the prolate spheroid.

We connect our problems with the dynamics of a particle on the background of black holes such as Kerr-(anti)de Sitter, Kerr-Newman, Kerr-Sen [4,7,8] black holes, and show that our equations and its solutions are identical to those of particles in curved spaces close to black holes. Because of the obvious difficulties in directly studying black holes, they are an ideal real-world target system for our quantum system as an analogue model. We note that it is possible to construct such a prolate spheroid in a laboratory, as shown by Rich and colleagues [9] and Hansen [10].

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