Time-evolution of perturbations in quasi-Schwarzschild black holes

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Presentation Overview

Introduction

- General relativity explains very well classical phenomena such as the precession of the perihelion of mercury.
- However, when it comes to the strong field, there seems to be a need for modifications to achieve a complete theory.
- We will discuss a power-law expansion of the effective potential around the Schwarzschild black hole.
- We study the time evolution of scalar, vector, and gravitational perturbations.

Master Equation

The action describing these fields is

$$S = \frac{1}{2}\int \sqrt{-g} d^4x \left[R - g^{\mu\nu}(\partial_\mu \Phi)(\partial_\nu \Phi) - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right], \tag{1}$$

Let us take the scalar field as example:

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial \Phi}{\partial x^\nu} \right) = 0, \tag{2}$$

$$\Phi(t, r, \theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{\Psi^0_{lm}(r, t)}{r} Y_{lm}(\theta, \phi), \tag{3}$$

A similar procedure can be made for the other fields.

Master Equation

Perturbations induced by scalar, vector, or gravitational fields satisfy the "master equation":

$$\frac{\partial^2 \Psi_\ell^s}{\partial r_*^2} - \frac{\partial^2 \Psi_\ell^s}{\partial t^2} - V_s \Psi_\ell^s = 0, \tag{4}$$

where the effective potential is given by:

$$V_s = \left(1 - \frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}(1 - s^2)\right]. \tag{5}$$

- $s = 0, 1, 2$ is the spin of perturbing field.

Evolution of field perturbation in General Relativiy

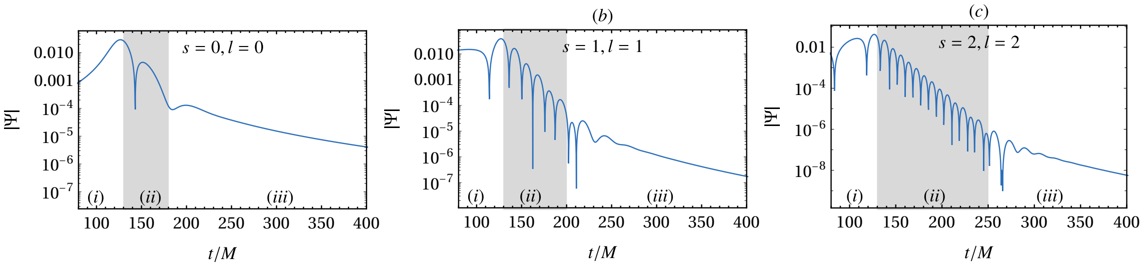

Figure 1: For gaussian package, selected the following constants: $a = 0.15$, $b = 0.05$ starting at $t_0 = 0$ s in $x_0 = 50$. With the observer at $r_* = 50$. The three cases above correspond to $s = 0, 1$ e 2. The shaded region indicates the ringdown stage.

Table 1: Frequencies of QNMs extracted with the Mathematica built in function FindFit, with the Prony method, and computed using Leaver's method .

s	ℓ	FindFit	Prony	Leaver (ver e.g. [1])
0	0	$0.104220 - 0.102807i$	$0.110599 - 0.098465i$	$0.110455 - 0.104896i$
	1	$0.289450 - 0.091829i$	$0.291083 - 0.095101i$	$0.292936 - 0.097660i$
	2	$0.471881 - 0.091806i$	$0.487207 - 0.097036i$	$0.483644 - 0.096759i$
	3	$0.673992 - 0.095215i$	$0.676620 - 0.096577$	$0.675366 - 0.096500i$
	4	$0.869246 - 0.094304i$	$0.867469 - 0.096600i$	$0.867416 - 0.096392i$
1	1	$0.265882 - 0.082399i$	$0.248569 - 0.093090i$	$0.248263 - 0.092488i$
	2	$0.448588 - 0.086194i$	$0.461500 - 0.101623i$	$0.457596 - 0.095004i$
	3	$0.646155 - 0.081963i$	$0.659227 - 0.088161i$	$0.656899 - 0.095616i$
	4	$0.850759 - 0.092493i$	$0.850170 - 0.093437i$	$0.853095 - 0.095860i$
2	2	$0.370612 - 0.088103i$	$0.374178 - 0.084071i$	$0.373672 - 0.088962i$
	3	$0.598397 - 0.085130i$	$0.599680 - 0.093052i$	$0.599443 - 0.092703i$
	4	$0.810658 - 0.090810i$	$0.809148 - 0.093680i$	$0.809178 - 0.094164i$

Evolution of fields in quasi-Schwarzschild

We assume that any modifications in the gravity theory can be analyzed through a deviation in the potential of the form

$$V = V_s + \delta V_s \longrightarrow \delta V_s = \frac{f}{(2M)^2} \sum_{j=0}^{\infty} \beta_j^s \left(\frac{2M}{r}\right)^j \tag{6}$$

- The form of the quasi-Schwarzschild potential suggests that it is instructive to separate the dependence of each term in the power-law individually. We have

$$V_s = \left(1 - \frac{2M}{r}\right) \left[\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}(1 - s^2) + \frac{\beta}{(2M)^2} \left(\frac{2M}{r}\right)^j\right]. \tag{7}$$

- We should expect changes in the ringdown.
- We will use different values for s and j, focus in $s = \ell$.

$j = 0$ terms and the presence of the quasibound spectrum

$$V_s = \left(1 - \frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}(1 - s^2) + \frac{\beta}{(2M)^2}\left(\frac{2M}{r}\right)^0\right] \longrightarrow \mu^2 = \frac{\beta}{(2M)^2} \quad (8)$$

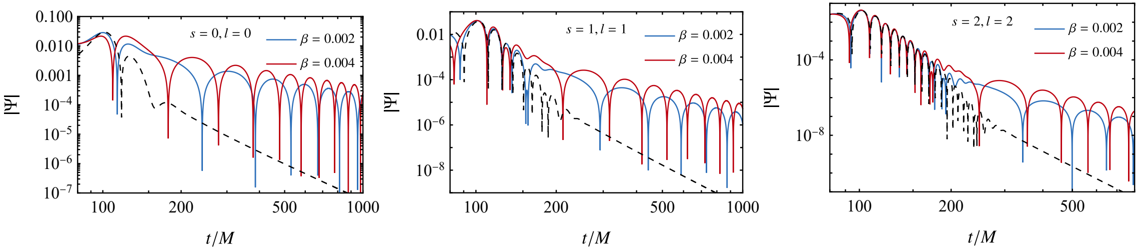

Figure 2: Time evolution of the gaussian, considering different values of the $s = \ell$ for $j = 0$. Dashed line denote the result of Schwarzschild evolution and at late times we can see an oscillation, which is modulated by the value of the parameter β.

$j = 0$ term and the presence of the spectrum quasibound

- All the literature on massive fields can be applied to this case
- Massive fields have a two distinct spectra related to the asymptotic behavior of the perturbations
  - Photon Sphere Modes
  - Quasibound Modes
- The QBMs have the characteristic of trapping perturbations

$j = 1$ terms and oscillations in the tail

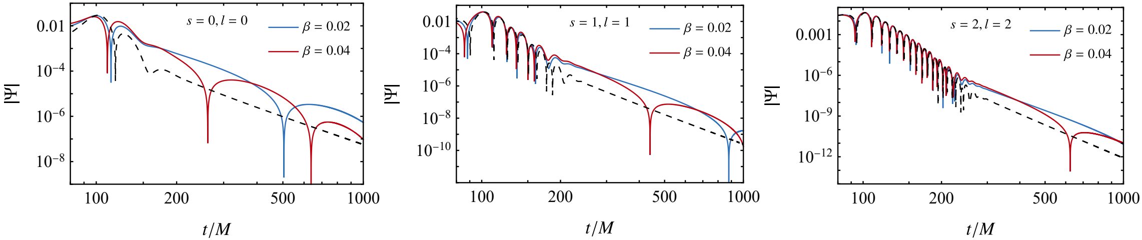

Figure 3: Similar to Fig. 2, we consider $j = 1$ in this case. We observe that the term $j = 1$ introduces an oscillatory behavior in the tail.

- The fields decline considerably faster than $j = 0$.
- Tail is modified, showing an oscilatory behavior.
- Behavior is similar to what happens with massive fields in Schwarzschild-de Sitter spacetime.

$j = 2$ terms and deviations in the power-law tail

$$V_s = \left(1 - \frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}(1-s^2) + \frac{\beta}{(2M)^2}\left(\frac{2M}{r}\right)^j\right] \longrightarrow \frac{\ell(\ell+1)+\beta}{r^2} \qquad (9)$$

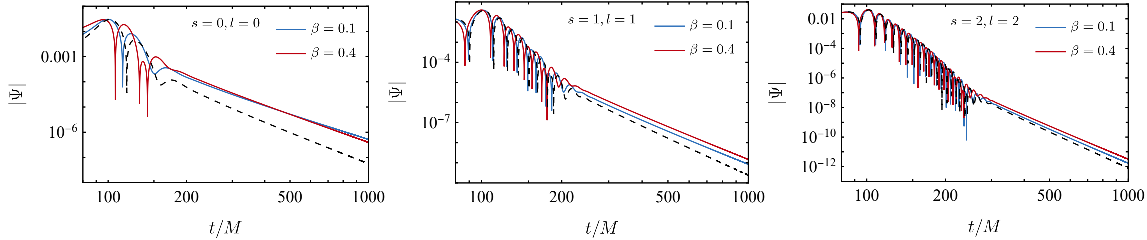

Figure 4: Similar to Fig. 2, but considering $j = 2$. This term changes the behavior of power-law in the tail due to the modification in the potential, which is of the same order as the term proportional to $\ell(\ell+1)$.

$j = 3, 4...$ terms and the ringdown

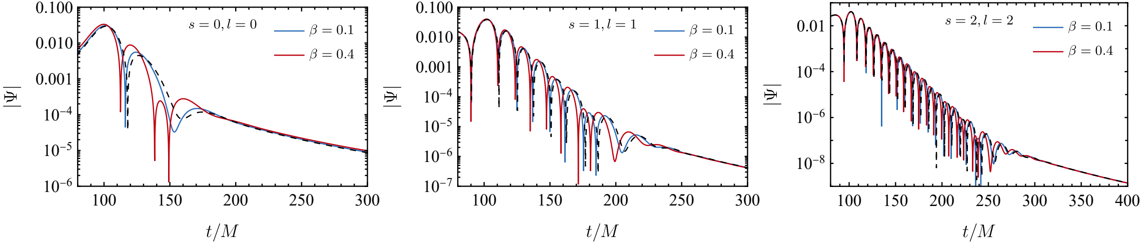

Figure 5: Same as Fig. 2, but considering $j = 3$. For terms $j > 2$, we see that the main contribution is to parametrically modify the ringdown frequency of the signals.

Conclusion

- We analyze the evolution of field perturbations in General Relativity and evolution of fields in quasi-Scwarzschild.

Perspectives:

- A deeper investigation for $j = 1$ terms involving evolution over longer times.
- Negative values of β: Instabilities.
- Theory mapping and detectability.

References

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Acknowledgements

Thank you