EXOTIC COMPACT OBJECTS in 2+1 DIMENSIONS

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MOTIVATIONS I : basic reasons & some facts

There are many reasons to explore the phenomenology of exotic compact objects:

1) **Are we really observing the BHs of GR?**

2) **Should we pay attention to what is observable, or only to what seems reasonable (today/to someone)?**

In the search for **alternative nonsingular scenarios** for compact objects, one can go beyond GR or consider exotic matter sources.

For time efficiency, it is always best to consider new problems in simplified form. If analytical, much better.

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MOTIVATIONS II: basic reasons & some facts

In the search for **exotic compact objects**, we observed that **wormholes** arise quite generically in **modified gravity theories beyond GR**.

Scalar geons in Born-Infeld gravity

V.I. Afonso[1,2], Gonzalo J. Olmo[2,3] and D. Rubiera-Garcia[4]

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New scalar compact objects in Ricci-based gravity theories

Victor I. Afonso[1,2], Gonzalo J. Olmo[2,3], Emanuele Orazi[4,5] and Diego Rubiera-Garcia[6]

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Citation Victor I. Afonso *et al* JCAP12(2019)044

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Compact objects in quadratic Palatini gravity generated by a free scalar field

Renan B. Magalhães, Luís C. B. Crispino, and Gonzalo J. Olmo

Phys. Rev. D **105**, 064007 – Published 4 March 2022

Birth of baby universes from gravitational collapse in a modified-gravity scenario

Andreu Masó-Ferrando[1], Nicolas Sanchis-Gual[2,3], José A. Font[2,4] and Gonzalo J. Olmo[1,5]

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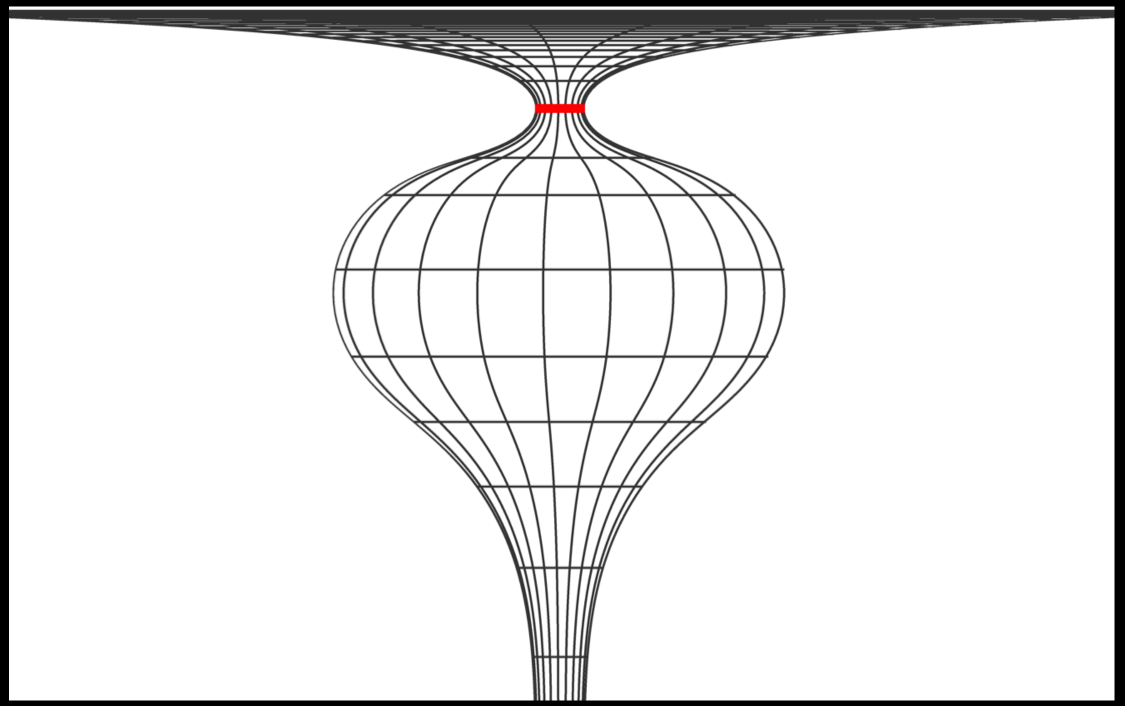
Echoes from bounded universes

Renan B. Magalhães,[1,2,*] Andreu Masó-Ferrando,[2,†] Flavio Bombacigno,[2,‡] Gonzalo J. Olmo,[2,3,§] and Luís C. B. Crispino[1,4,¶]



With a 2+1 **scalar field model**, we will see that even more exotic structures may arise within GR.

FIRST: THE MODEL

DOI: https://doi.org/10.3390/universe10060258

2. (2 + 1)-Einstein Theory with Nonlinear Scalar Field

Let us start by defining the action for (2 + 1)-Einstein gravity coupled to a scalar field as

$$\mathcal{S} = \int d^3x\sqrt{-g}\left(\frac{1}{2\kappa}\mathcal{R} - \frac{1}{2}L(Y)\right), \qquad (1)$$

where $\mathcal{R} \equiv g^{\mu\nu}R_{\mu\nu}$ is the usual curvature scalar of a space-time metric $g_{\mu\nu}$ and Ricci tensor $R_{\mu\nu}$, while $L(Y)$ is an arbitrary function of the scalar field invariant $Y \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, and

Work by R.V. Maluf, G. Mora-Pérez, GJO, and D. Rubiera-García, Universe 2024, 10, 258

We consider standard GR but with a **noncanonical scalar field.**

Impose staticity and circular symmetry. This can be done by hand.

$$T_{\mu\nu} = \left(L_Y\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}L\right), \qquad \partial_x\left(\sqrt{-g}g^{xx}L_Y\phi_x\right) = 0,$$

$$ds^2 = -e^{A(x)}dt^2 + e^{A(x)}\frac{L_Y^2}{W^2(x)}dx^2 + \frac{1}{W^2(x)}d\theta^2, \qquad \phi_{xx} = 0$$

The structure of the line element is chosen to simplify the scalar field equation

SECOND: THE EQUATIONS

$$
\begin{aligned}
R^t{}_t &= Y\left(\frac{A_x L_{Y_x}}{2L_Y} - \frac{A_{xx}}{2}\right) = \kappa(L - YL_Y), \\
R^x{}_x &= Y\left(\frac{A_x L_{Y_x}}{2L_Y} - \frac{A_x W_x}{W} - \frac{L_{Y_x} W_x}{L_Y W}\right. \\
&\quad \left. - \frac{W_x^2}{W^2} - \frac{A_{xx}}{2} + \frac{W_{xx}}{W}\right) = \kappa L, \\
R^\theta{}_\theta &= Y\left(-\frac{L_{Y_x} W_x}{L_Y W} - \frac{W_x^2}{W^2} + \frac{W_{xx}}{W}\right) \\
&= \kappa(L - YL_Y),
\end{aligned}
$$

The first equation can be written as:

$$A_{xx} - A_x \frac{L_{Y_x}}{L_Y} = -2\kappa \frac{(L - YL_Y)}{Y}$$

And a formal solution is:

$$L_{Y_x} \equiv \frac{\partial L_Y}{\partial x}$$

$$A_x = L_Y \left(c_1 - 2\kappa \int dx \frac{(L - YL_Y)}{YL_Y} \right)$$

If we take $L(Y) = \lambda Y^\alpha$ then: $A_x = -\lambda \alpha Y^{\alpha-1} \Delta_1$ where $\Delta_1 \equiv \frac{2\kappa(1-\alpha)}{\alpha}(x - x_1)$

The combination $R^x_x - R^t_t - R^\theta_\theta$ leads to $\frac{A_x W_x}{W} = \kappa \frac{(L - 2YL_Y)}{Y}$ which allows us to obtain

$$\gamma \equiv \frac{(2\alpha - 1)}{(1 - \alpha)} \qquad W(x) = \frac{1}{r_0}\left(\frac{x - x_1}{x_0}\right)^{\gamma/2}$$

Recall that W(x)=1/r(x)

Shift symmetry in x

THIRD: MORE EQUATIONS & SOLUTIONS

Using W(x) in the angular equation we find:

$$Y_x - \frac{2\kappa}{\alpha\Delta_1}Y = \frac{\lambda\alpha\Delta_1}{(2\alpha-1)}Y^\alpha$$

This **Bernoulli equation** can be solved exactly and allows to solve for $A_x = -\lambda\alpha Y^{\alpha-1}\Delta_1$

$$e^{A(x)} = \Delta_2^{-\gamma}$$

where

$$\Delta_2 \equiv c_2\left[1 + \frac{2\lambda\kappa(1-\alpha)x_0}{\gamma c_2}\left(\frac{r_0}{r}\right)^{2/\gamma}\right] \equiv \left[1 \pm \left(\frac{R_0}{r}\right)^{2/\gamma}\right]$$

Two families of **asymptotically flat** solutions: $\lambda = \pm|\lambda|$

$$ds^2 = -\Delta_2^{-\gamma}dt^2 + \sigma_0^2\Delta_2^{-(2+\gamma)}dr^2 + r^2d\theta^2$$

$\frac{1}{2} < \alpha < 1$ corresponds to the interval $\gamma > 0$

$$\gamma \equiv \frac{(2\alpha-1)}{(1-\alpha)}$$

Attractive:
$\lambda = +|\lambda|$

$$ds^2 = -\frac{1}{\left(1+\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^\gamma}dt^2 + \frac{\sigma_0^2dr^2}{\left(1+\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{2+\gamma}} + r^2d\theta^2$$

Repulsive:
$\lambda = -|\lambda|$

$$ds^2 = -\frac{1}{\left(1-\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^\gamma}dt^2 + \frac{\sigma_0^2dr^2}{\left(1-\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{2+\gamma}} + r^2d\theta^2$$

FOURTH: CURVATURES

Attractive:

$\lambda = +|\lambda|$

$$ds^2 = -\frac{1}{\left(1+\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{\gamma}}dt^2 + \frac{\sigma_0^2 dr^2}{\left(1+\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{2+\gamma}} + r^2 d\theta^2$$

Repulsive:

$\lambda = -|\lambda|$

$$ds^2 = -\frac{1}{\left(1-\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{\gamma}}dt^2 + \frac{\sigma_0^2 dr^2}{\left(1-\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{2+\gamma}} + r^2 d\theta^2$$

$$R = \pm\frac{2(\gamma+4)\left(1\pm\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{\gamma+1}\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}}{\gamma\sigma_0^2 r^2}$$

$$K = \frac{4(3\gamma^2+8\gamma+8)\left(1\pm\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{2(\gamma+1)}\left(\frac{R_0}{r}\right)^{\frac{4}{\gamma}}}{\gamma^2\sigma_0^4 r^4}$$

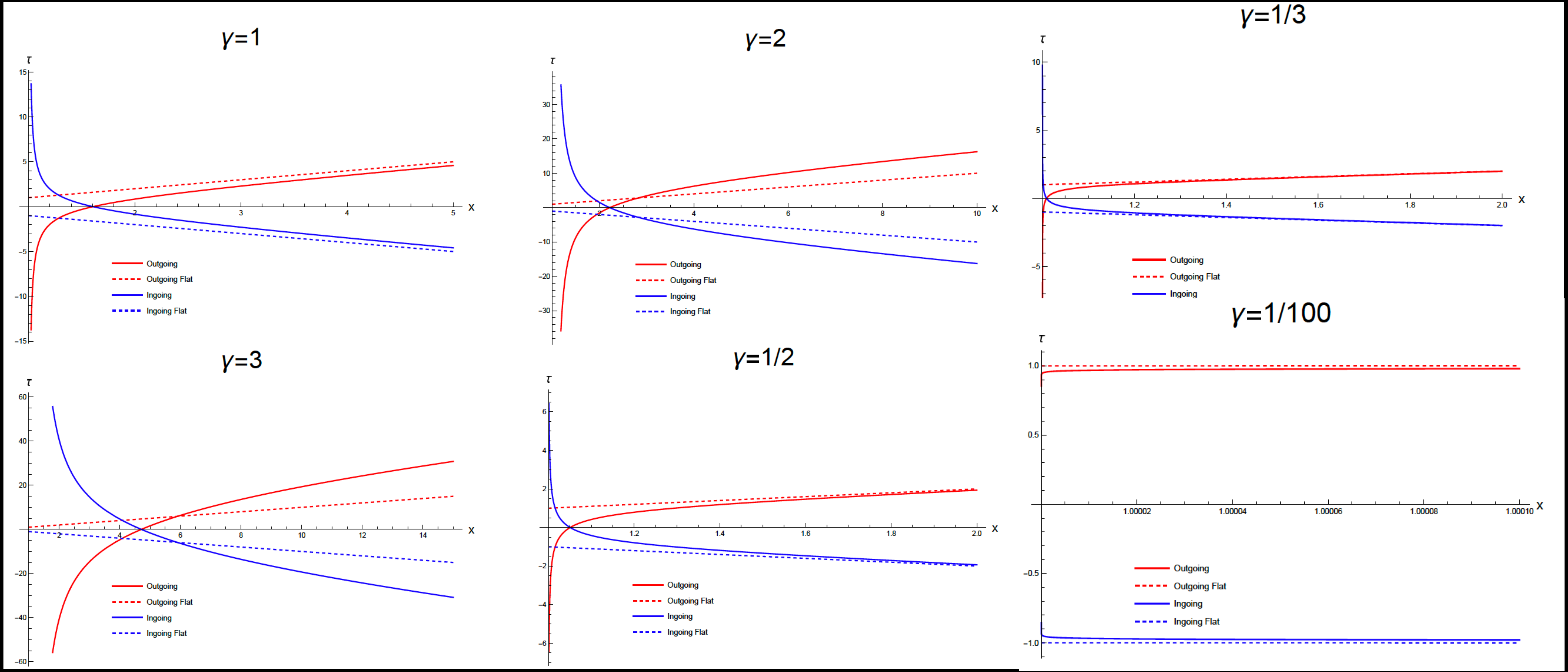
The attractive case has divergent curvatures as $r \to 0$ and **incomplete geodesics**.

The repulsive case has vanishing curvatures as $r \to R_0$ and **complete geodesics**.

FIFTH: GEODESICS

Only the case $\lambda < 0$ is of interest

$$ds^2 = -\frac{1}{\left(1-\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{\gamma}}dt^2 + \frac{\sigma_0^2 dr^2}{\left(1-\left(\frac{R_0}{r}\right)^{\frac{2}{\gamma}}\right)^{2+\gamma}} + r^2 d\theta^2$$

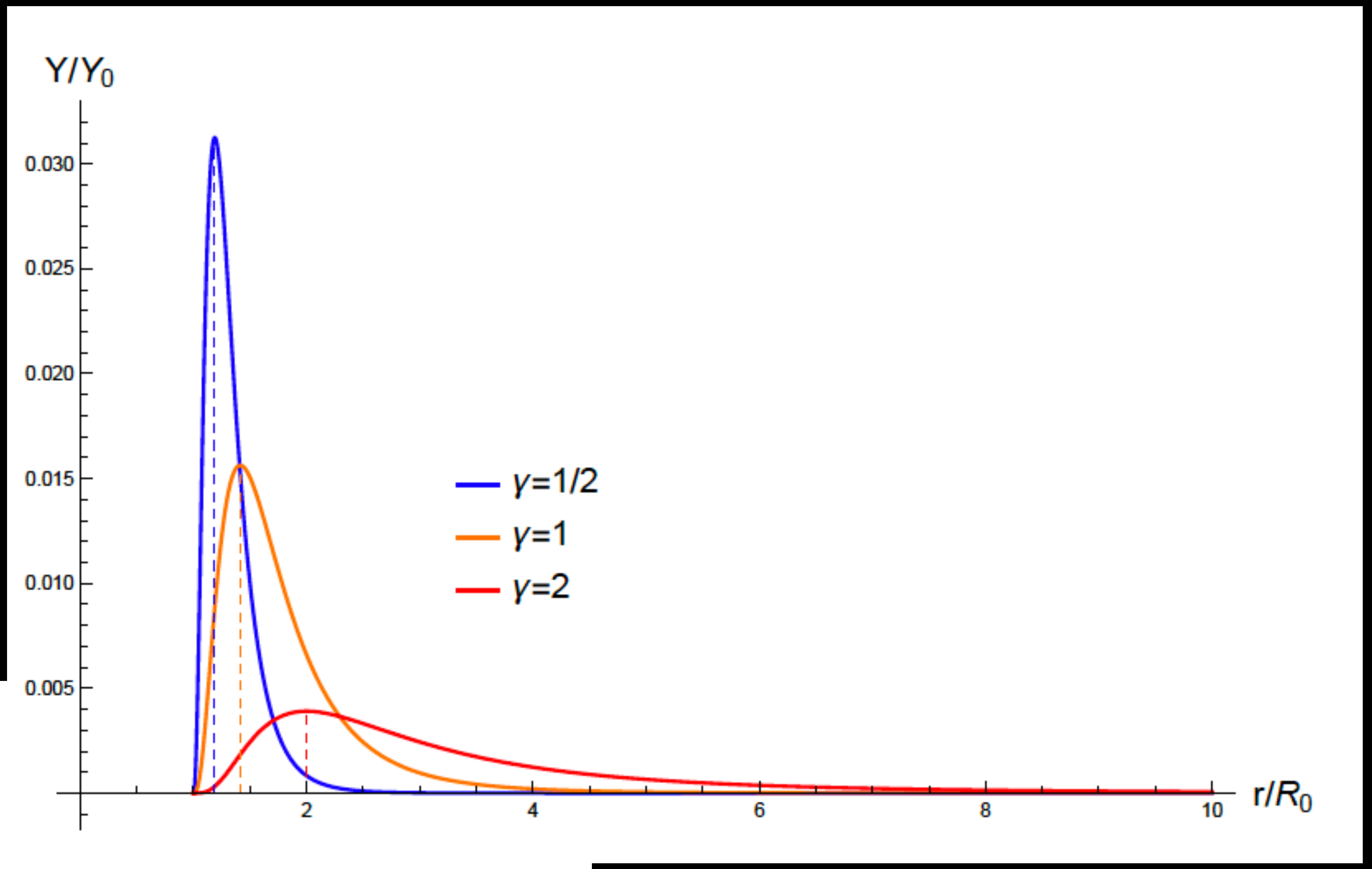


All geodesics are complete. $r = R_0$ is a boundary of the manifold

SIXTH: ENERGY DENSITY

Considering $\lambda < 0$:

$$\Upsilon = \Upsilon_0 \frac{\left(1 \pm \hat{r}^{-2/\gamma}\right)^{2+\gamma}}{\hat{r}^{\frac{2(2+\gamma)}{\gamma}}}$$



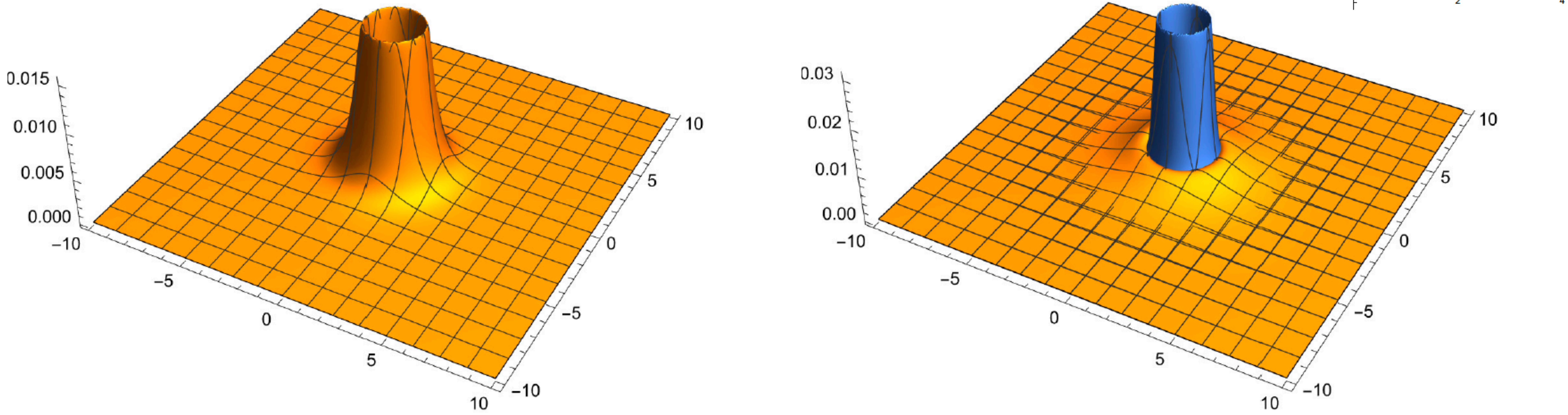


Figure 3. Left: three-dimensional representation of the kinetic term $\Upsilon = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ when $\lambda < 0$ for the case $\gamma = 1$. Right: same representation for $\gamma = 1/2$ (blue) and $\gamma = 2$ (orange). Note how the more compact solution $\gamma = 1/2$ is always hidden by the $\gamma = 2$ one except at the innermost region. The different amplitudes of the maxima are also evident in this plot.

Work by R.V. Maluf, G. Mora-Pérez, GJO, and D. Rubiera-García, Universe 2024, 10, 258

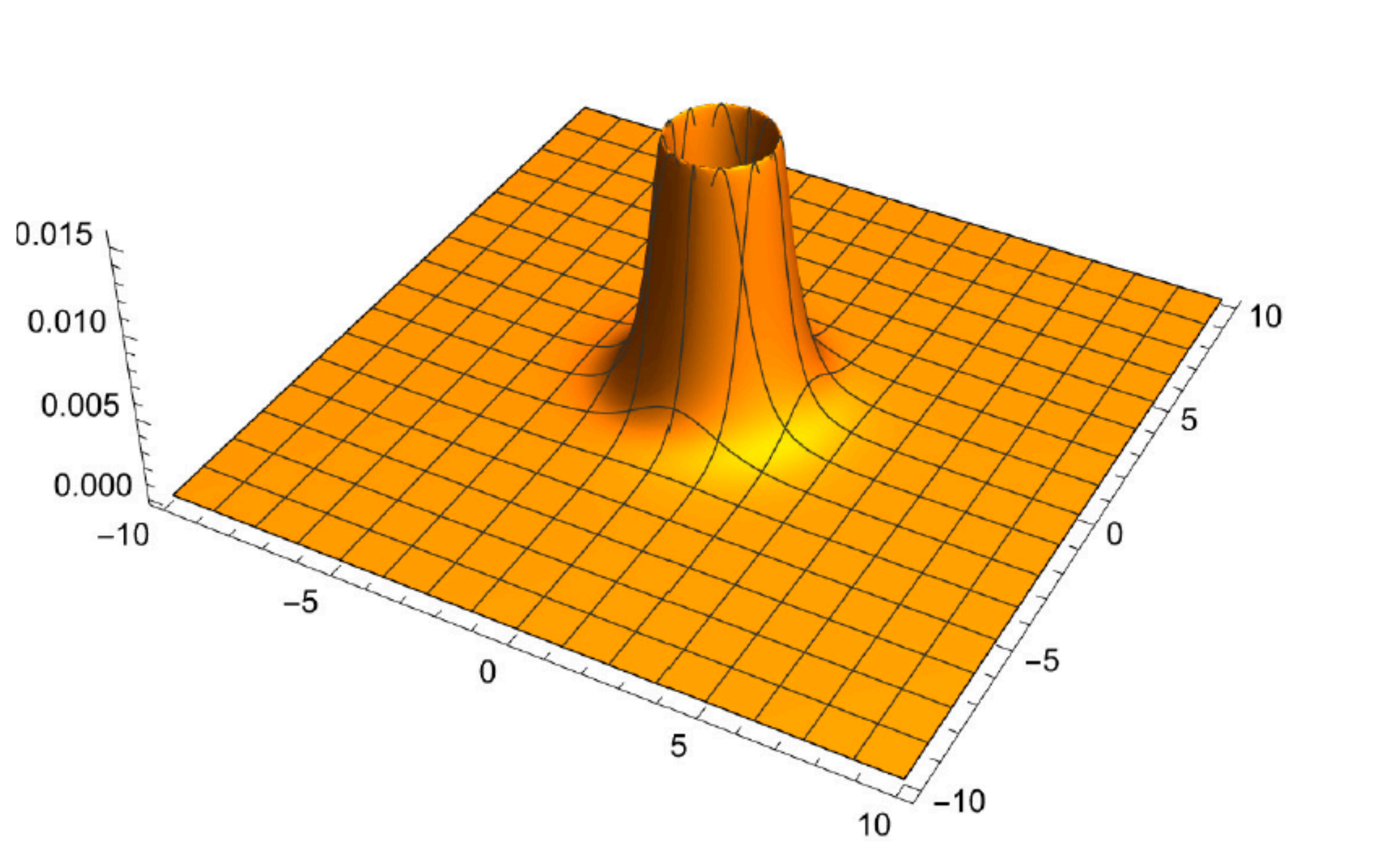
DOI: https://doi.org/10.3390/universe10060258

All geodesics are complete. $r = R_0$ is a boundary of the manifold

CONCLUSION

Nonsingular space-time with an internal boundary found in a 2+1 scalar field model built on *mathematical simplicity* arguments.

Geodesic completeness is achieved non-perturbatively in a compact region.



OPEN QUESTIONS

- Are these configurations stable?
- Can we add regular attractive matter on top of this field?
- Could we tune the sign of λ to localize the repulsive region?
- Can objects of this kind exist in 3+1 dimensions?
- Can current codes support these boundary conditions?
- What would the observable features of such objects be?

Thank you!!!

Questions ... Comments ???