Radial stability of spherical bosonic stars and critical points

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[arXiv:2404.07257]

Introduction

- Formation mechanism
- Model for dark matter
- Stability?

Gleiser(1988)
Jetzer(1989)
Seidel&Suen(1990)
Brito+(2016) [arXiv:1508.05395]

Framework

$$\mathcal{S}_s = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{R}{16\pi G} + \mathcal{L}_s\right]$$

$$\mathcal{L}_0 = -g^{ab}\nabla_a\Phi\nabla_b\bar{\Phi} - V_0(\Phi\bar{\Phi})$$

$$V_0(\Phi\bar{\Phi}) = \begin{cases} \mu^2\Phi\bar{\Phi} & \rightarrow \text{mini} \\ \mu^2\Phi\bar{\Phi}\left(1 - \frac{2\Phi\bar{\Phi}}{v_0^2}\right)^2 & \rightarrow \text{solitonic} \\ \frac{2\mu^2 f_a^2}{B}\left[1 - \sqrt{1 - 4B\sin^2\left(\frac{\sqrt{\Phi\bar{\Phi}}}{2f_a}\right)}\right] & \rightarrow \text{axionic} \end{cases}$$

$$\mathcal{L}_1 = -\frac{1}{4}\mathcal{F}_{ab}\bar{\mathcal{F}}^{ab} - V_1(\mathcal{A}_a\bar{\mathcal{A}}^a)$$

$$V_1(\mathcal{A}_a\bar{\mathcal{A}}^a) = \frac{1}{2}\mu^2\mathcal{A}_a\bar{\mathcal{A}}^a$$

Framework

$$E_{ab} \equiv G_{ab} - 8\pi G T_{ab}^{[s]} = 0$$

$$T_{ab}^{[0]} = \nabla_a \Phi \nabla_b \bar{\Phi} + \nabla_b \Phi \nabla_a \bar{\Phi} - g_{ab} \left[\frac{1}{2} g^{cd} \left(\nabla_c \Phi \nabla_d \bar{\Phi} + \nabla_d \Phi \nabla_c \bar{\Phi} \right) + V_0(\Phi\bar{\Phi}) \right] ,$$

$$T_{ab}^{[1]} = \frac{1}{2} g^{cd} \left(\mathcal{F}_{ac} \bar{\mathcal{F}}_{bd} + \bar{\mathcal{F}}_{ac} \mathcal{F}_{bd} \right) - \frac{1}{4} g_{ab} F_{cd} \bar{F}^{cd} + \frac{1}{2} \mu^2 \left(\mathcal{A}_a \bar{\mathcal{A}}_b + \bar{\mathcal{A}}_a \mathcal{A}_b - g_{ab} \mathcal{A}_c \bar{\mathcal{A}}^c \right) ,$$

$$\nabla_a \nabla^a \Phi - \frac{\partial V_0}{\partial(\Phi\bar{\Phi})} \Phi = 0 ,$$

$$\nabla_a \mathcal{F}^{ab} - \mu^2 \mathcal{A}^b = 0 .$$

Equilibrium solutions

$$\mathrm{d}s^2 = g^{(0)}_{ab}\,\mathrm{d}x^a\mathrm{d}x^b = -\sigma(r)^2 N(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{N(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

$$N(r) = 1 - \frac{2G\mathcal{M}(r)}{r}$$

$$\Phi^{(0)} = e^{-i\omega t}\,\phi(r)\,,$$

$$\mathcal{A}^{(0)}_a\mathrm{d}x^a = e^{-i\omega t}\left[f(r)\,\mathrm{d}t + ig(r)\,\mathrm{d}r\right]$$

Perturbed solutions

$$g_{ab}^{(1)}\mathrm{d}x^a\mathrm{d}x^b = \sigma(r)^2 N(r)\tilde{H}_0(t,r)\,\mathrm{d}t^2 + \frac{\tilde{H}_2(t,r)}{N(r)}\,\mathrm{d}r^2$$

$$\Phi^{(1)} = e^{-i\omega t}\,\phi_1(t,r)\;,$$

$$\mathcal{A}_a^{(1)}\mathrm{d}x^a = e^{-i\omega t}\,[f_1(t,r)\,\mathrm{d}t + ig_1(t,r)\,\mathrm{d}r]\;,$$

$$\tilde{H}_0(t,r) = (e^{-i\Omega t} + e^{+i\Omega t})H_0(r)\;,$$

$$\tilde{H}_2(t,r) = (e^{-i\Omega t} + e^{+i\Omega t})H_2(r)\;,$$

$$\phi_1(t,r) = e^{-i\Omega t}\,\phi_+(r) + e^{+i\Omega t}\,\phi_-(r)\;,$$

$$f_1(t,r) = e^{-i\Omega t}\,f_+(r) + e^{+i\Omega t}\,f_-(r)\;,$$

$$g_1(t,r) = e^{-i\Omega t}\,g_+(r) + e^{+i\Omega t}\,g_-(r)\;.$$

$\Omega^2 > 0$ → Stable

$\Omega^2 < 0$ → Unstable

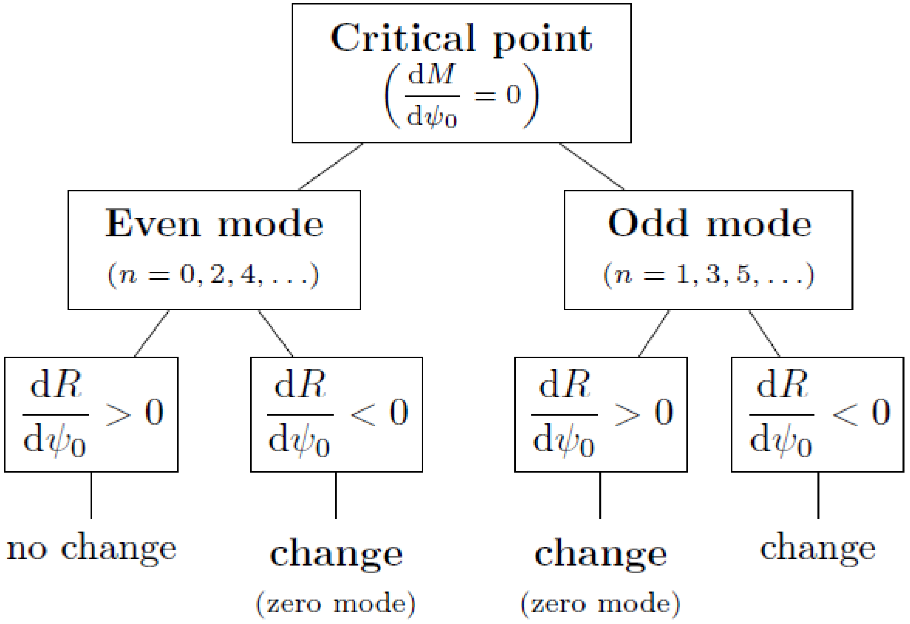
$$\{\Omega_n^2\}_{n=0}^{\infty}$$

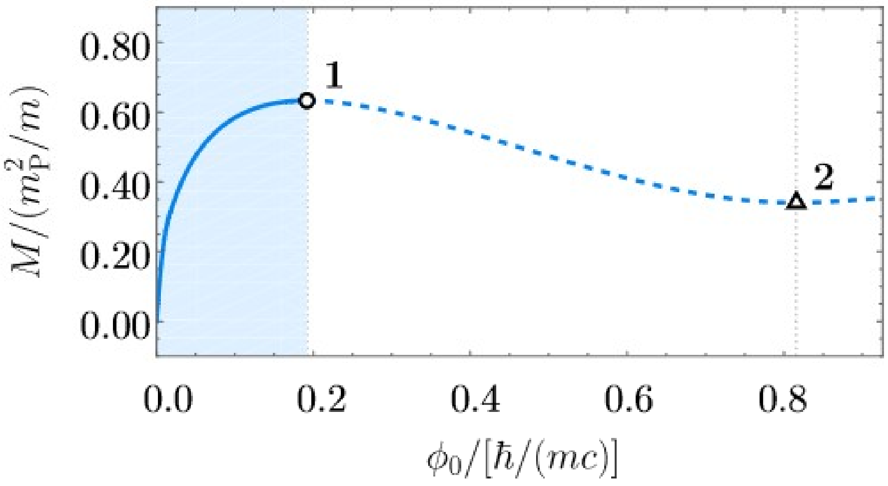
$$\Omega_0^2 < \Omega_1^2 < \Omega_1^2 \ldots$$

Boundary conditions

	Metric functions		Matter functions				
	$\mathcal{M}$	σ	ϕ	ϕ'	f	f'	g
Inner BCs ($r = 0$)	0	σ_0	ϕ_0	0	f_0	0	0
Outer BCs ($r = \infty$)	M	1	0	0	0	0	0
	H_0	H_2	$\phi_\pm$	$\phi'_\pm$	$f_\pm$	$f'_\pm$	$g_\pm$
Inner BCs ($r = 0$)	h_0	0	$\phi_\pm(0)$	0	$f_\pm(0)$	0	0
Outer BCs ($r = \infty$)	h_∞	0	0	0	0	0	0

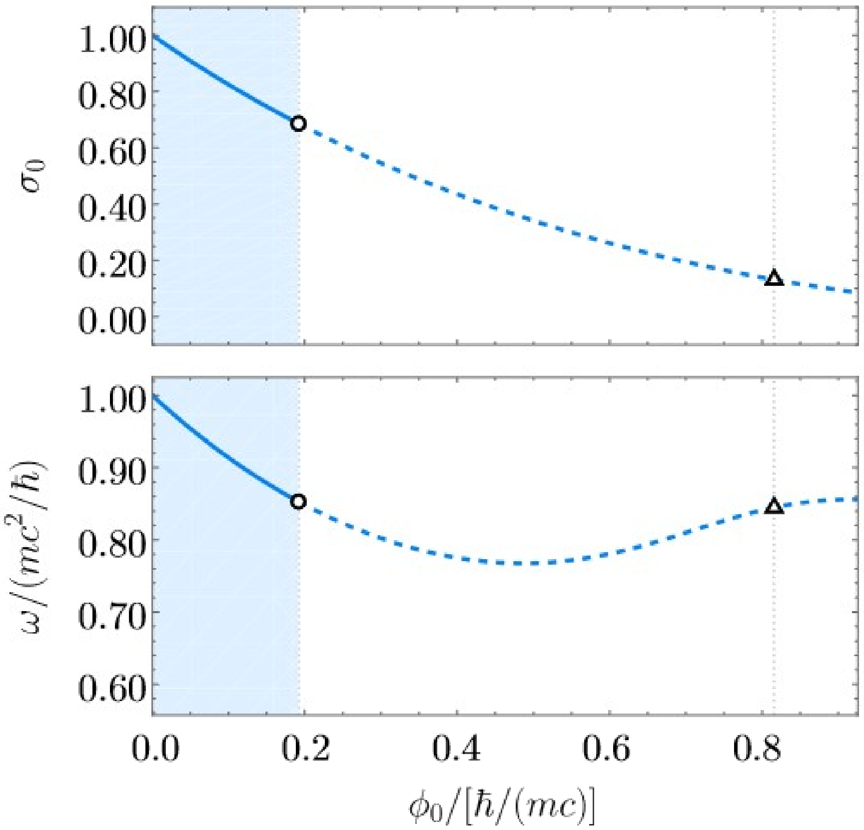
Critical point method

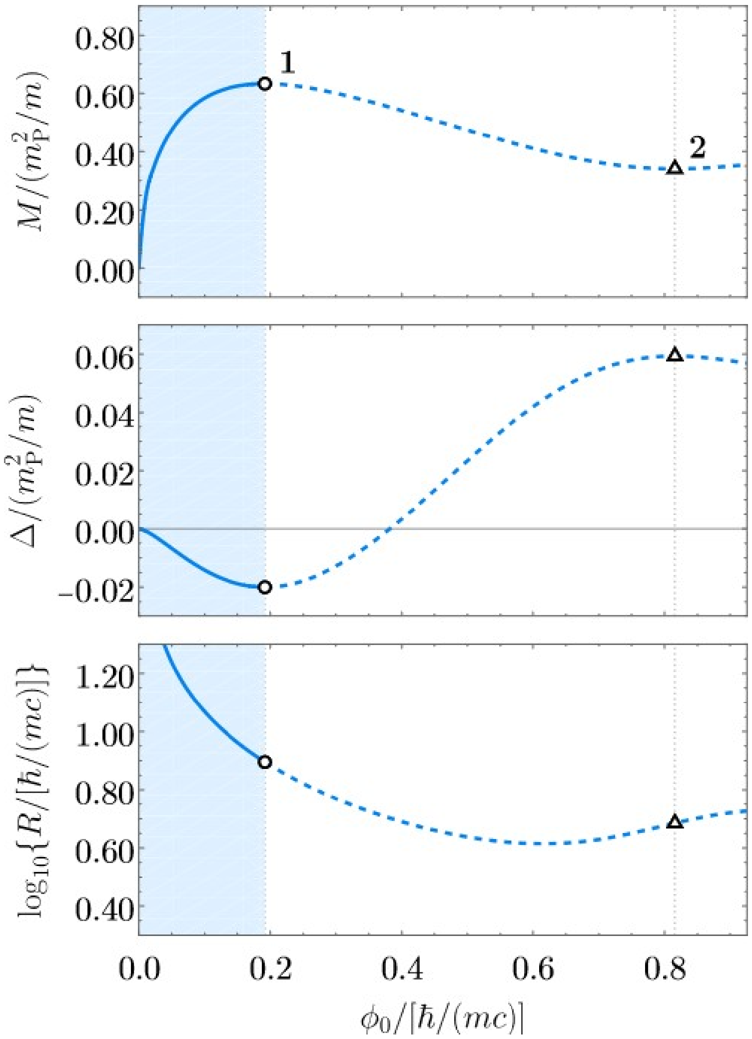




Results

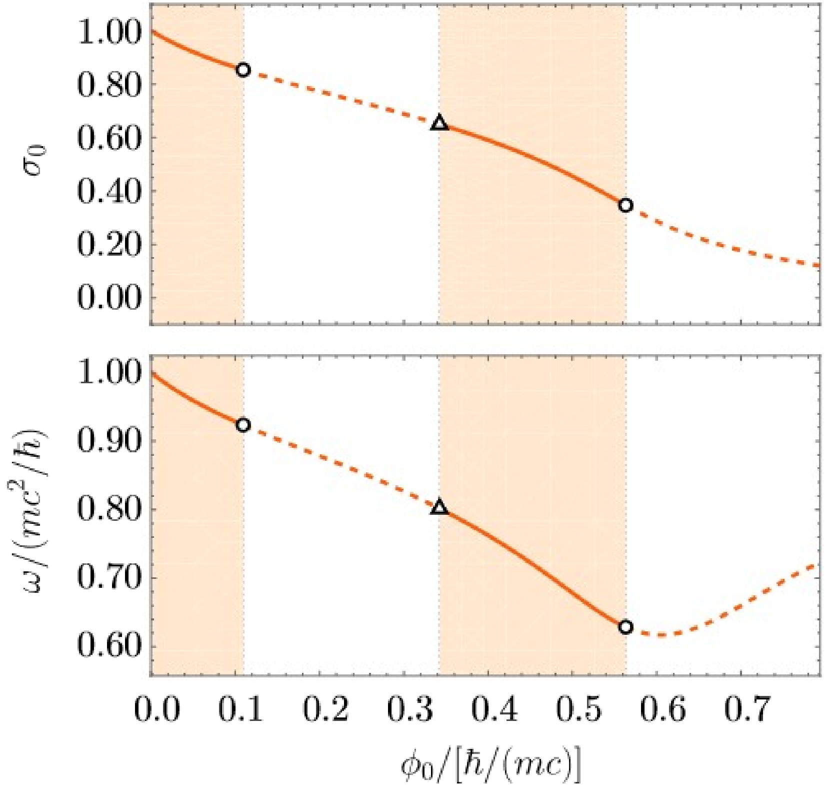
Mini-boson stars

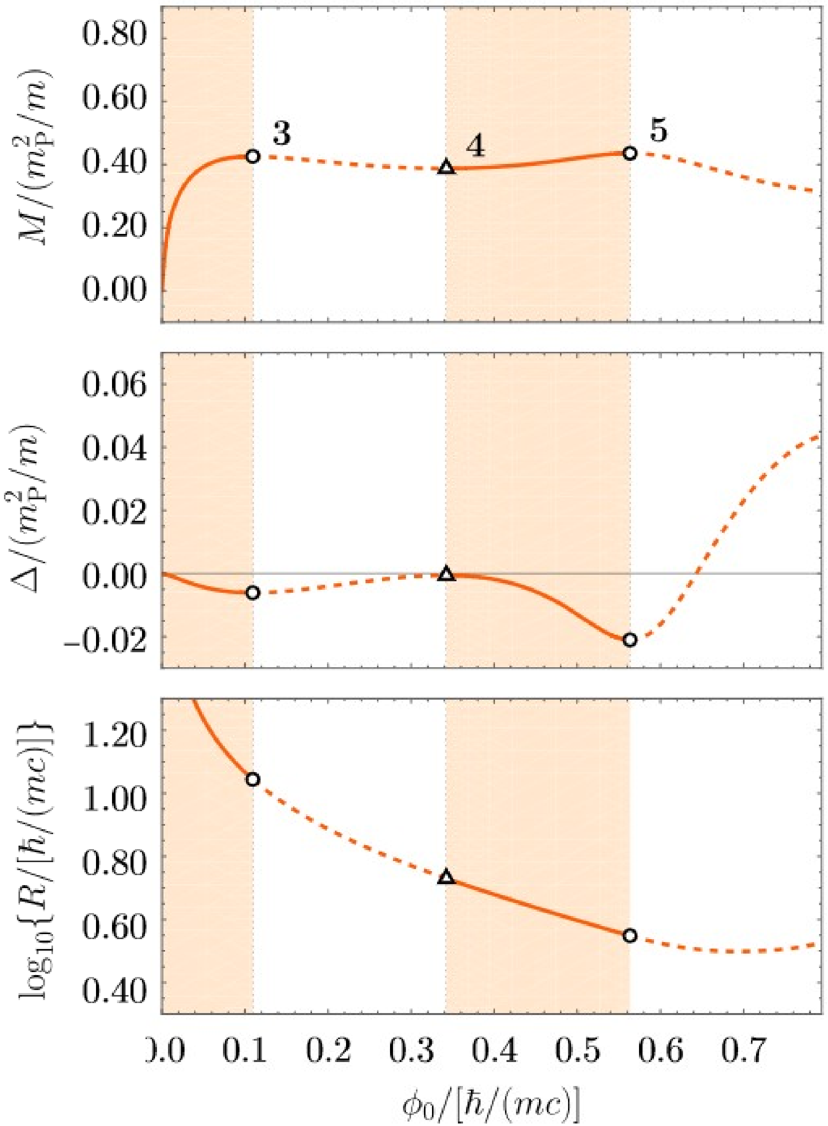




Results

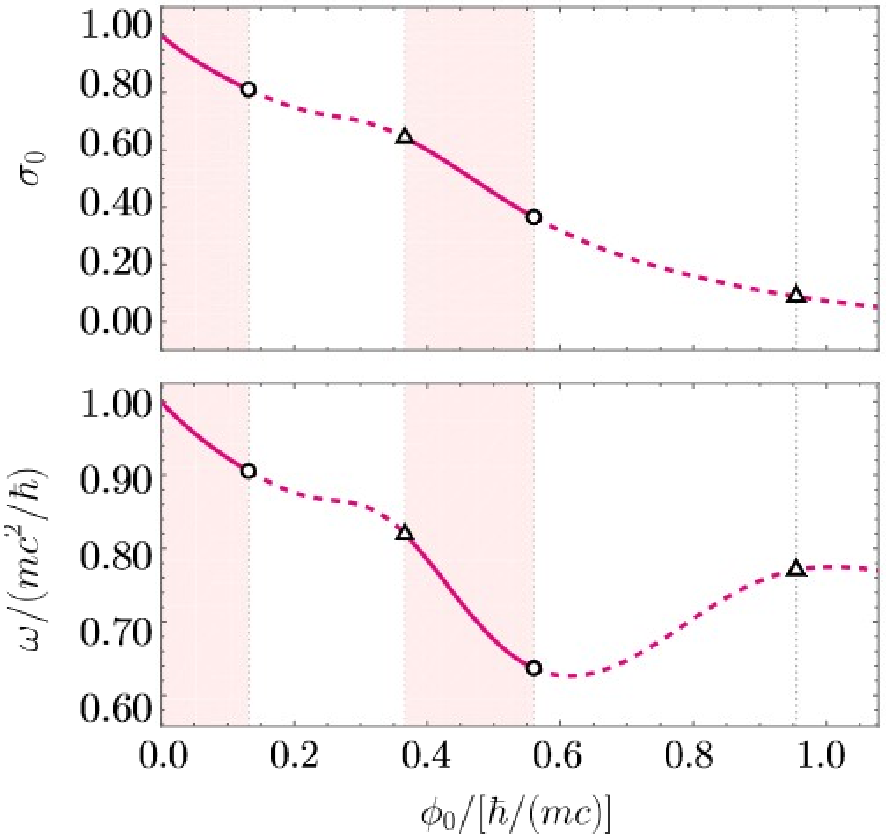
Solitonic boson stars with $v_0 = 0.20$

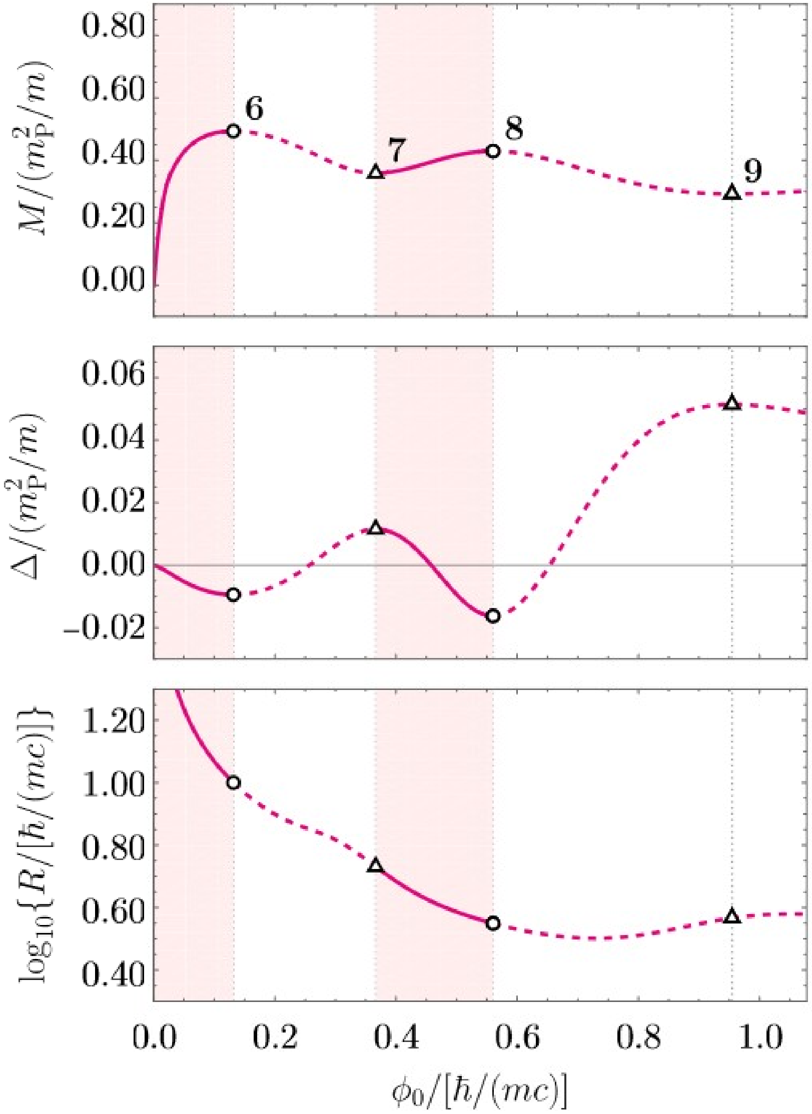




Results

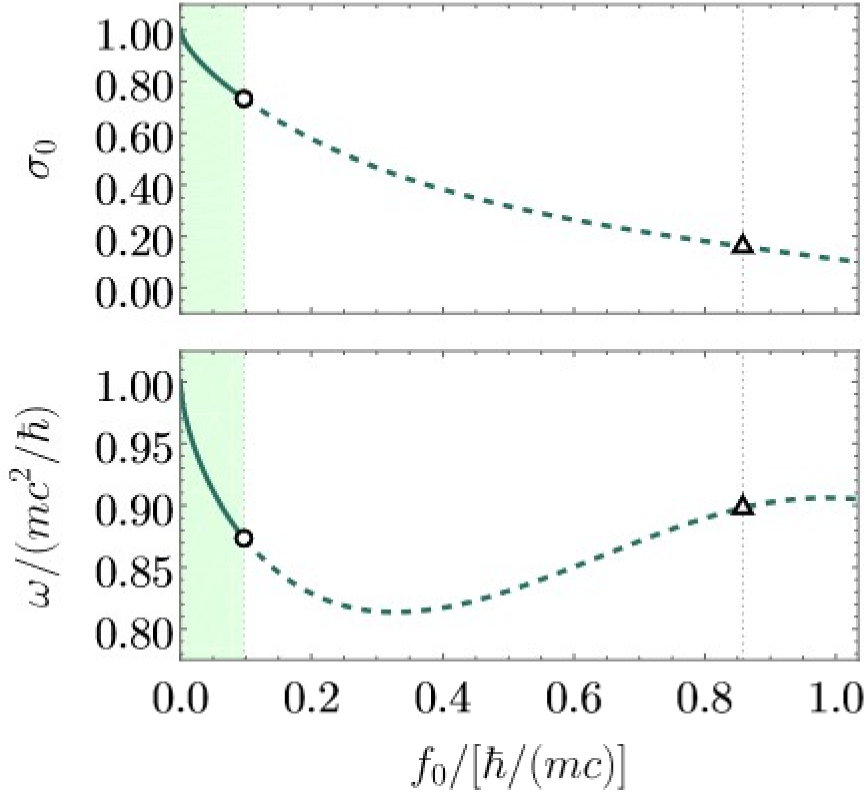
Axion boson stars with $f_a = 0.08$

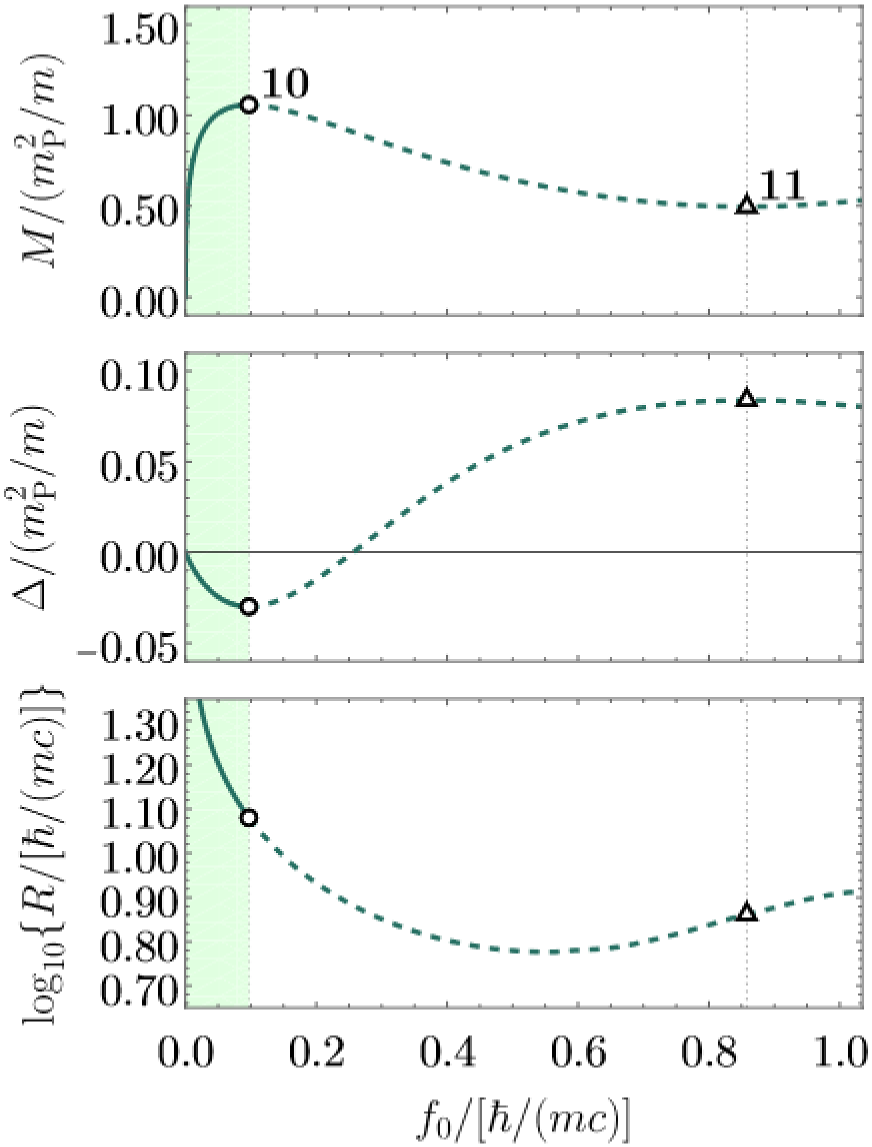




Results

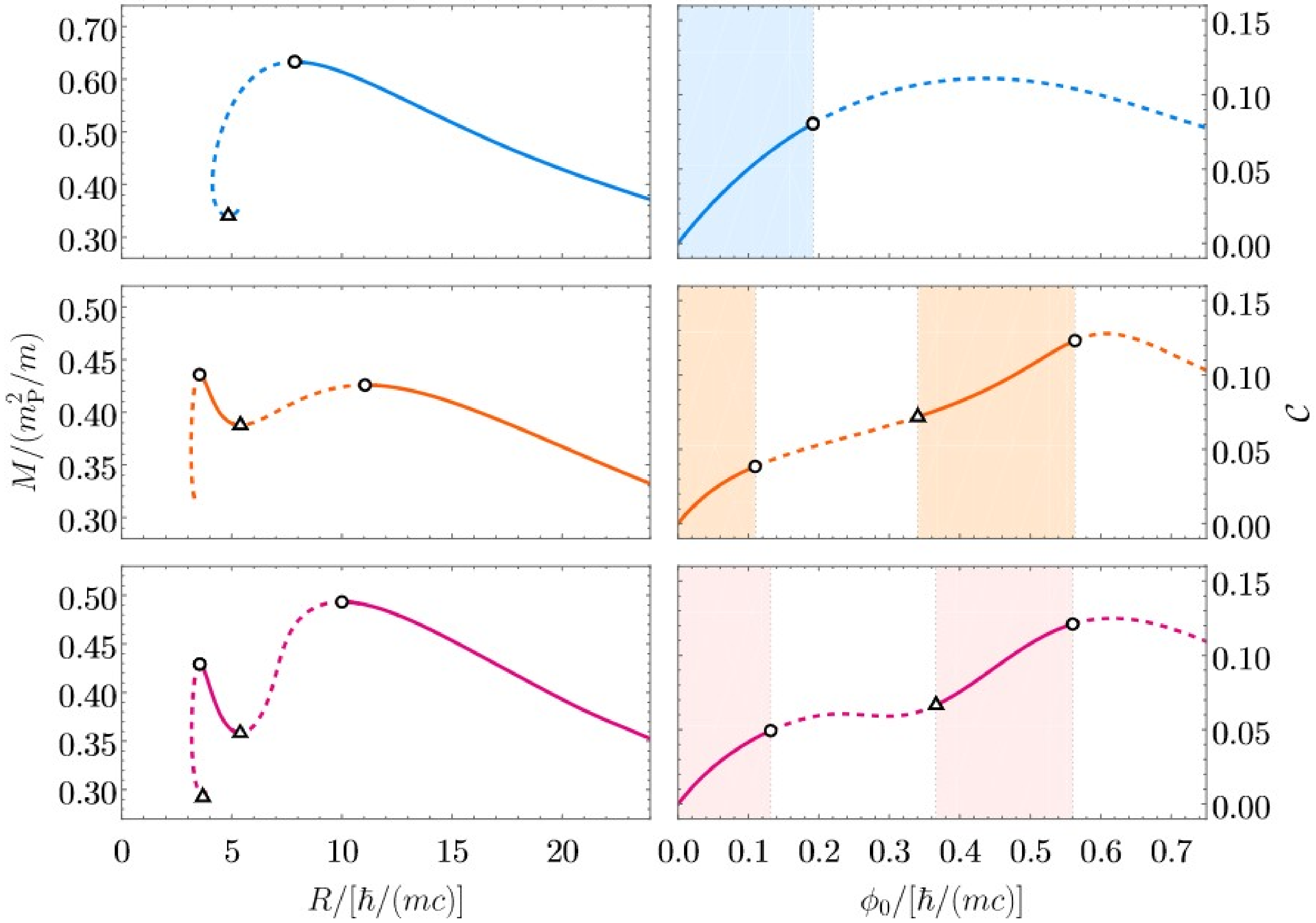
Proca stars





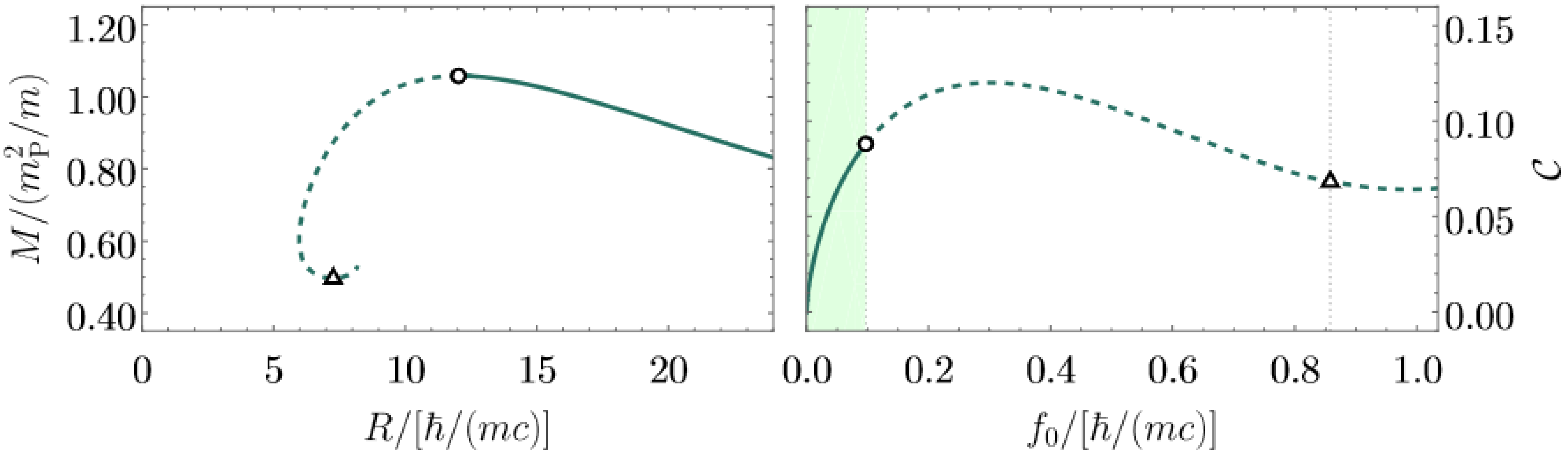
Results

Boson stars



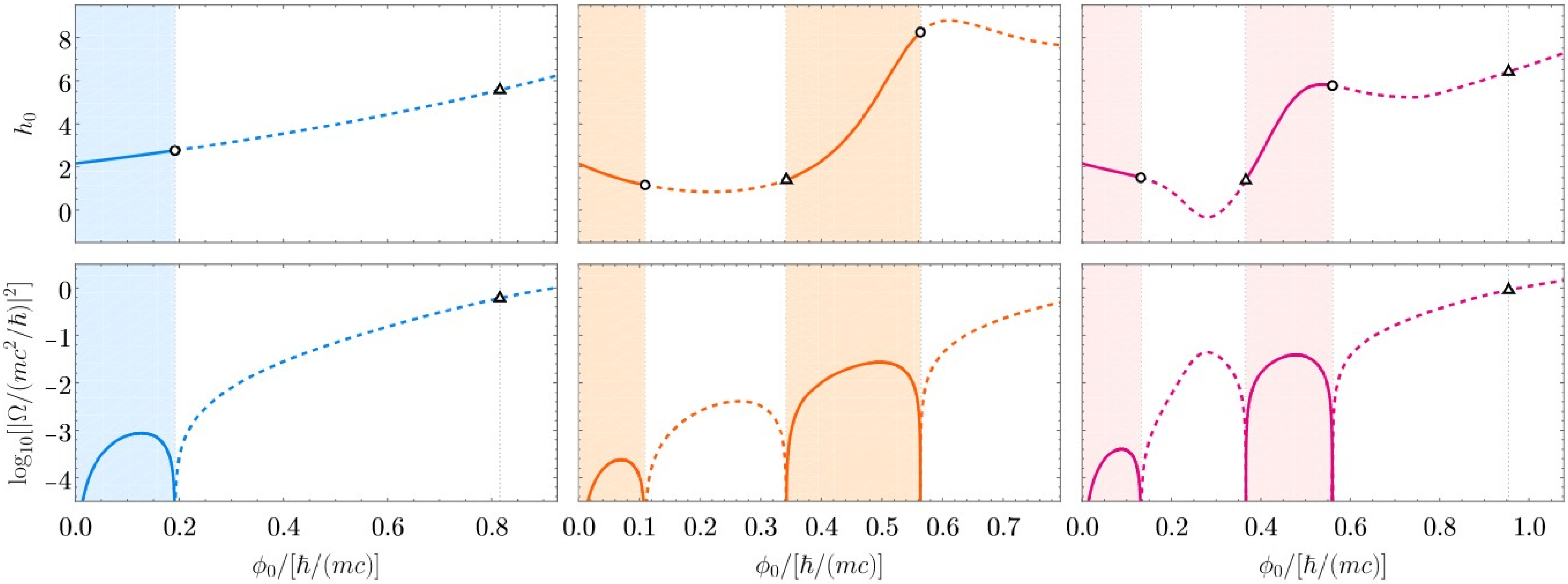
Results

Proca stars



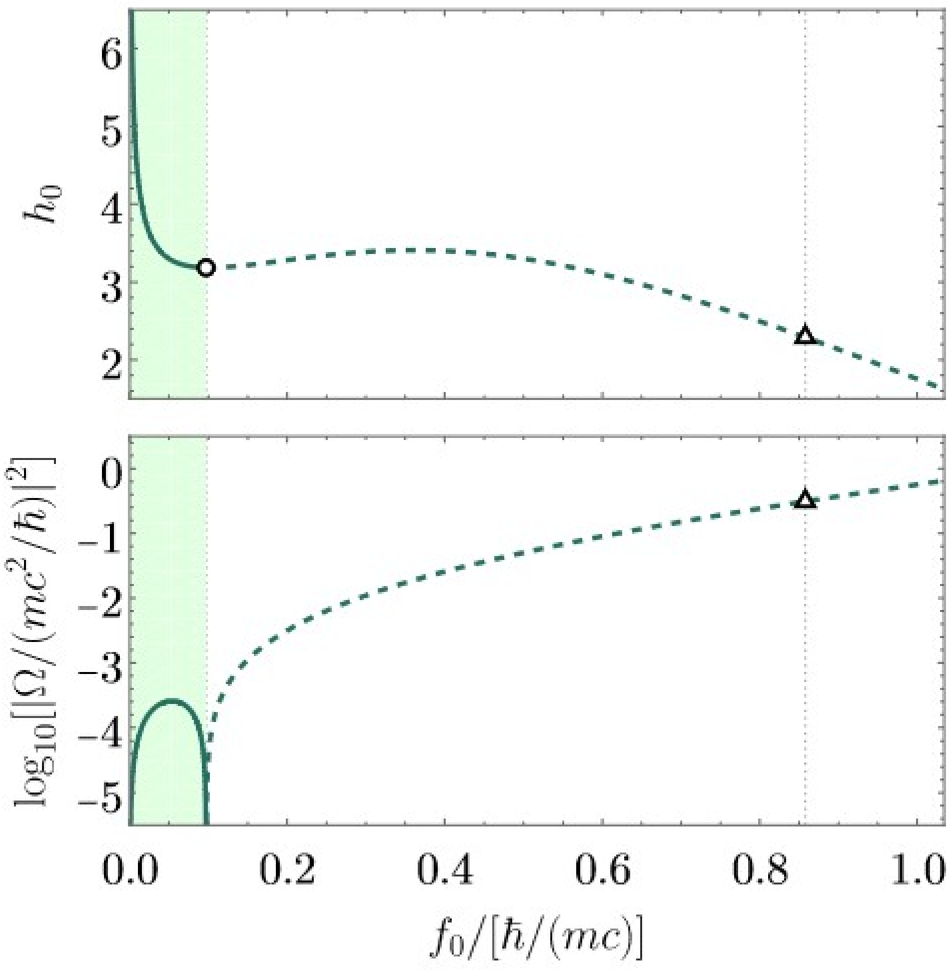
Results

Boson stars



Results

Proca stars



Results

Critical point	$\frac{\psi_0}{\hbar/(mc)}$	$\frac{\omega}{mc^2/\hbar}$	$\frac{M}{m_P^2/m}$	$\frac{Q}{m_P^2/m^2}$	$\frac{R}{\hbar/(mc)}$	$\frac{dR}{d\psi_0}$	$\left(\frac{\Omega_0}{mc^2/\hbar}\right)^2$
1	0.192	0.853	0.633	0.653	7.86	< 0	0
2	0.816	0.845	0.341	0.281	4.84	> 0	−0.606
3	0.109	0.924	0.426	0.432	11.1	< 0	0
4	0.342	0.802	0.388	0.388	5.37	< 0	0
5	0.563	0.629	0.436	0.457	3.54	< 0	0
6	0.132	0.905	0.493	0.503	10.0	< 0	0
7	0.366	0.820	0.358	0.347	5.39	< 0	0
8	0.560	0.637	0.429	0.445	3.55	< 0	0
9	0.955	0.770	0.294	0.241	3.69	> 0	−0.900
10	0.0971	0.874	1.06	1.09	12.0	< 0	0
11	0.858	0.898	0.496	0.412	7.28	> 0	−0.312

Final remarks

- Self-interactions leads to a more complex parameter space;
- Spherically symmetric Proca stars along the stable branch are not dynamically stable;
- $dM/d\psi_0 = 0$ is not a sufficient condition for the existence of a zero-frequency mode.

Obrigada! Thank you!

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