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Kaluza-Klein monopole with scalar hair

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Gr@v

based on JHEP 01 (2024) 181 (e-Print: 2312.02280)
*(with **Y. Brihaye, C. Herdeiro** and **J. Novo)***

Introduction

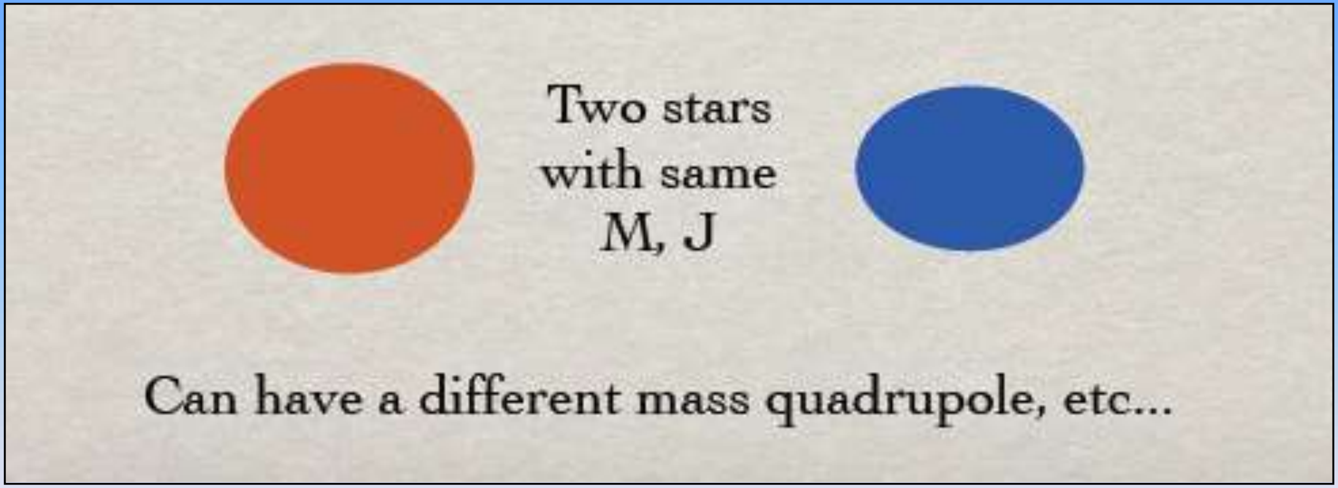
<u>The “no-hair” original idea (1971):</u>

the (equilibrium) Black Holes are uniquely determined by **(M,J,Q)** -

(asymptotically measured quantities subject to a Gauss law)

and no other independent characteristics (**hair)**

The idea is motivated by the *<u>uniqueness theorems</u>*
and indicates that black holes are **very special objects**



Introduction

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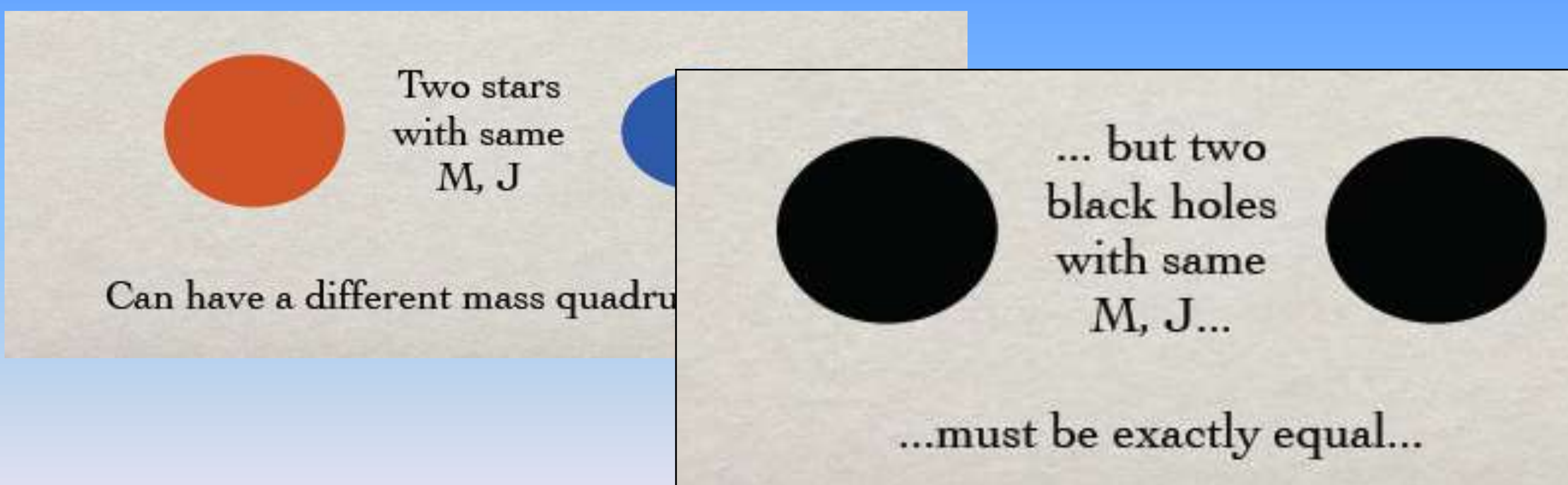


Introduction

mass

The "no-hair" original idea (1971):

the Black Holes are uniquely determined by **(M,J,Q)**

angular momentum
(Kerr black hole)

electric charge
(Reissner-Nordström black hole)

- *various no-hair theorems*
- *loopholes?*
- *hairy Black Holes*
- *active field of research*

the "no-hair" conjecture:

the Black Holes are uniquely determined by **(M,J,Q)**

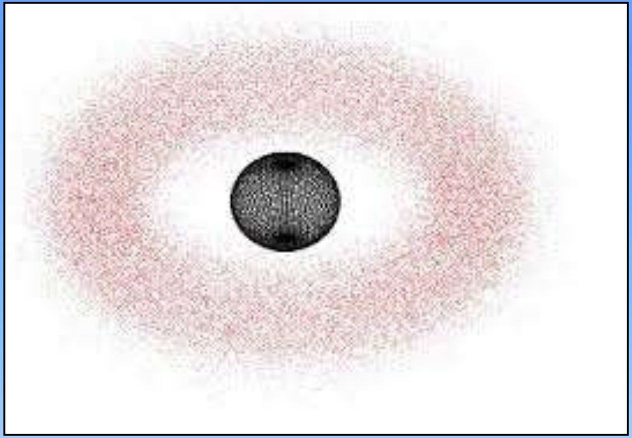
no hair theorem:
Pena-Sudarsky (1997)

(i) rotation: Kerr black hole with scalar hair

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \partial_\alpha\Psi^*\partial^\alpha\Psi - U(|\Psi|)\right]$$

numerics
(Herdeiro and Radu 2014)

existence proof:
Chodosh&Shlapentokh-Rothman



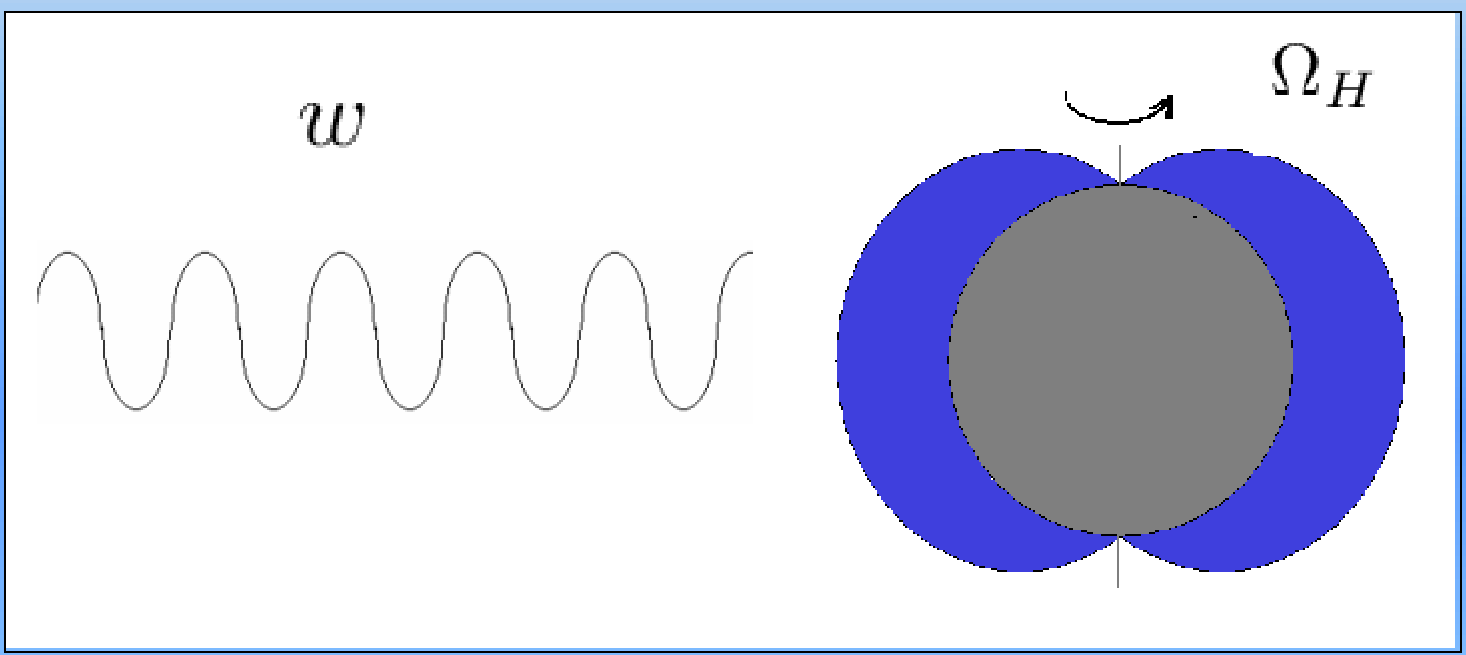
vacuum Kerr is a solution (Ψ=0)

regular, stationary, asymptotically flat black holes
<u>with scalar hair</u>

likely the simplest example of hairy Black Hole

spinning black holes only!

$$\Phi \sim e^{i(m\varphi - wt)}$$



synchronization condition:

$$w = m\Omega_H$$

(zero flux)

event horizon angular velocity

general mechanism!

The "no-hair" conjecture:

the Black Holes are uniquely determined by (**M,J,Q**)

no hair theorem:
Mayo-Bekenstein (1996)

(ii) electric charge: Reissner-Nordström black hole with (gauged) scalar hair

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - D_\alpha\Psi^* D^\alpha\Psi - U(|\Psi|)\right]$$

$$D_\alpha\Psi = (\partial_\alpha - iqA_\alpha)\Psi$$

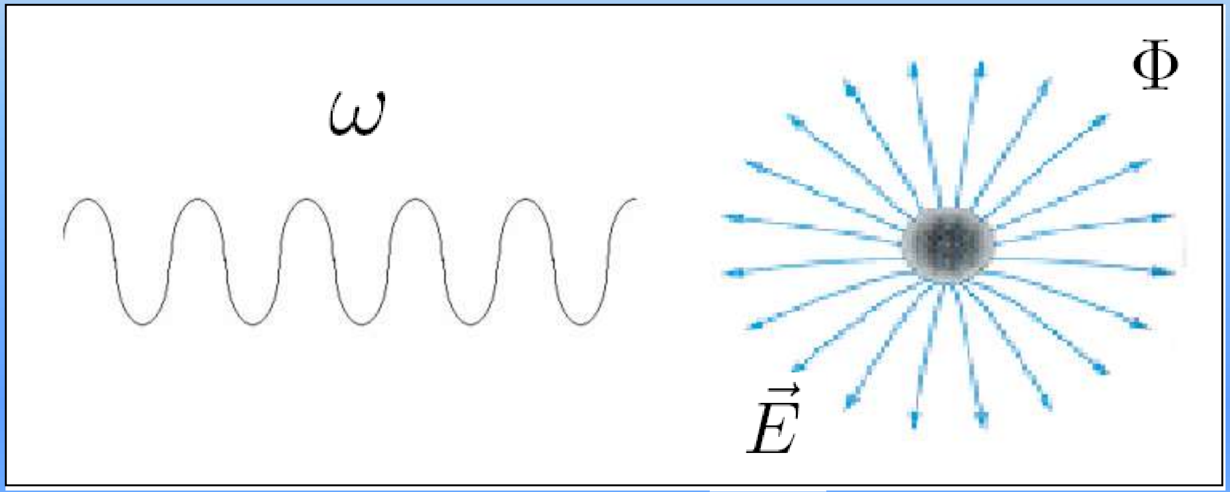
numerics
(Herdeiro and Radu 2020)

Reissner-Nordström black hole is a solution (y=0)

regular, static, asymptotically flat black holes
with gauged scalar hair

$\Psi \sim e^{-i\omega t}$

charged black holes only!



general mechanism!

resonance condition:

$$\omega = q\Phi$$

gauge coupliing constant

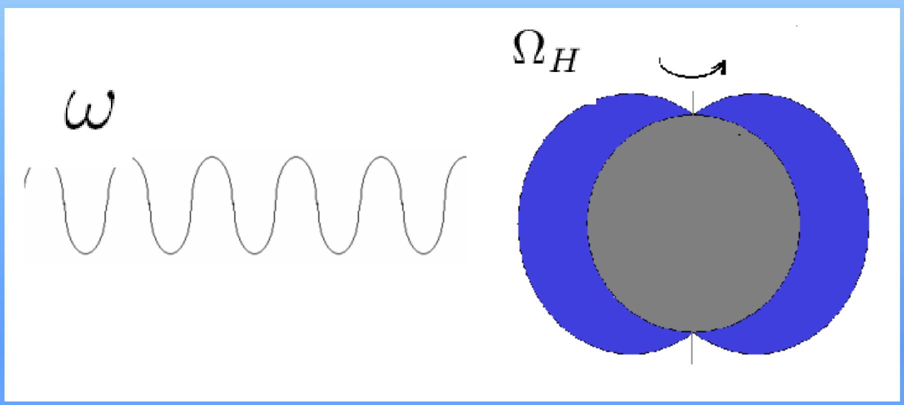
electrostatic (chemical) potential

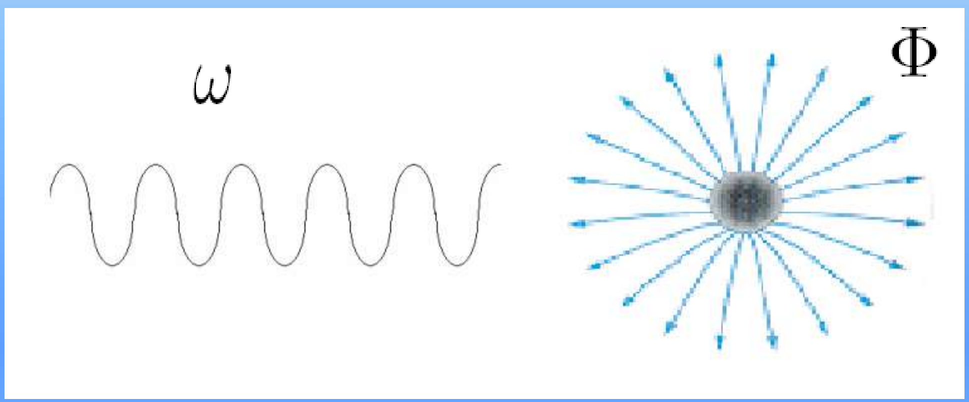
folklore:

Rotation ⟺ *Electric charge*

rotating black hole with scalar hair

charged black hole with scalar hair





synchronization condition:

$$w = m\Omega_H$$

resonance condition:

$$\omega = q\Phi$$

Q: any similarities? common origin?

A: Yes! In a Kaluza-Klein setting

general idea:

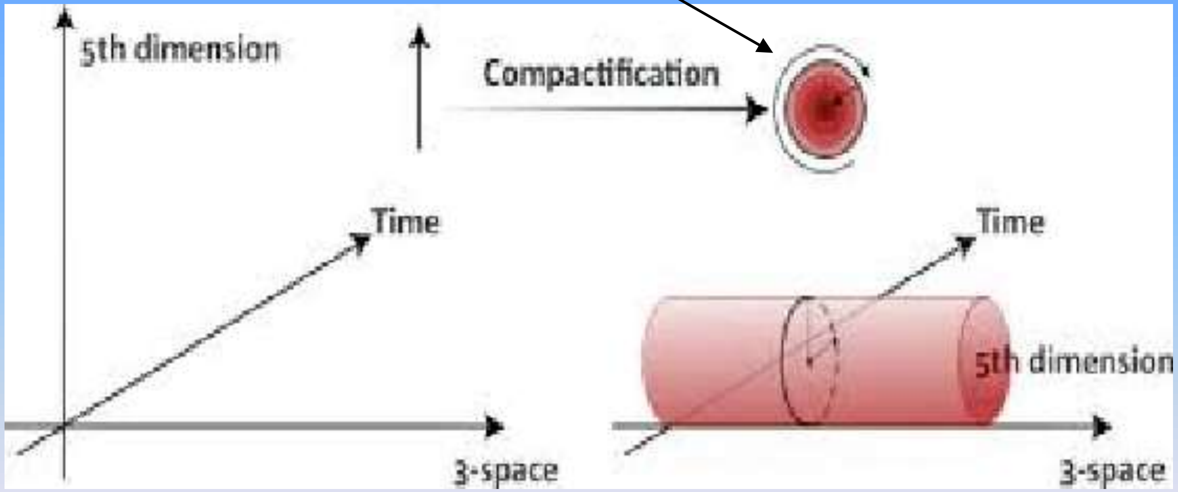
use a Kaluza-Klein framework!

$$ds_5^2 = e^{-a\psi(x)} ds_4^2 + e^{2a\psi(x)} (dz + 2A_i(x) dx^i)^2$$

D=4:*dilaton*

D=4:*U(1) potential*

extra-dimension



Kaluza and Klein (1920s)

general idea:

use a Kaluza-Klein framework!

$$ds_5^2 = e^{-a\psi(x)} ds_4^2 + e^{2a\psi(x)} (dz + 2A_i(x)dx^i)^2$$

dilaton

D=4: *U(1) potential*

$$S_5 = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} R$$

Kaluza and Klein (1920s)

no z-dependence

$$\mathcal{S}_4 = \frac{1}{4\pi G_4} \int d^4x \sqrt{-g^{(4)}} \left[\frac{1}{4} R^{(4)} - \frac{1}{4} e^{3a\psi} F_{ij} F^{ij} - \frac{1}{2} \partial_i \psi \partial^i \psi \right]$$

$G_4 = G_5 / L$

general idea:

use a Kaluza-Klein framework!

D=5: *rotation*

$$ds_5^2 = e^{-a\psi(x)} ds_4^2 + e^{2a\psi(x)}(dz + 2A_i(x)dx^i)^2$$

dilaton

D=4: *U(1) potential*

Rotation ⟺ Electric charge

D=5 black holes with synchronized scalar hair

$$w = m\Omega_H$$

⇒

D=4 black holes with resonant scalar hair

$$\omega = q\Phi$$

extend the KK model for a D=5 complex scalar field:

$$\mathcal{S} = \frac{1}{4\pi G_5}\int_{\mathcal{M}} d^5x\sqrt{-g}\left[\frac{1}{4}R - \frac{1}{2}g^{ab}\left(\Psi^{\dagger}_{,a}\Psi_{,b} + \Psi^{\dagger}_{,b}\Psi_{,a}\right) - \mu^2\Psi^{\dagger}\Psi\right]$$

scalar:

$$\Psi = \Phi(x)e^{ikz}$$

$$k = 2\pi m/L$$

$$m = 0, \pm 1, \pm 2, ..$$

line element:

$$ds_5^2 = e^{-a\psi(x)}ds_4^2 + e^{2a\psi(x)}(dz + 2A_i(x)dx^i)^2\,,$$

$$\text{with}\quad ds_4^2 = g^{(4)}_{ij}(x)dx^i dx^j \quad\text{and}\quad a = \frac{2}{\sqrt{3}}$$

$$w = m\Omega_H$$

$$\mathcal{S}_4 = \frac{1}{4\pi G_4}\int_{\mathcal{M}} d^4x\sqrt{-g^{(4)}}\left[\frac{1}{4}R^{(4)} - \frac{1}{4}e^{3a\psi}F_{ij}F^{ij} - \frac{1}{2}\partial_i\psi\partial^i\psi - \frac{1}{2}g^{ij(4)}\left(D_i\Phi^{\dagger}D_j\Phi + D_j\Phi^{\dagger}D_i\Phi\right) - U(|\Phi|,\psi)\right]$$

$$\omega = q\Phi$$

$$q_s = 2k$$

$$U(|\Phi|,\psi) = \mu^2|\Phi|^2 e^{-a\psi} + k^2 e^{-3a\psi}|\Phi|^2$$

$$\partial_a \to D_j = \partial_a - iq_s A_a$$

gauge derivative

the concrete construction:

use a particular ansatz which factorizes the angular dependence ***(ODEs)***

$$ds^2 = -\mathcal{F}_0(r)dt^2 + \mathcal{F}_1(r)dr^2 + \mathcal{F}_2(r)\left(\sigma_1^2 + \sigma_2^2\right) + \mathcal{F}_3(r)(\sigma_3 - 2W(r)dt)^2$$

rotation

$$\sigma_1 = \cos\psi d\theta + \sin\psi \sin\theta d\varphi$$
$$\sigma_2 = -\sin\psi d\theta + \cos\psi \sin\theta d\varphi$$
$$\sigma_3 = d\psi + \cos\theta d\varphi.$$

$$\Psi = \phi(r)\begin{pmatrix} \sin\frac{\theta}{2} e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2} e^{i\frac{\varphi}{2}} \end{pmatrix} e^{i\left(\frac{\psi}{2} - wt\right)}$$

Klehaus, Kunz, Hartmann (2014)

no scalar field ⇒ *the Kaluza-Klein monopole (vacuum background)*

NUT parameter

$$ds^2 = \left(1 + \frac{2N}{r}\right)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] + \frac{4N^2}{1 + \frac{2N}{r}}(d\psi + \cos\theta d\varphi)^2 - dt^2$$

the Taub-NUT instanton (Hawking 1977) *the (vacuum)Taub-NUT instanton uplifted to D=5* ⇨ ***soliton***

$$ds^2 = \left(1 + \frac{2N}{r}\right)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] + \frac{4N^2}{1+\frac{2N}{r}}(d\psi + \cos\theta d\varphi)^2 - dt^2$$

dilaton

D=4: *U(1) potential*

$$S_5 = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}R$$

no ψ-dependence

$$\mathcal{S}_4 = \frac{1}{4\pi G_4}\int d^4x\sqrt{-g^{(4)}}\left[\frac{1}{4}R^{(4)} - \frac{1}{4}e^{3a\psi}F_{ij}F^{ij} - \frac{1}{2}\partial_i\psi\partial^i\psi\right]$$

- ***the Gross-Perry--Sorkin magnetic monopole (1983)***

$$A = N\cos\theta d\varphi$$

• ***the (vacuum)Taub-NUT instanton uplifted to D=5***
solitonic solution (no horizon, no singularities)

$$ds^2 = \left(1+\frac{2N}{r}\right)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] + \frac{4N^2}{1+\frac{2N}{r}}(d\psi + \cos\theta d\varphi)^2 - dt^2$$

• ***adding an horizon: a static Black Hole in Gross-Perry-Sorkin background***

$$ds = -\left(1-\frac{r_h}{r}\right)dt^2 + \left(1+\frac{2\bar{N}}{r}\right)\left[\frac{dr^2}{1-\frac{r_h}{r}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] + \frac{4N^2}{1+\frac{2\bar{N}}{r}}(d\psi + \cos\theta d\varphi)^2$$

• ***a rotating Black Hole in Gross-Perry-Sorkin background (simplest case)***

$$ds^2 = -\mathcal{F}_0(r)dt^2 + \mathcal{F}_1(r)dr^2 + \mathcal{F}_2(r)\left(\sigma_1^2 + \sigma_2^2\right) + \mathcal{F}_3(r)(\sigma_3 - 2W(r)dt)^2$$

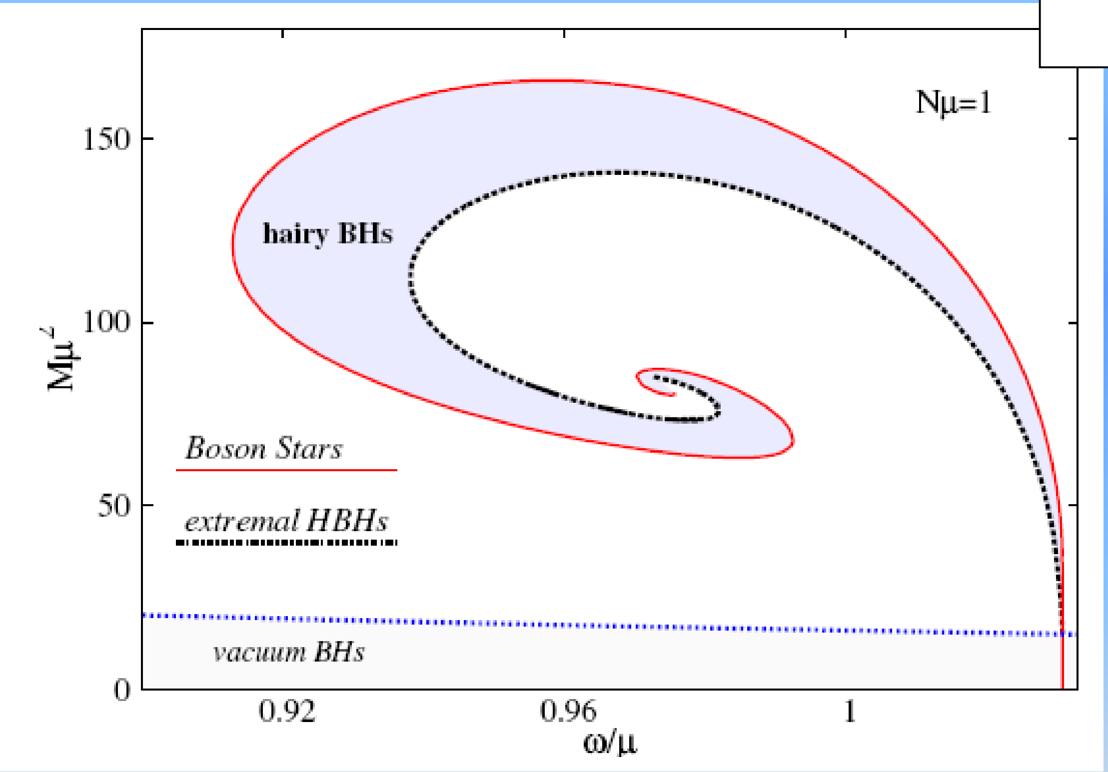
• *our work:*
add ***scalar hair***

$$\Psi = \phi(r)\begin{pmatrix} \sin\frac{\theta}{2}e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2}e^{i\frac{\varphi}{2}} \end{pmatrix} e^{i\left(\frac{\psi}{2}-wt\right)}.$$

the ansatz used in numerics ⟹ ***a set of ODEs***

$$ds^2 = -e^{2F_0(r)}\frac{\left(1-\frac{r_H}{r}\right)^4}{\left(1+\frac{r_H}{r}\right)^2}dt^2 + e^{2F_1(r)}H(r)\left(1+\frac{r_H}{r}\right)^4\left[dr^2 + r^2\left(\sigma_1^2+\sigma_2^2\right)\right] + e^{2F_2(r)}\frac{4N^2}{H(r)}[\sigma_3 - 2W(r)dt]^2$$

$$\Psi = \phi(r)\begin{pmatrix} \sin\frac{\theta}{2}e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2}e^{i\frac{\varphi}{2}} \end{pmatrix} e^{i\left(\frac{\psi}{2}-wt\right)}$$



$$w = m\Omega_H$$

synchronization condition

the domain of existence of solutions
(some universal features)

D=5

rotation

$$ds^2 = -\mathcal{F}_0(r)dt^2 + \mathcal{F}_1(r)dr^2 + \mathcal{F}_2(r)\left(\sigma_1^2 + \sigma_2^2\right) + \mathcal{F}_3(r)(\sigma_3 - 2W(r)dt)^2$$

rotating EKG

$$\Psi = \phi(r)\begin{pmatrix} \sin\frac{\theta}{2}e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2}e^{i\frac{\varphi}{2}} \end{pmatrix} e^{i\left(\frac{\psi}{2} - wt\right)}$$

$$ds_5^2 = e^{-a\psi(x)}ds_4^2 + e^{2a\psi(x)}(dz + 2A_i(x)dx^i)^2$$

D=4

$$ds_4^2 = -S_0(r)dt^2 + S_1(r)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]$$

$$V(r) = -2NW(r)$$

$$A = V(r)dt + N\cos\theta d\varphi$$

spherically symmetric EMKGd

one can map the properties

$$\partial_a \to D_j = \partial_a - iq_sA_a$$

$$\Phi = \phi(r)\begin{pmatrix} \sin\frac{\theta}{2}\, e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2}\, e^{i\frac{\varphi}{2}} \end{pmatrix} e^{-i\omega t}$$

explicit construction

$$\omega = m\Omega_H = W\big|_{r_H} = q_s\Phi$$

playing with EKG vortices

(i) D=4 gauged Boson Stars from D=5 (ungauged) solutions

D=5

$$ds^2 = e^{-a\psi(r)}\left(-e^{-2\delta(r)}N(r)dt^2 + \frac{dr^2}{N(r)} + r^2 d\Omega_2^2\right) + e^{2a\psi(r)}dz^2$$

$$\Psi = \phi(r)e^{-i\omega t}$$

$$\begin{cases} t = \cosh\alpha\ T - \sinh\alpha\ Z \\ z = \cosh\alpha\ Z - \sinh\alpha\ T \end{cases}, \quad \text{where} \quad \alpha \in \mathbb{R}$$

boost

D=4

$$ds_4^2 = -\frac{e^{-2\delta(r)}N(r)}{\sqrt{S(r)}}dT^2 + \sqrt{S(r)}\left(\frac{dr^2}{N(r)} + r^2 d\Omega_2^2\right), \quad A = V(r)dt$$

$$S(r) = \cosh^2\alpha - e^{-\delta(r)-3a\psi(r)}N(r)\sinh^2\alpha$$

$$V(r) = \left(e^{-2\delta(r)-3a\psi(r)}N(r) - 1\right)\frac{\sinh\alpha\cosh\alpha}{2S(r)}$$

$$g_s = 2\omega\sinh\alpha$$

$$\Psi = \phi(r)e^{-i\tilde{\omega}T}$$

$$\tilde{\omega} = \omega\cosh\alpha$$

$$\partial_a \to D_j = \partial_a - iq_s A_a$$

one can map the properties

gauged Boson Stars: EMKGd theory

playing with EKG vortices

(ii) D=4 (ungauged) Boson Stars in a Melvin magnetic universe

D=5

$$ds^2 = e^{-a\psi(r)}\left(-e^{-2\delta(r)}N(r)dt^2 + \frac{dr^2}{N(r)} + r^2 d\Omega_2^2\right) + e^{2a\psi(r)}dz^2$$

$$\Psi = \phi(r)e^{-i\omega t}$$

twist

$$\varphi \to \varphi + B_0 z$$

Boson Stars: EMKGd theory

D=4

$$ds_4^2 = \sqrt{\Lambda(r,\theta)}\left(\frac{dr^2}{N(r)} + r^2 d\theta^2 - N(r)e^{-2\delta(r)}dt^2\right) + \frac{r^2\sin^2\theta d\varphi^2}{\sqrt{\Lambda(r,\theta)}}$$

one can map the properties

$$\Lambda(r,\theta) = 1 + e^{-3a\psi(r)}B_0^2 r^2 \sin^2\theta.$$

$$\Psi = \phi(r)e^{-i\omega t}$$

$$A = \frac{e^{-3a\psi(r)}B_0 r^2\sin^2\theta}{2\Lambda(r,\theta)}d\varphi, \quad \psi(r,\theta) = \psi_i(r) + \frac{1}{2a}\log\Lambda(r,\theta)$$

to summarize:

main message:

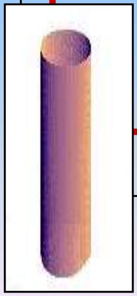
- ***seemingly different classes of solutions can be related***
- ***an example: synchronized vs. resonant hairy Black Hole***

- rich landscape of Kaluza-Klein solutions

an example:

- **D=5 Einstein-scalar field**

boson vortices

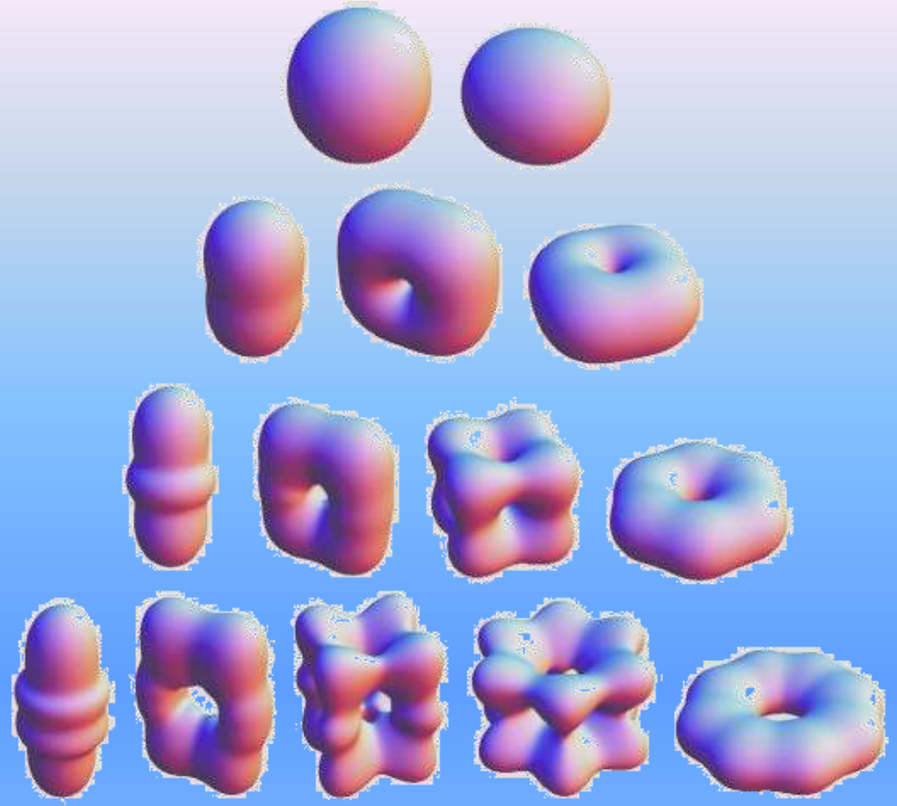


new solutions
(generation technique)

D=4 gauged Boson Stars
(Minkowski asymptotics)

D=4 (ungauged) Boson Stars in Melvin Universe

many open questions…



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